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The Local Acceleration Due to Gravity as Determined with a Cart and Track

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Abstract

The aim of this lab is to determine an experimental value for the local acceleration due to gravity. In order to do this, a cart was released down a track and allowed to pass through two photogates recording the entrance and exit times of the cart. These times along with the length of a light blocking strip on the cart, were used to calculate the acceleration of the cart down the track at various angles, and through linearization, the experimental value for the local acceleration due to gravity was determined to be 10.027 ± 0.312 m/s^2. This value has a percent error of only 2.2% from the accepted value of 9.8 m/s^2, which proves that this method of determining local acceleration due to gravity can be effective and accurate. Additionally, this experimental value shows how similar the approximation $g = 10\ \text{m/s}^2$ is to the accepted value.

Keywords

Gravity, Local Acceleration Due to Gravity

1. Introduction

Local acceleration due to gravity is a measure of the acceleration a body in free fall experiences near the surface of the earth. Technically, the law of universal gravitation states that the magnitude of the force of gravity that two masses exert on each other is proportional to the inverse square of the distance between them, but $g = 9.8\ \text{m/s}^2$, gravitational acceleration measured at sea-level serves as an accurate approximation for most points on earth [1]. In this lab we determined an experimental value for the local acceleration due to gravity.

The goal of this lab is to determine an experimental value for *g*, the local acceleration due to gravity. There are several ways to empirically determine *g*, the local acceleration due to gravity, from pendula to satellites ([2] [3]). These measurements can have civilian applications, like monitoring changes in mass distributions which affect the Earth's gravitational field [4]. In this lab, a cart, low-friction track, and two photogates were used. The times the cart entered and exited each photogate were measured, and these times and the length of the cart were related to the experimental acceleration of the cart with the equation from **Appendix A3**. Additionally, the expected acceleration of the cart on the track for some angle is related to the local acceleration due to gravity is determined with the equation in **Appendix A4**.

2. Methods

Firstly, the track was attached to a ring stand and angled at 10, measured with an angle-measurement pendulum attached to the track. Then, the photogates were attached to their own ring stands and were suspended above the track, with care taken to ensure that they were perpendicular to the track, that they were high enough that they would only detect the 2.5 cm photogate-blocking strip on the cart, and that there was enough space before the first photogate so the car could be released. These steps were taken so that the photogates could most clearly detect the cart, and thus give the clearest measurements. Then the photogates were activated and the cart was released. After the cart passed through both photogates and reached the bottom of the track, the photogates were disabled and the initial and final times for both photogates were recorded. This process was repeated twice more for 10˚, and then three times for angles of 15˚, 20˚, 25˚, and 30˚, for a total of three trials at five different angles.

3. Results

The raw data collected from the photogates (see **Appendix A1**) were processed to yield **Table 1**. Since the acceleration of the cart, $a = g\sin\theta$, is linear with respect to $\sin\theta$, the slope of the plot of *a* vs. $\sin\theta$ is the experimental value of *g*. Plots of both average acceleration vs. angle of inclination (see **Figure 1**) and average acceleration vs. sine of angle of inclination (see **Figure 2**) were created. The slope of the latter plot found using a linear regression (see **Figure 2**), leading to an experimental value of $g = 10.027\ \mathrm{m/s^2}$, with an uncertainty of 0.312 $\mathrm{m/s^2}$. This results in a percent error of only 2.2% (see **Appendix A5**), with the expected value lying within the range.

4. Discussion

According to the data obtained, the experimental acceleration due to gravity is 10.027 ± 0.312 $\mathrm{m/s^2}$. This yields a percent error of 2.2% when compared to the accepted value of $g = 9.8\ \mathrm{m/s^2}$, and it lies within its range. One likely source of error can be found in the measurement of the angle of inclination θ. In order to measure the angle of inclination, an upside down protractor parallel to the track with a pendulum strung from the origin was used. When at rest, the string of the pendulum indicates the angle of the track. However, in this experiment, the pendulum was not weighted very heavily, which resulted in a rather unstable measurement, which could have led to greater angle measurements. This is supported by the nonzero, positive y-intercept of the trend line. In order to make the angle measurements more precise, this pendulum could be weighted more heavily, or an alternative angle measurement device could be used, such as an angle ruler.

Table 1. Average acceleration and corresponding error in relation to angle of inclination and the sine of the angle of inclination.

Angle of Inclination (°)	Sine of Angle of Inclination	Average Acceleration (m/s^2)	Standard Deviation (m/s^2)
10	0.17	1.88	0.03
14	0.24	2.61	0.01
20	0.34	3.48	0.01
25	0.42	4.29	0.03
30	0.50	5.22	0.20

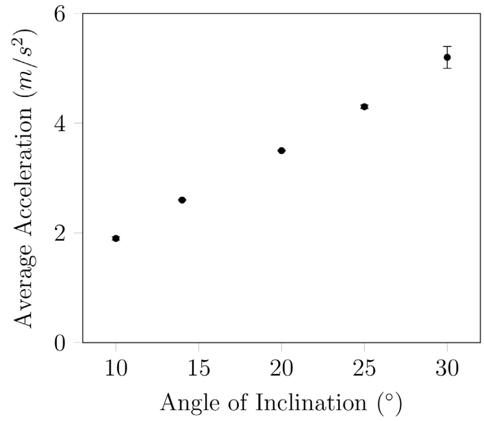

Figure 1. Angle of inclination vs. average experimental acceleration.

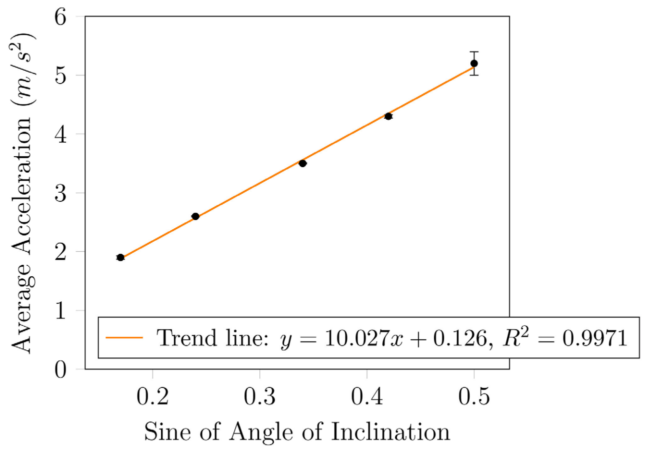

Figure 2. Sine of angle of inclination vs average experimental acceleration.

Another interesting source of error is the exclusion of friction and air resistance. In this situation, the acceleration experienced by the cart due to friction can be modeled by $\mu g \cos\theta$. With small values of θ this friction acceleration is significant, but as θ increases, the cosine becomes smaller and the acceleration down the track due to gravity becomes more dominant. As a result, data points with smaller θ values are more downward skewed, leading to an inflated trend line slope. This accounts for the higher than expected experimental value.

5. Conclusions

One interesting conclusion from this lab is that even with several steps taken to reduce potential error, the experimental acceleration achieved was about 2.2% greater than the accepted value, although the accepted value did lie in the range of the experimental value. This goes to show that while there is certainly a difference between $g = 9.8\ \mathrm{m/s^2}$ and 10.027 ± 0.312 $\mathrm{m/s^2}$, in small scale settings like this lab they are not too different. While this variation in g was certainly due to experimental error, how much could g vary if measured from different elevations and places? While it would be a small amount, it would be interesting to repeat this procedure at different elevations and compare the data.

More generally, while obvious already, this lab illustrates how gravity acts on objects not in free fall, and how this interaction with gravity can be isolated and measured. It would be worthwhile to see how much more accurate an experimental value accounting for air resistance and friction would be.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix A

Appendix A1. Raw Data

Table A1. Raw data.

θ (˚)	t_{1i} (s)	t_{1f} (s)	t_{2i} (s)	t_{2f} (s)	Δt_1 (s)	Δt_2 (s)	Δt (s)	$a_{experimental}$ (m/s^2)	$a_{expected}$ (m/s^2)	% Error	Trial
10	1.069286	1.102686	1.564586	1.579685	0.033400	0.015099	0.461900	1.87	1.7	0.097	1
10	0.124522	0.155024	0.592058	0.606790	0.030502	0.014732	0.437034	1.91	1.7	0.122	2
10	0.491386	0.519485	0.946086	0.960603	0.028099	0.014517	0.426601	1.86	1.7	0.092	3
14	1.365066	1.389608	1.747582	1.760087	0.024542	0.012505	0.357974	2.60	2.4	0.099	1
14	0.687213	0.711985	1.071186	1.083700	0.024772	0.012514	0.359201	2.62	2.4	0.104	2
14	0.251403	0.275284	0.628490	0.640925	0.023881	0.012435	0.353206	2.59	2.4	0.094	3
20	1.126685	1.149614	1.484385	1.495186	0.022929	0.010801	0.334771	3.48	3.4	0.039	1
20	0.649510	0.670985	0.997912	1.008494	0.021475	0.010582	0.326927	3.49	3.4	0.042	2
20	0.841985	0.863385	1.190285	1.200885	0.021400	0.010600	0.326900	3.47	3.4	0.036	3
25	0.787685	0.810410	1.127013	1.136929	0.022725	0.009916	0.316603	4.27	4.1	0.031	1
25	3.606286	3.626586	3.926300	3.935989	0.020300	0.009689	0.299714	4.29	4.1	0.035	2
25	5.731386	5.752126	6.054584	6.064284	0.020740	0.009700	0.302458	4.32	4.1	0.043	3
30	2.947885	2.963713	3.214493	3.222884	0.015828	0.008391	0.250780	5.33	4.9	0.087	1
30	1.414086	1.430886	1.689109	1.697624	0.016800	0.008515	0.258223	5.35	4.9	0.091	2
30	9.882728	9.899309	10.166608	10.174697	0.016581	0.008611	0.266777	5.00	4.9	0.019	3

Appendix A2. Calculation of Δt_1, Δt_2, and Δt

$$\text{time in photogate 1} = \text{time photogate 1 entered} - \text{time photogate 1 left}$$
$$\text{time in photogate 2} = \text{time photogate 2 entered} - \text{time photogate 2 left}$$
$$\text{time between photogates} = \text{time photogate 1 left} - \text{time photogate 2 entered}$$
$$\Delta t_1 = t_{1f} - t_{1i} \quad \Delta t_2 = t_{2f} - t_{2i} \quad \Delta t = t_{2i} - t_{1f}$$
$$\Delta t_1 = 1.102686\ \text{s} - 1.069286\ \text{s} = 0.033400\ \text{s}$$
$$\Delta t_2 = 1.760087\ \text{s} - 1.747582\ \text{s} = 0.012505\ \text{s}$$
$$\Delta t = 0.628490\ \text{s} - 0.275284\ \text{s} = 0.353206\ \text{s}$$

Appendix A3. Calculation of Experimental Acceleration

$$\text{acceleration} = \frac{\frac{\text{cart length}}{\text{time in gate 2}} - \frac{\text{cart length}}{\text{time in gate 1}}}{\frac{\text{time in gate 1} + \text{time in gate 2}}{2} + \text{time between gates}}$$

$$a = \frac{\frac{L}{\Delta t_2} - \frac{L}{\Delta t_1}}{\frac{\Delta t_2 + \Delta t_1}{2} + \Delta t}$$

$$a = \frac{\frac{0.025\ \text{m}}{0.010600\ \text{s}} - \frac{0.025\ \text{m}}{0.021400\ \text{s}}}{\frac{0.010600\ \text{s} + 0.021400\ \text{s}}{2} + 0.326900\ \text{s}} = 3.5\frac{\text{m}}{\text{s}}$$

Appendix A4. Calculation of Expected Acceleration

$$a = g\sin\theta$$

For case $\theta = 25$

$$a = \left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right)\sin 25 = 4.1\frac{\mathrm{m}}{\mathrm{s}^2}$$

Appendix A5. Calculation of Percent Error

$$\text{Percent Error} = \frac{\left|\text{Accepted Value} - \text{Experimental Value}\right|}{\text{Accepted Value}} \cdot 100\%$$

$$\%\text{Error} = \frac{\left|9.8 - 10.027\right|}{9.8} \cdot 100\% = 2.2\%$$