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The Evolving Bipartite Network and Semi-Bipartite Network Models with Adjustable Scale and Hybrid Attachment Mechanisms

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Abstract

The bipartite graph structure exists in the connections of many objects in the real world, and the evolving modeling is a good method to describe and understand the generation and evolution within various real complex networks. Previous bipartite models were proposed to mostly explain the principle of attachments, and ignored the diverse growth speed of nodes of sets in different bipartite networks. In this paper, we propose an evolving bipartite network model with adjustable node scale and hybrid attachment mechanisms, which uses different probability parameters to control the scale of two disjoint sets of nodes and the preference strength of hybrid attachment respectively. The results show that the degree distribution of single set in the proposed model follows a shifted power-law distribution when parameter r and s are not equal to 0, or exponential distribution when r or s is equal to 0. Furthermore, we extend the previous model to a semi-bipartite network model, which embeds more user association information into the internal network, so that the model is capable of carrying and revealing more deep information of each user in the network. The simulation results of two models are in good agreement with the empirical data, which verifies that the models have a good performance on real networks from the perspective of degree distribution. We believe these two models are valuable for an explanation of the origin and growth of bipartite systems that truly exist.

Keywords

Bipartite Networks, Evolving Model, Semi-Bipartite Networks, Hybrid Attachment, Degree Distribution

1. Introduction

With the development of complexity theory, complex networks, as a powerful research tool, provide a new approach for the study of complexity science [1]-[8]. The function of complex network is to abstract objects or individuals into nodes, and the connection between objects is expressed as edges between nodes, so as to build a complex network model. With network models, we can explain the complexity phenomenon in reality, and help us better understand the topological structure, evolution mechanism and dynamics behavior of real system. It's of great significance for the design, control, optimization and management in the real networks.

Small-world [9] and scale-free [10] properties of real network was found in the end of last century, which attracted wide attention of scholars in complex networks and related scientific fields around the world. In 1999, Barabási and Albert found that many real networks have self-organizing property, and the degree of nodes follows the power-law, thus called BA scale-free network model [11]. The research [12] shows that the growth of nodes and preferential attachment are key factors power-law appears. The so-called preferential attachment means that when a node links to the other node, the probability of the other node being selected is proportional to its own degree. In fact, this mechanism exists in many real-world networks. Subsequently, some other network models were established based on real systems, such as EBA model [13], fitness model [14], etc., all of which have scale-free characteristics, which can be regarded as the extension model of BA scale-free network.

Bipartite network is a kind of network with special structure, which is composed of two different types of nodes sets and the edges between the two distinct types of sets (No edges are allowed inside the same set). A large number of complex systems in reality can be described by bipartite network models. For instance, in the user-movie scoring system [15], users and movies constitute different types of sets respectively, and the edge between the two types of nodes represents users' scoring behaviors for movies, thus forming the user-movie binary network. Also like the producer-supplier network [16], the scientific collaboration network [17] and so on, these models are widely applied to the recommender system, has high commercial value. The early research on bipartite networks mainly focused on the construction and property of theoretical models. A survey is reported in [18] briefly summarized the research status of bipartite networks, introduced several kinds of common real networks, and gave the definitions of some basic topological properties. Guillaume *et al.* [19] argued that all complex networks contain basic bipartite structures. In [20], Tian *et al.* set up a local-world-like evolving bipartite network model, and used node saturation restrictions and preferential attachment to generate a model with small degree value interval in the local-world. In addition, they also proposed a general evolving model [21], and rigorously derived the distribution and correlation judgments of degree. As research continues, attention has been paid to the simulation and characterization of bipartite network models for real networks, and the various properties they exhibit. Zhang *et al.* [22] proposed an online network model, which built different evolution situation of objects and users sets based on the proactive selection activity of users, and verified that the degree distribution of objects accords with power-law distribution, while the degree distribution of users follows shifted power-law distribution [23]. Anita Chandra *et al.* [24] proposed a user-object bipartite network growth model by analyzing the selection behavior of online users, which considered different attachment procedures for external edges of new users and internal edges of old users, and verified the performance with nine real data sets. Compared with the earlier models, the above models are closer to reality, and have achieved nice performance in daily life, especially in the field of recommender algorithms. The existing research has made great progress in the way of using the bipartite network model to analyze and apply to the actual object, however, the research on the evolution nature and the fit relations for the models and actual networks is still lacking. Therefore, we shall pertinently solve related challenges in the paper.

In this paper, we propose the evolving bipartite network model with Adjustable Scale and Hybrid Attachment Mechanisms (ASHAM bipartite network model) and the extended semi-bipartite network model with Hybrid Attachment Mechanisms (EHAM semi-bipartite network model). In the early studies on bipartite network models [19], we find that most of the models only limited to the adjustment and innovation of the link mechanism, and rarely paid attention to the difference in the growth of two distinct types of sets in the real network. The majority of models defaulted to the balanced growth of the size of the two sets. In this way, the scale of the node set grows up in the bipartite network presents a fixed 1:1 ratio. However, in fact, the scale of the bipartite networks is usually unbalanced, such as the scientific collaboration network [17], in which the number of articles is much larger than that of researchers. In addition, link mechanisms generally consist of preferential attachment and randomness. Hence, we consider using multiple probability parameters to severally adjust the scale difference of two sets and the strength of hybrid links attachment. The results of numerical simulation and empirical analysis prove that the two proposed models have good representation for real network.

The rest of the paper is organized as follows: In Section 2, we propose the ASHAM model for capturing the nature of actual bipartite systems, and analyze the degree distribution functions of two distinct sets in ASHAM model. In Section 3, we establish the EHAM semi-bipartite network model on the basis of the present work. We compare two models with empirical data through numerical simulation in Section 4. At last, we give some concluding remarks in Section 5.

2. The ASHAM (Adjustable Scale and Hybrid Attachment Mechanisms) Bipartite Network Model and Its Topological Property

In this section, we shall propose an evolving bipartite network model to have flexible performance. As we all know, the actual bipartite systems generally have the following features:

- The scale of two distinct sets is as often as not equivalent. For example, in online user-goods networks, the growing number of users and goods is determined by many different factors, thus the scale of sets is different.
- In terms of selection behaviors, one user may select objects based on their popularity, or pick objects randomly. This selection tendency is related to the user's attitude towards objects. For instance, one user may choose hot songs, or randomly play an unknown one in the audio player. Moreover, an item can be recommended by the seller to active customers or arbitrarily recommended to a common one. The final recommended target users of different items are generally determined by a mixture of these two ways.

According to characteristics above, we propose the ASHAM model. The evolving model is mainly based on two rules: the unbalanced growth mechanism of nodes and the hybrid link mechanism with preferential attachment and randomness link. Consequently, the detailed modelling method is described below.

2.1. The Method of ASHAM Modelling

ASHAM bipartite graph is represented as triplet $G=\langle A,B,E\rangle$, where A and B are two disjoint sets of nodes, and $E\subseteq A\times B$ is the set of edges. Before the network evolves, we assume a_0 nodes in A, b_0 nodes in B and e_0 edges in E. Given a node i in A and a node j in B, we denote k_i as the degree of i and k_j as the degree of j, thus $e_0=\sum k_i=\sum k_j$ ($k_i,k_j\geq 1$, $i=1,2,\cdots,a_0$, $j=1,2,\cdots,b_0$). Thereafter, the model evolves at each time step according to the following rules:

1) Add a new node to the network, through the adjustable probability parameters p and q determine the set to which the new node belongs: add to A with probability p, or add to B with probability q. the probabilities p and q can be varied in the interval $0 < p, q < 1$ and $p + q = 1$.

2) If the new node belongs to A, this new node will be linked to m old nodes in B. The attachment mechanism is determined by the probability r: with probability r, by preferential probability $k_j/\sum_{N_B}k_j$ select m old nodes in B; or with probability $1-r$, by probability $1/N_B$ randomly choose m old nodes in B. Where $0\leq r\leq 1$, and we denote N_B as the number of existing nodes in B.

3) If the new node belongs to B, the new node will be linked to n old nodes in A. The attachment mechanism is determined by the probability s: with probability s, by preferential probability $k_i/\sum_{N_A}k_i$ select n old nodes in A; or with probability $1-s$, by probability $1/N_A$ randomly choose n old nodes in A. Where $0\leq s\leq 1$, and we denote N_A as the number of existing nodes in A.

Table 1 shows some basic properties of ASHAM model, where N_A, N_B and E

Table 1. Some basic properties of ASHAM bipartite network model.

Model	N_A	N_B	E	ρ
ASHAM	a_0+pt	b_0+pt	$(mp+nq)t+e_0$	$\frac{(mp+nq)t+e_0}{(a_0+pt)(b_0+pt)}$

respectively represent the number of nodes in A, B and the total number of edges in the network. $\rho = E/(N_A \times N_B)$ denotes the sparsity of the edges in the network. And the process of ASHAM modelling is shown in Figure 1.

2.2. The Degree Distributions of ASHAM Networks

In the ASHAM bipartite network model, the construction in two sets A and B is peer-to-peer and only differ in the given parameters. Therefore, the degree distributions of nodes of set A or B shall be equivalent under the same values of corresponding parameters, and the degree distribution expressions of two sets shall show strong correlation. Hence, we only derive the degree distribution of one single set.

Firstly, let's talk about the degree distribution $P(k)$ of A in the case of $s \neq 0$. In time step t, the degree of i is $k_i(t)$. According to the continuum theory [13] and mean field theory [25], discrete time steps and node degrees can be regarded as continuous values. When a new node is added to A, the degree of old nodes in A won't change. Thus, when the new node is added to B, we have change in degree of i as

$$\frac{\partial k_i}{\partial t} = nqs\frac{k_i}{\sum_{N_A} k_i} + nq(1-s)\frac{1}{N_A}, \tag{1}$$

where $N_A = a_0 + p(t-1)$, $\sum_{N_A} k_i = e_0 + (mp+nq)(t-1)$. We assume that $t \gg a_0, m, n$. Thus, Equation (1) can be rewritten as

$$\frac{\partial k_i}{\partial t} = \frac{nqsk_i}{(mp+nq)t} + \frac{nq(1-s)}{pt}. \tag{2}$$

As we know, the initial degree of i is $k_i(t_i) = m$, where t_i is the time node i is added into the set A. Then we obtain the degree of i at step t as

$$k_i(t) = \frac{mx+y}{Xx}\left(\frac{t}{t_i}\right)^{\frac{x}{z}} - \frac{y}{x}, \tag{3}$$

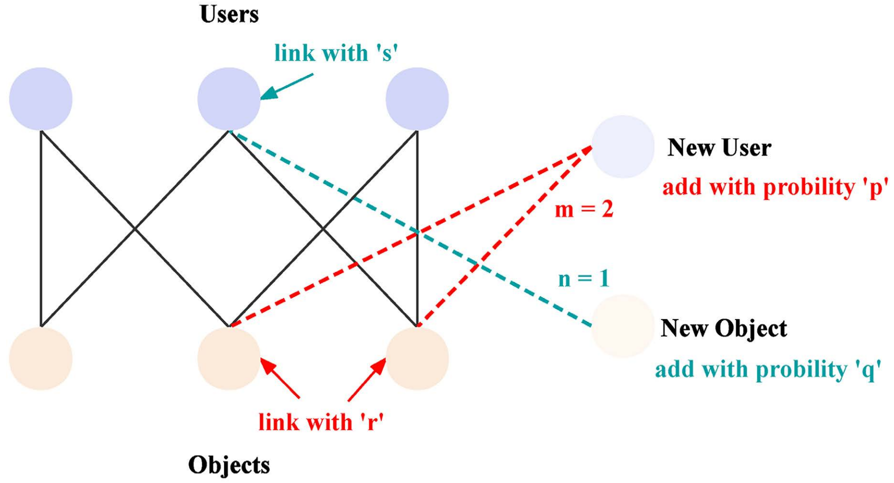

Figure 1. The schematic diagram of ASHAM modelling.

where $x = npqs$, $y = (1-s)(mp+nq)nq$, $z = p(mp+nq)$.

Let $k_i(t) < k$, then $t_i > t\left[(xk+y)/(xm+y)\right]^{-z/x}$. Thus, the cumulative probability $P(k_i(t) < k)$ can be expressed as

$$P(k_i(t) < k) = P\left(t_i > t\left(\frac{xk+y}{xm+y}\right)^{-\frac{z}{x}}\right). \tag{4}$$

In this model, we assume that nodes are added with equal time interval, then the probability that node arrives at time t_i is

$$P(t_i) = \frac{1}{a_0 + pt}. \tag{5}$$

Combining Equation (4) and Equation (5), we get

$$P(k_i(t) < k) = 1 - \frac{t}{a_0 + pt}\left(\frac{xk+y}{xm+y}\right)^{-\frac{z}{x}}. \tag{6}$$

Since t is very large, we assume that $a_0 + pt \approx pt$. We can achieve the degree distribution of nodes in set A as

$$P(k) = \frac{\partial P(k_i(t) < k)}{\partial k} = \frac{z}{px}\left(m + \frac{y}{x}\right)^{\frac{z}{x}}\left(k + \frac{y}{x}\right)^{-\left(1+\frac{z}{x}\right)}. \tag{7}$$

In Equation (7), we notice that distribution follows shifted power law distribution, which is known as Mandelbrot Law [26], where shifting coefficient $c_A = y/x$ and scale exponent $\gamma_A = -(1 + z/x)$.

Similarly, the degree distribution function of node in B can also be derived as

$$Q(k) = \frac{w}{qu}\left(n + \frac{v}{u}\right)^{\frac{w}{u}}\left(k + \frac{v}{u}\right)^{-\left(1+\frac{w}{u}\right)}, \; r \neq 0, \tag{8}$$

where $u = mpqr$, $v = mp(mp+nq)(1-r)$, $w = q(mp+nq)$, with shifting coefficient $c_B = v/u$ and scale exponent $\gamma_B = -(1 + w/u)$. The combination of parameters in Equation (7) and Equation (8) reflects a mirror corresponding relationship, *i.e.*, $p \leftrightarrow q$, $m \leftrightarrow n$, $x \leftrightarrow u$, $y \leftrightarrow v$, $z \leftrightarrow w$.

In particular, when $s = 0$, the node newly added to B is randomly linked to other nodes, thus Equation (1) can be rewritten as

$$\frac{\partial k_i}{\partial t} = nq\frac{1}{N_A}, \tag{9}$$

When $t \gg a_0$, and with $N_A = a_0 + p(t-1)$ we get

$$\frac{\partial k_i}{\partial t} = \frac{nq}{pt}. \tag{10}$$

At t_i, the node i is added in set A with the degree $k_i(t_i) = m$, then we have

$$k_i(t) = \frac{nq}{p}\ln\frac{t}{t_i} + m. \tag{11}$$

Let $k_i(t) < k$, the cumulative probability is given as

$$P\left(k_i\left(t\right)<k\right)=P\left(t_i>te^{-\frac{p(k-m)}{nq}}\right). \tag{12}$$

Assuming that nodes are arrived with the same time interval, so t_i have a constant probability

$$P\left(t_i\right)=\frac{1}{a_0+pt}. \tag{13}$$

Inserting Equation (13) into Equation (12), we get

$$P\left(k_i\left(t\right)<k\right)=1-\frac{t}{a_0+pt}e^{-\frac{p(k-m)}{nq}}. \tag{14}$$

Finally, with assuming at a large *t*, the degree distribution can be written as

$$P\left(k\right)=\frac{\partial P\left(k_i\left(t\right)<k\right)}{\partial k}=\frac{1}{nq}e^{-\frac{p(k-m)}{nq}},\ k\geq m. \tag{15}$$

Equation (15) is the degree distribution function in *A* set in the case of $s=0$, and it clearly shows the form of an exponential distribution. In the same way, the degree distribution of set *B* follows an exponential form in the extreme case $r=0$.

3. The EHAM (Extended Hybrid Attachment Mechanisms) Semi-Bipartite Network Model

In the process of generating ASHAM model, we find that the bipartite network model can indeed simulate the actual network on the structural perspective. However, same as other models, bipartite models can only represent the network structure and weight information, while the abundant information hidden in the actual network, especially the association information within the node set, is not expressed. In fact, in many true bipartite relations, there is also a case of internal connections in one single set. Such as, in the online user-object network, in addition to the purchase or rating between the user and the object, there is also an internal connection between the user and the user (like friendship or kinship). These connections indirectly affect the association between users and objects. What's more, we can also dig out the interest similarity association among users from the purchase or rating records. If the information of these deep associations is applied in the recommender system, the accuracy of the recommendation algorithm may be greatly improved. China's Pinduoduo shopping platform is a good example, which not only takes advantage of users' purchase or rating information for product recommendation, but also makes use of social platform information to achieve more accurate recommendation effect. In order to enable the model to carry more abundant information, we extend the ASHAM model and propose the EHAM semi-bipartite network model: Based on the evolution of ASHAM network, add links between the internal nodes of *A* to form an internal network (assuming in set *A*), and ensure that the original bipartite network of the existing links have not change. For the convenience of statement, in the semi-bipartite network, the links between nodes in set A are referred to as internal edges, and the links between A and B are referred as external edges.

We denote $H = \langle G, E_{in} \rangle$ as EHAM semi-bipartite network, where $G = \langle A, B, E \rangle$ is the proposed ASHAM bipartite network, and E_{in} is the internal edges in A. With the evolving of G, when the number of nodes in A grows to $l \ge a_0$, the l nodes form a globally coupled initial internal network with e_0^{in} edges. Denoting k_i^{in} as the degree of i in the internal network of A, then we have $e_0^{in} = \sum k_i^{in}$ ($k_i^{in} \ge 1$, $i = 1, 2, \cdots, l$). Since then, whenever a new node is added to A, the connection rules for this node are as follows (also see in Figure 2).

The newly added node i in A is not only externally linked to nodes in B according to the way of ASHAM bipartite model, but also internally linked to the existing nodes in A. The internal attachment is determined by the probability parameter p_{in}: with probability p_{in}, by preferential probability $k_i^{in} \Big/ \sum_{N_A} k_i^{in}$ internally linked to most l old nodes in A (the number of links follows uniform distribution $U(1, l)$); with probability $1 - p_{in}$, by random probability $1/N_A$ linked to most l old nodes in A. Where $0 \le p_{in} \le 1$, and N_A represents the number of nodes that already exist in the set A.

4. Simulation and Empirical Analysis

In this section, in order to verify the characterization effect of the proposed two models, we perform numerical simulation of the two models, and establish bipartite network and semi-bipartite network by real data respectively, and then compare the degree distribution of the theoretical model with the empirical networks.

4.1. Empirical Data

We use two data sets, *MovieLens* [15] and *Netflix* (https://konect.cc/), for

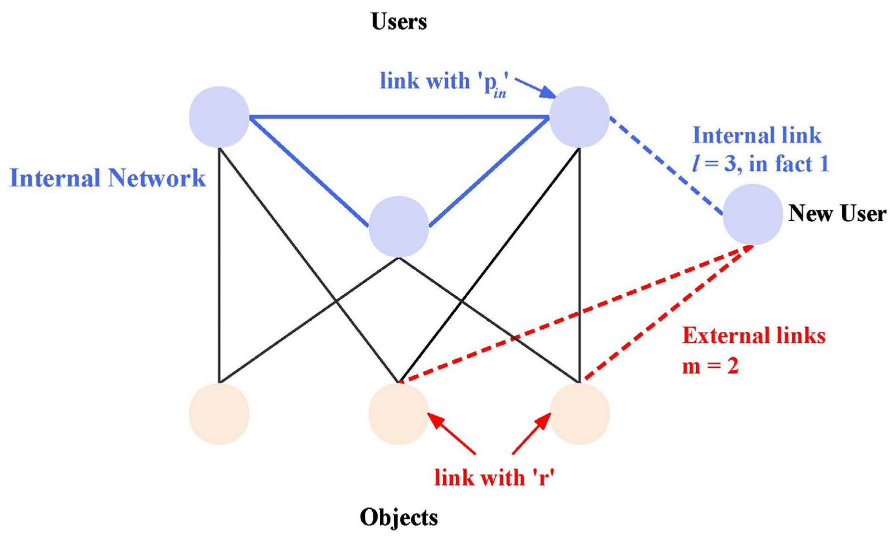

Figure 2. The schematic diagram of EHAM modelling.

evaluating the performance of two models. The above data sets are all user-movie rating records. Here, users and movies are two disjoint sets and if one user rates movies then it will form connections between them. Like many rating systems, both data sets are rated on discrete values of 1 to 5.

In fact, in the scoring system, the rating information from 2 to 4 contains the swing or random consciousness of the user's preference attitude towards the object to a certain extent, and it is difficult to reflect the user's exact selection intention. Most of these rating choices represent a fuzzy and uncertain choice tendency; On the other hand, the rating of 1 or 5 can clearly express the user's firm attitude towards the object, and clearly reflect the user's likes and dislikes. Therefore, in the numerical simulation, taking score values 1 and 5 as conditions, we filter and form four small data sets in two data sets, then analyze the bipartite network and semi-bipartite network formed by these four cube data sets with ASHAM and EHAM model. Table 2 provides the basic statistical properties of the two original data sets and the four filtered small data sets, and this information help us to determine the values of parameters in the theoretical models. In the second column of the table, we provide the name of the dataset. The third column displays the rating scores present in the dataset; $|U|$, $|O|$ and $|E|$ in the fourth to sixth columns refer to the number of users, objects, and edges in the dataset, respectively. In the last column of Table 2, the density of the dataset is given, which is defined as $\rho = \frac{|E|}{|U| \times |O|}$.

4.2. Degree Distributions of ASHAM Bipartite Network and Empirical Networks

In the simulation experiment, the parameters of ASHAM model are determined by the scale of real bipartite network, which is the number of nodes in each of two disjoint sets and links between them. By adjusting parameters r and s, the degree distribution of ASHAM model and the real network achieve a good agreement. Table 3 presents the specific simulation parameter values of the ASHAM model for four filtered small data sets. In the last two columns of the table, the mean square error (MSE) in the user or movie set's degree between

Table 2. Basic statistical properties of two original datasets and four filtered small datasets in *MovieLens* and *Netflix*.

No.	Dataset	rating	$\|U\|$	$\|O\|$	$\|E\|$	$\|\rho\|$
1	*MovieLens*	1~5	943	1682	100,000	0.06305
2	*MovieLens*	1	723	1363	6110	0.00620
3	*MovieLens*	5	928	1172	21201	0.01949
4	*Netflix*	1~5	3000	2779	197,248	0.02366
5	*Netflix*	1	1941	1662	9069	0.00281
6	*Netflix*	5	2914	1793	45,247	0.00866

data and model is given. Therefore, the lesser values of MSE indicate a good agreement between data and model. The results of the degree distribution simulation experiments of the ASHAM bipartite network model are shown in **Figure 3**, where the blue dots in (a)-(d) are the data of *MovieLens* and the blue dots in (e)-(h) are the data of *Netflix*. Besides, the red dots in **Figure 3** represent the degree of the user set in the model, while the yellow dots represent the degree of the object set in the model. It can be observed that the simulation results fit well with the real data in **Figure 3**, and it proves that the ASHAM model can represent the characteristics of the real data in terms of the degree distribution. Furthermore, we notice that all the degree distributions are similar to shifted power-law distribution, which is also consistent with the theoretical derivation results in Section 3. This phenomenon also satisfies the result caused by the selection behaviour of preferential attachment 11, that is, the presence of preferential attachment leads to the formation of power-law distribution.

4.3. Degree Distributions of EHAM Semi-Bipartite Network and Empirical Networks

In the studies of bipartite networks, a common method is to obtain the internal edges of a particular set through one-mode projecting [27]. However, these methods are too simple, and if projected directly into the rating network, information such as differences and similarities in ratings will be lost. In this paper, the method we use to set up the internal network is different from the general one-mode projecting method. Based on the empirical data, we use the node similarity algorithm to establish the internal edge of a single set, and embed the similar information among nodes of a particular set into the internal network of the semi-bipartite network, so that the new semi-bipartite network is able to carry more abundant deep information. Here, the specific construction procedure of establishing the user interest similarity network (namely the internal network of user set) in the user-movie rating empirical network is as follows:

Step 1: Grab all ratings in the data set and reveal the user's interests and preferences based on the comprehensive rating information. Firstly, the Euclidean distance between each user is calculated. The greater the distance, the lower the interest similarity between users, and vice versa. The interest Euclidean distance (*IED*) between users u_i and $u_{i'}$ is given as

$$IED_{ii'} = \sqrt{\sum_{j=1}^{|O|}\left(s_{ij} - s_{i'j}\right)^2}, \tag{16}$$

Table 3. The specific simulation parameter values of the ASHAM model for real networks.

No.	Dataset	rating	*m*	*n*	*r*	*s*	*p*	*q*	*t*	*MSE_U*	*MSE_M*
1	*MovieLens*	1	6	6	0.76	0.79	0.35	0.65	2074	0.000368	0.000483
2	*MovieLens*	5	17	19	0.65	0.72	0.44	0.56	2064	0.000086	0.000142
3	*Netflix*	1	6	5	0.61	0.64	0.54	0.46	3592	0.000549	0.000113
4	*Netflix*	5	17	18	0.56	0.69	0.62	0.38	4672	0.000175	0.000074

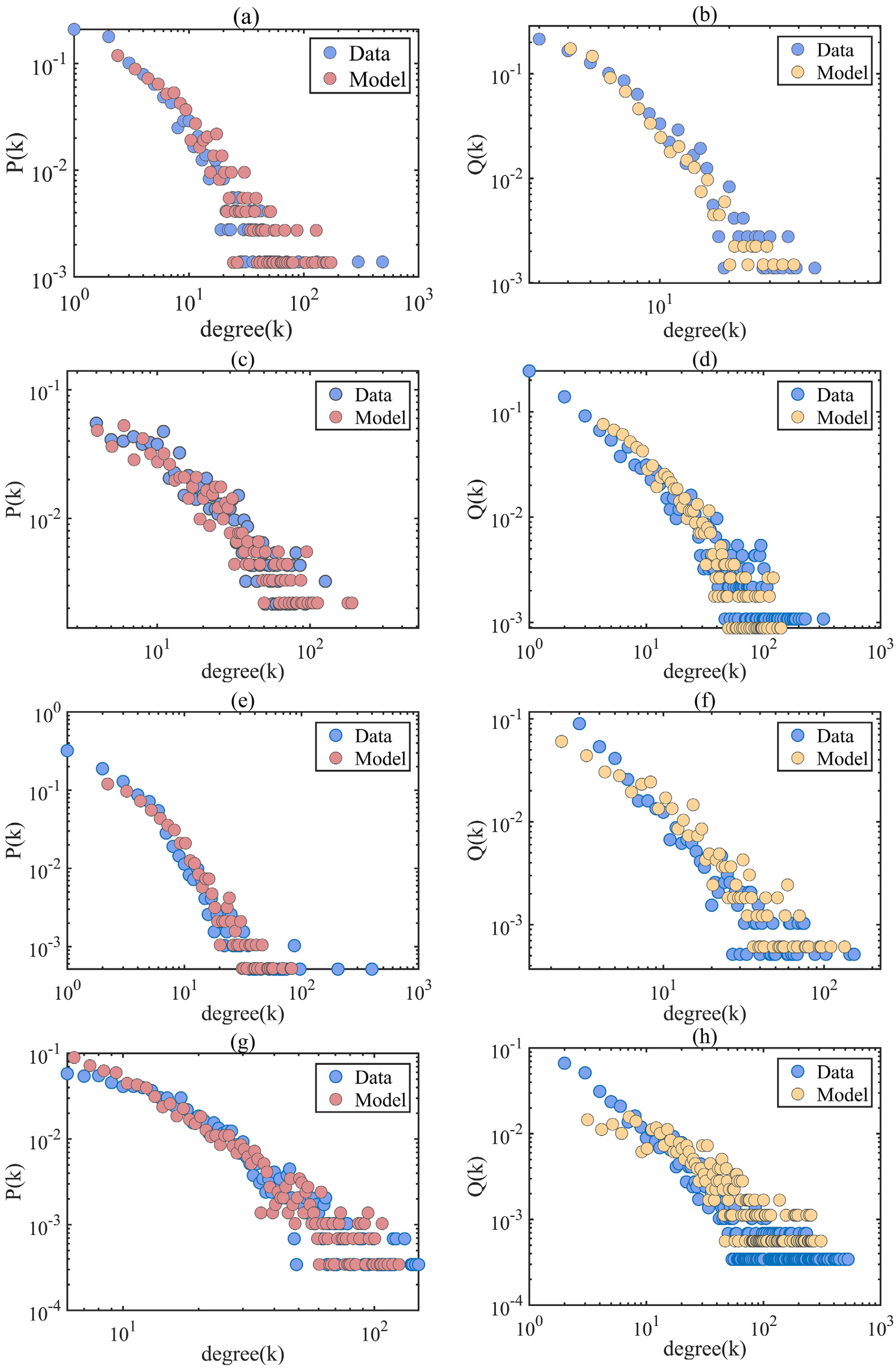

Figure 3. The result of degree distribution simulation of the ASHAM model and real networks is shown. (a), (e) The user set in rating = 1. (c), (g) The user set in rating = 5. (b), (f) The movie set in rating = 1. (d), (h) The movie set in rating = 5.

where $|O|$ is the total number of movies in the data set, and $s_{ij}, s_{i'j}$ represents the rating values of user u_i and $u_{i'}$.

Step 2: Establish the adjacency matrix of the interest similarity network (*ISN*). From Equation (16), the interest distance between all users can be achieved, and then their mean interest Euclidean distance (IED_{mean}) can be calculated. Let

$$ISN_{ii'} = \begin{cases} 1, & IED_{ii'} < IED_{mean} \\ 0, & \text{others} \end{cases}, \tag{17}$$

While the interest Euclidean distance of two users *i* and *i'* is less than the mean interest Euclidean distance, $ISN_{ii'}$ equals one indicates that *i* and *i'* have high similarity in interest about movies, otherwise $ISN_{ii'}$ equals zero.

When keeping the parameter settings of ASHAM bipartite network model unchanged (that is, the parameter settings of EHAM model are same as Table 2), the parameters *l* and p_{in} are adjusted to make the density of internal network density in EHAM semi-bipartite network model basically consistent with the density of user interest similarity network in the real network, and the obtained parameter values of EHAM model are shown in Table 4. In the last column of Table 4, we calculate the mean square error (MSE) between data and model, and observe that the difference between them is very small. This indicates that when the network scale and density of data and model are consistent, the internal network of EHAM model can be reliably fitted to the real user interest similarity network by appropriately adjusting the probability parameter p_{in}. Figure 4 shows the degree distribution plots of the user interest similarity network of four filtered small data sets (blue triangles) and the internal network of the EHAM model (red triangles). The good agreement shows that the internal network of EHAM model is consistent with the user interest similarity network of the real data in terms of the degree distribution, which also indicates that the EHAM semi-bipartite network model can well display the characteristics of the empirical semi-bipartite network, and has the effect of displaying the interest association information hidden in the empirical data. What's more, from the shape of the degree distribution in the plots, it can be seen that the user interest similarity tends to Poisson distribution, that is, the values are mostly concentrated near a special value.

5. Conclusions

In this paper, we propose the ASHAM evolving bipartite network model and

Table 4. The parameter values of EHAM model are tabulated, and error values between the model and real data are proposed.

No.	Dataset	rating	*l*	p_{in}	*MSE*
1	*MovieLens*	1	433	0.31	0.00002137
2	*MovieLens*	5	577	0.90	0.00000813
3	*Netflix*	1	1228	0.61	0.00000121
4	*Netflix*	5	1861	0.05	0.00000057

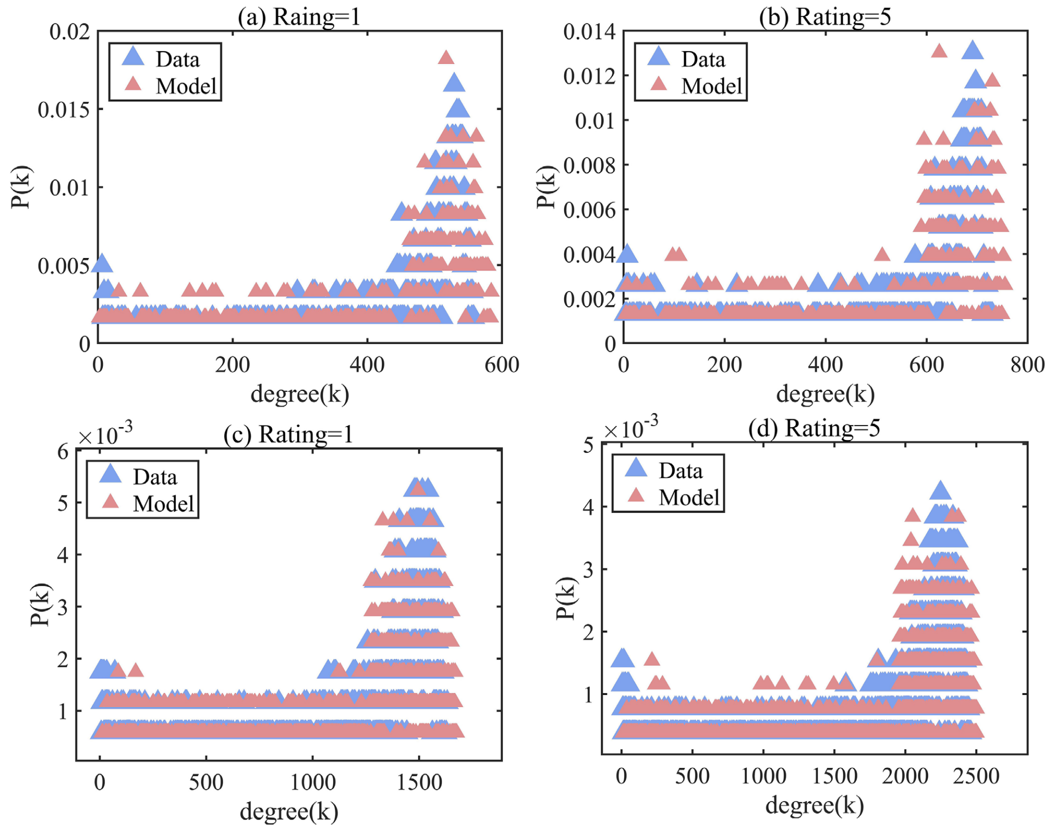

Figure 4. The degree distribution of the user interest similarity networks of four filtered small data sets (blue triangles) and the internal network (red triangles) of the EHAM model.

EHAM semi-bipartite network model based on the adjustable scale and hybrid attachment mechanisms. Compared with the models in [19] and [22], the ASHAM bipartite network model can control the growth scale of different sets of nodes through adjustable parameters, which is much close to the real situation and more flexible. In addition, the EHAM semi-bipartite network model can make the potential node similarity information emerging in the research of forming the user interest similarity network, which will help people to improve the design of the recommendation algorithms in the recommender system and improve the accuracy of algorithms.

The theoretical analysis of this paper shows that, in the case of r or s is not equal to zero, the degree distributions of two different kind of nodes in ASHAM model obey the shifted power-law distribution, otherwise follow the exponential distribution. In the numerical simulation, we find that for the point of view in degree distribution, the bipartite network model and the semi-bipartite network model evolving according to the unbalanced growth of sets of nodes and the hybrid attachment mechanism can be close to the real networks, and enable to emerge the deep information contained in the complex data. Under certain parameters condition, the ASHAM model can be regarded as a generalization of the evolving bipartite network model in [22]. Besides, the internal network of EHAM model has a good performance on simulating the user interest similarity network of rating systems. It indicates that, compared with the general bipartite network, the semi-bipartite network plays a complementary role in capturing deep information in the real network, and effectively compresses much of the hidden information into links in the internal network. The features of two proposed models help us to further discover the intrinsic nature of real networks and explore deep mechanisms in bipartite structure systems.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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