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A New Mathematical Justification for the Hypothesis of the Longevity of Jupiter's Great Red Spot

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Abstract

As is known, the Great Red Spot (GRS) is one of the most mysterious sights in the solar system and is a strong storm that is quite large. According to the laws of hydrodynamics and gas dynamics, it should have disappeared several centuries ago, but scientists still observe it and cannot accurately explain this phenomenon. Since turbulence and atmospheric waves in the GRS region absorb the energy of its winds, the vortex loses energy by radiating heat. In the work, it is proved with a mathematical and non-classical approach that the GRS and anticyclones will live for a long time; otherwise, we had to first of all prove that the vortex threads (loops) and ovals could not exist. Based on these supports, mathematical methods prove their existence forever by observing a large vortex (GRS); moreover, they are sources of heat. When proofs are obtained, the results are consistent with the previous hypotheses of the researcher. The introduction of the work gives a comparison of various hypotheses; for example, one of them states that the decrease in the size of the GRS is only an illusory observation. Next, we first consider the applicability conditions for the mathematical justification of the hypothesis of the longevity of the Great Red Spot. The wind equation and the GRS are energized by absorbing smaller eddies and ovals, and this total energy is constant. With the help of the KH mechanism in the case of Brunt Vaisala, the frequencies (which can be calculated by a program with given formulas) are determined using very strictly mathematical evidence to substantiate the validity of the hypothesis about the longevity of Jupiter's Great Red Spot.

Keywords

Mathematical Models, Jupiter, Turbulence, Longevity of GRS, Anticyclone, Hydrodynamic Equilibrium

1. Introduction

The Jupiter's Great Red Spot (GRS) is one of the most mysterious sights in the solar system and is a strong storm that is quite large. According to the laws of hydrodynamics and gas dynamics, it should have disappeared several centuries ago, but scientists still observe it and cannot explain this phenomenon. However, Pedram Hassanzadeh, a research fellow at Harvard University, and Philip Marcus, a professor of fluid dynamics at the University of California at Berkeley, believe they have found a partial explanation for this phenomenon. The Great Red Spot was supposed to disappear after several decades of existence. "Rather, it has been there for hundreds of years," says P. Hassanzadeh. Many processes are capable of dispersing atmospheric eddies like the Red Spot. Turbulence and atmospheric waves in the Red Spot region absorb the energy of its winds. The vortex loses energy by radiating heat. Finally, the Red spot is located between two strong jet streams that move in opposite directions and should slow its rotation. Some researchers argue that the GRS is energized by absorbing smaller eddies. "Some computer models show that large eddies can live longer if they merge with smaller vortices. But that doesn't happen often enough to explain the Red Spot's longevity," says P. Markus. To solve the mystery of the Red Spot's persistence, P. Hassanzadeh and P. Markus built a model that was completely 3D and of very high resolution. "The models focused on horizontal winds, where most of the energy is concentrated, eddies also have vertical currents, although they have much less energy, because modeling all this is quite difficult," explains P. Hassanzadeh. However, it is the vertical movement that may be the key to the mystery of the Red Spot. It should be noted that there is still no clear mathematical evidence that the GRS will live on for a long time. Although many authors give partial explanations for both the misconception and the positive side, for the sake of clarity, we will put this reasoning in the beginning of 2004, when its length was about half a century less than a year earlier, when it reached a size of 40,000 km (25,000 miles), which is about three times the diameter of the Earth. At the current rate of contraction, by 2040 it will be round. It is not known how long this spot will last or whether this change is the result of normal oscillations [1]. In 2019, the GRS began to "flake off" at the edge as fragments of the storm broke away and dissipated. The shrinking and "flaking" have led some astronomers to speculate that the GRS could dissipate within 20 years. However, other astronomers believe that the apparent size of the GRS reflects its cloud cover rather than the size of the actual underlying vortex, and they also believe that peeling events can be explained by interactions with other cyclones or anticyclones, including incomplete absorptions of smaller systems. If so, it would mean that the Great Red Spot is not threatened with extinction (e.g., [2] [3] [4]; these would seem to be admitted during observations as an illusion). Time-lapse of the 1979 Voyager 1 rendezvous with Jupiter, showed the movement of atmospheric bands and the circulation of the GRS. The top clouds of this storm are about 8 kilometers (5 miles) above the surrounding top clouds. The reason the storm has continued to exist for centuries is because the planet has no surface (only a hydrogen mantle) to provide friction; eddies of circulating gas persist in the atmosphere for a very long time because there is nothing to oppose their angular momentum. However, the upper atmosphere above the storm has significantly higher temperatures than the rest of the planet. Acoustic (sound) waves arising from the storm's turbulence below have been proposed as an explanation for the area's heating. Acoustic waves propagate vertically up to 800 km (500 miles) above the storm, where they break up in the upper atmosphere, converting wave energy into heat. However, in recent years, astronomers note the lack of model facts: "the existence of narrow stable bands and flows symmetrical about the equatorial flow from west to east (in the direction of the planet's rotation), the difference between zones and belts, and the origin and stability of large eddies, such as the Great Red Spot.". Until now, some scientists believed that the size of the Jupiter's GRS could be reduced by observation, but there was no rigorous justification for this. But NASA itself also denied that a decrease in the size of the GRS has not yet been proven, except as noted by professors at Boston University, where the results of observations on the GRS turned out to be the opposite, that it is a heat source (see **Figure 1**). The existing models to prove the durability of the GRS did not have a strictly mathematical justification. This paper is the first comprehensive justification that evidence for the longevity of the GRS has a rigorous justification using astrophysical hydrodynamics, a non-classical approach about the existence of a cycle, and conservation of energy balance for pumping energy from ovals and small vortices, which provides energy recovery for Jupiter's GRS. Simultaneously, our model provides hydrodynamics about the equilibrium of Jupiter with the GRS. In order to present a model about the dynamic stability of the movement of the GRS and its mathematical justification for the longevity hypothesis, which with a non-classical approach proved.

2. Conditions of Applicability and Mathematical Substantiation of the Hypothesis about the Longevity of the Great Red Spot

1) We need to explain what we meant when we noticed the phrase: "But this does not happen often enough to explain the longevity of the GRS," says fluid dynamics professor P. Markus.

2) We also need to explain what he meant when he noticed the phrase: "Previously, researchers simply ignored vertical flows because they did not know about their importance or used simpler equations, since it is quite difficult to model all this," explains P. Hassanzadeh.

3) How to apply hydro-aerodynamics, as well as gas-dynamic modeling approaches, so that the laws of astrophysical hydrodynamics and geophysical hydrodynamics are also preserved in the dynamic equilibrium of the GRS and Jupiter itself.

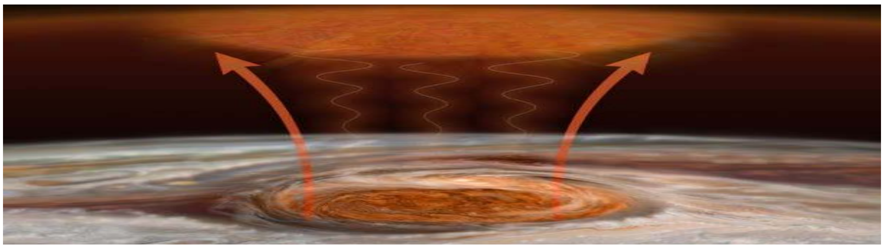

Figure 1. Close up of the Great Red Spot imaged by the Juno Spacecraft in April 2018 From NASA Artist's concept of the heating mechanism from Jupiter's Great Red Spot NASA.

4) In order to reveal the secret of the GRS survivability, it is necessary to determine what movements are possible on the GRS and how to describe mathematical models.

5) GRS: Can they provide additional energy for hundreds of years?

6) From what steel is a mysterious heat source?

First, note the observations made by NASA: NASA's Juno spacecraft, which recently arrived at Jupiter, will have several opportunities to observe the Great Red Spot and its surrounding turbulent region during its 20-month mission. Juno will look hundreds of miles down into the atmosphere with its microwave radiometer, which passively captures heat from within the planet. This capability will allow Juno to reveal the deep structure of the Great Red Spot as well as other Jupiter features such as colorful cloud bands. Just then, as a result of the NASA observation, we substantiated theoretical models associated with characteristic features such as vortex, turbulence, heat, and energy balances. Therefore, it is appropriate for us to mention the terms of energy, which we will use in proving the dynamical equilibrium of Jupiter's GRS. In addition, for more explanations on the account of the fact that the BKP is like a heat source, we recall the latest hypotheses of lead researcher James O'Donoghue. That is, according to an article published in the journal Nature on July 27 (see Figure 1), Jupiter's Great Red Spot is probably the same mysterious heat source that causes surprisingly high temperatures in the atmosphere of the fifth planet from the Sun. The study of the gas giant was sponsored by NASA. Scientists are baffled as to why the temperature in Jupiter's upper atmosphere is comparable to that found on Earth since Jupiter is nearly five times farther from the Sun than our planet. The researchers decided to find the answer to the question, what then is the source of heat for the fifth planet if it is not the Sun? The foundation for solving this riddle was laid by researchers from the Center for aero-physics at Boston University (USA), who analyzed temperatures at an altitude much higher than the cloud tops on Jupiter. A hurricane in the GRS produces two types of turbulent energy waves—gravitational and sonic—that collide and heat the atmosphere above them. The principle of operation of gravitational waves resembles the movement of a guitar string after it is touched by a musician while strumming, and sound waves are gas compression. It is believed that the heating of the atmosphere 800 km above the Great Red Spot is caused by the collision of these two types of waves (see, Figure 1).

Note 1. We will soon finish the results of the study with mathematical modeling of the fact that supposedly sound (see Figure 1) on the GRS is given as a heat source.

3. Mathematical Substantiation of the Hypothesis about the Lon-Gevity of the Jupiter's Great Red Spot

3.1. About the New Jupiter GRS Models

First of all, let's give a simple form to the integrative way of the GRS. Most natural phenomena, such as vortex motions, can be imagined as integrative imaginations in the following way: let's say that two disks rotate with different angular velocities. At the same time, they come into contact with each other, and a disk with a higher angular velocity will entrain a disk with a lower angular velocity, which will lead to the occurrence of vortex movements. This disk must be considered a zone, and the other a belt, similarly to zones and belts (see, Figure 2) on Jupiter. Then these swirls give us an idea of Jupiter's GRS.As is known, the GRS of Jupiter is a solitary vortex. Such vortices, apparently, arise due to the effect of wind twisting by the Coriolis force and retain their shape for a long time. Although the observation of astronomers does not fully explain the dynamic motions, the stability of the local dynamics of this phenomenon However, the mathematical description of this phenomenon is always of great interest. It is pertinent to note that the dynamics of Jupiter's atmosphere need a mathematical apparatus that would explain a comprehensive theory of the dynamics of Jupiter's atmosphere. Astronomers noted that such a theory does not yet exist. Because such a theory should explain the following facts: "the existence of narrow stable bands and flows symmetrical with respect to the equatorial flow from west to east (in the direction of rotation of the planet), the difference between zones and belts, as well as the origin and stability of large eddies, for example, the Great Red Spots." In this direction, the noted problems at some level are more or less solved (by mathematical modeling and justification) by the works of the authors (see, [5] [6]-[11]). However, earlier existing theories of atmospheric models can be divided into two classes: surface and deep. The first assumes that observations of circulation are largely due to the thin outer (weather) layer of the atmosphere and that the inner part is stable. The second postulate is that the observed flows are a manifestation of processes occurring in the deep layers of Jupiter's atmosphere. Each theory has both strengths and weaknesses; therefore, most planetary scientists and astronomers believe that the true theory will include elements of both models. In our opinion, if we ensure the dynamic stability and the theory of equilibrium of Jupiter itself, then we can interpret new models that will give mathematical justifications that will include both the inner and outer atmospheres of Jupiter. By including them, the satellites will, under our conditions, properly fulfill the conditions in order to preserve the stability of Jupiter.

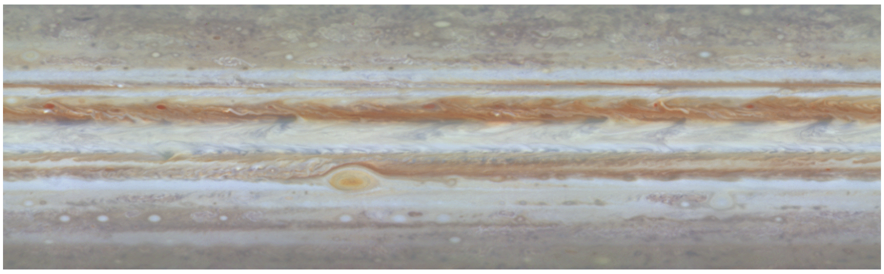

Figure 2. Zones belts and vortices on Jupiter (see [5] [7] [8]) NASA.

In works (e.g., [11] [12]), a system of equations was introduced that describes waves in a rotating atmosphere, the depth of which is sufficiently less than the wavelength:

$$\frac{\partial \boldsymbol{v}}{\partial t}+\boldsymbol{v}(\boldsymbol{v}\nabla)=-g\nabla h+\Omega[\boldsymbol{v},\boldsymbol{n}]. \quad (1)$$

$$\frac{\partial H}{\partial t}+div(H\boldsymbol{v})=0, \Omega=2\omega_0\sin\alpha \quad (2)$$

where,

$$div(\boldsymbol{v})=\frac{\partial v}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial v}{\partial z} \text{ and } \nabla=\frac{\partial}{\partial x}\boldsymbol{i}+\frac{\partial}{\partial y}\boldsymbol{j}+\frac{\partial}{\partial z}\boldsymbol{k} \quad (3)$$

Here the fluid depth is *H*, -horizontal is the velocity, *g* is the uprooting force of gravity, is the angular velocity of Jupiter's rotation, and is the unit vector along the vertical. All quantities depend only on horizontal coordinates: meridional angle and latitudinal angle.

In this situation, for processing the cyclones and anticyclones, it is more suitable to apply the Rossby models (following as in [7] [10]). Since, within shallow water theory, the equation describing the dispersion of waves on the planet rotating with the planet around the Z-axis is clockwise. Here, the equation describing this process is expressed in the local Cartesian coordinate system and has the form [7]:

$$\omega\left(\omega^2-4\Omega_z^2-k_\perp^2c_z^2\right)-\frac{2k_z\Omega_y}{R_{jupiter}}c_z^2=0 \quad (4)$$

where is the projection of the rotation velocity of the system on the local vertical; is the projection of the rotation velocity of the system on the meridian; The adiabatic velocity of sound, the radius of the planet Jupiter, is the wavenumber along latitude; the wavenumber taken along the meridian Here, as a high-frequency solution, is the dispersion law of gravitational-gyroscopic waves, and as a low-frequency solution, is the dispersion law of Rossby waves (a suitable solution):

$$\omega_R=-\frac{2k_z\Omega_y}{R_{jupiter}\left(k_\perp^2+4\Omega_z^2/c_z^2\right)}. \quad (5)$$

It can be seen from (5) that Rossby waves [10] are similar to drift waves in plasma [11]. The similarity is due to the fact that in a rotating atmosphere and a magnetized plasma, the Coriolis force and the Lorentz force have similar properties. In this case, the Rossby radius coincides with the expression for the Larmuir radius of ionic plasma [11] [12]. For drift waves, a stationary solution of solitary vortex, was found in [11] [12]. The main difference between a vortex and drift solitons in plasma [11] is that in Rossby waves, the main factor is the gradient of the Coriolis force, while in drift solitons, it is the temperature and density gradients of the plasma between the angular velocity and the pressure gradient and the Coriolis parameter $f = 2\Omega\left(\sin(\varphi)\right)$. To ensure the constant action of the cyclone and the anticyclone, the relations between the angular velocity and the pressure gradient and the Coriolis parameter are given. Follow as [6], the Coriolis force, or the deflecting force of rotation, appears in the equations of relative motion and is a fictitious force that describes the effect of the movement of the coordinates system associated with Jupiter(remember important fragments from [6]): $\boldsymbol{K} = -2\Omega \times \boldsymbol{V}$. The component $-2\Omega \times \boldsymbol{V}$ along the coordinate axes: $x: -2\Omega v\left(\sin(\varphi)\right)$, $y: 2\Omega u\left(\sin(\varphi)\right)$, if the *x*-axes is directed to the East, but *y*-to the North, *z*- vertically upwards and the wind speed component *U*, *V*, *W* along these axes. In this case $w \ll u$. The quantity $f = 2\Omega\left(\sin(\varphi)\right)$ is called the Coriolis parameter $K_x = -2\Omega v\left(\sin(\varphi)\right) = fv$, $K_y = 2\Omega u\left(\sin(\varphi)\right) = -fu$. Where Ω - the rotation velocity of Jupiter's, φ - along latitude. The ratio of the inertial force to the Coriolis force is called the Rossby number: $Ro = \frac{\left(\mathrm{d}V/\mathrm{d}t\right)_x}{fU} = \frac{U}{Lf}$, scales, horizontal *L*, vertical *H*, (the atmosphere is anisotropic, and these scales differ significantly), the velocity scale *U*, the time scale for horizontal displacements LU^{-1} for vertical HU ones, and the characteristic Coriolis parameter $2\Omega\left(\sin(\varphi)\right) = f$. The variation of pressure in the radial is given by $\frac{\mathrm{d}P}{\mathrm{d}r} = r\omega^2\rho$ The pressure at the axes of rotation is P_c Therefore, the required pressure at the point *r* is $P = P_c + \frac{1}{2}r^2\omega^2\rho$. It means that one of model locomotive gradient of pressure as it unmentioned above which guarantied cyclone and anticyclone if Rossby regime satisfied.

3.2. Existence of Circulation on This Circuit, Vortex Lines for Long-Live of Jupiter's GRS

As it shown in the work (e.g., [6] [9]), external affecting on it $\boldsymbol{F}$ (Coriolis, gravitational, and other possible) has the potential *U*: $\boldsymbol{F} = gradU$, for example, a fluid located in a gravity field and directed along the *Z*-axis, in this case, $U = -gZ$ could be taken since the GRS is projected on the plane (*x*, *y*, 0). Then circulation acceleration of motion GRS is

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\int_{(L)}\left(V, \mathrm{d}x\right) = \int_{(L)} a_x\mathrm{d}x + a_y\mathrm{d}y + a_z\mathrm{d}z \tag{6}$$

Therefore, we can able to formulate the following theorem.

Theorem1 Around of GRS on a closed fluid circuit the time derivatives of the velocity circulation are equal to the acceleration circulation on this circuit

Proof of Theorem1. It is sufficient to differentiate under the integral expressions in equality (6), and taking into account $\frac{1}{2}\left(\upsilon_x^2 + \upsilon_y^2 + \upsilon_z^2\right) = \frac{1}{2}|\upsilon|^2$ From here consider that the contour (L) is an ellipse, then $\frac{1}{2}|\upsilon|^2 = 0$. Further, since the density is a single-valued function of the pressure P (e.g., [6]), and taking the notations $\Phi(\rho) = \int \frac{\mathrm{d}\rho}{\varphi(\rho)}$ and account into compound derivatives we get: $grad\Phi = \frac{1}{\rho} gradP$. So, by Newton's law, it is justified that the basic equation of hydrodynamics $\rho \mathrm{d}V\boldsymbol{a} = \boldsymbol{F}\rho \mathrm{d}V - \mathrm{d}V gradP$ is satisfied and hence it follows that $\boldsymbol{a} = \boldsymbol{F} - \frac{1}{\rho} gradP$. Consequently from the acceleration circulation (5) it follows that

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}t} = \int_{(L)} a_x \mathrm{d}x + a_y \mathrm{d}y + a_z \mathrm{d}z = \int_{(L)} \mathrm{d}(U - \phi) = 0. \tag{7}$$

Then from equality of (7) we obtain that $\Gamma = const$. Thus, Theorem1 is proved.

Theorem 2 (existence of vortex lines) Particles of liquid, around GRS forming vortex lines, at any time, and at all times of motion form vortex lines, coming from their origin, through ovals (see, **Figure 2**, **Figure 3**) and swirled parts of liquid.

Proof of Theorem 2. Since the motion on fluid circuits around the GRS refers to the approximate, so-called, quasi-laminar, then under the condition of incompressibility, $div\boldsymbol{V} \to 0$, $rot\boldsymbol{V} = \varepsilon$, for $\varepsilon \to 0$. Where, $div\boldsymbol{V}$ as it shown in formula (3), and $rot\boldsymbol{V} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$, $\boldsymbol{V} = (V_x V_y V_z)$, and $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are the unit vectors for the x, y, and z axes, respectively. Because of center of GRS a small compressible solid and having rotation, particles of liquid, around GRS forming vortex lines, at any time, and at all times of motion form vortex lines, coming from their origin, through ovals and swirled parts of liquid. In order to proof theorem 2, it suffices to start relatively vortex thin surfaces [7]. Let (σ_0) there be a surface at the $t = t_0$; moment of time, then in each of its $\boldsymbol{\Omega} = rot\boldsymbol{\upsilon}$ vortex velocities $\Omega_n = 0$. If we take on (σ_0) surface any (L_0) closed contour bounding a part of surface, then by Stokes formula it will be true.

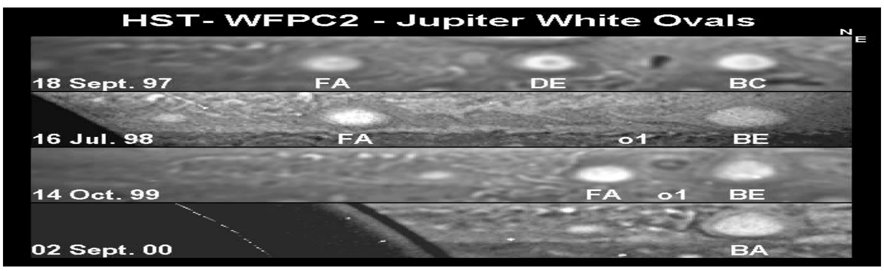

Figure 3. Zones belts and vortices on Jupiter. The wide equatorial zone is visible in the center surrounded by two dark equatorial belts (SEB and NEB). NASA (see, [6]).

$$\int_{(L)} \upsilon_x \mathrm{d}x + \upsilon_y \mathrm{d}y + \upsilon_z \mathrm{d}z = \iint_{(L_0)} \Omega_n \mathrm{d}\sigma = 0. \quad (8)$$

At the moment of time the surface (σ_0) will pass to surface (σ), and its part $(s_0) \to (s)$ but, liquid (L_0) contour will pass to liquid contour (L). Again by Stokes formula from the equality of (8), we have that $\iint_{(L)} \Omega_n \mathrm{d}\sigma = 0.$ Since, being arbitrary (σ), we easily get that along (σ), is identical $\Omega_n = 0.$ So, (σ), surface turns to be vertical. Hence, taking into account that vortex lines can be always considered as an intersection of two vortex surfaces, the theorem is proved.

Theorem 3. Let conditions of Theorem 1, 2 are fulfilled, and the Rossby conditions of free, cyclone, anticyclone (see, Figure 4). If quasi-laminar and turbulent (see Figure 5, Figure 6) fluid flow around the GRS exist, then the necessary and sufficient conditions for the existence of stability of constant GRS rotation and the Jupiter' s balance, the internal and external energy balances of Jupiter are preserved.

Proof of Theorem 3. According to Stokes formula, the intensity of the vortex flows, across a cross section is given by the circulation velocity of that section, then by asserts of Theorem1 and proves Theorem2 can be assertion the Theorem3. It means that the velocity circulation around GRS in a closed fluid circuit is constant.

Hence, the existence conditions of ovals, vortices, laminar flows and turbulent transition (e.g., [5] [6] [9]), which is provide for the constancy of velocity circulation around GRS, on a closed fluid circuit and will be for many years.

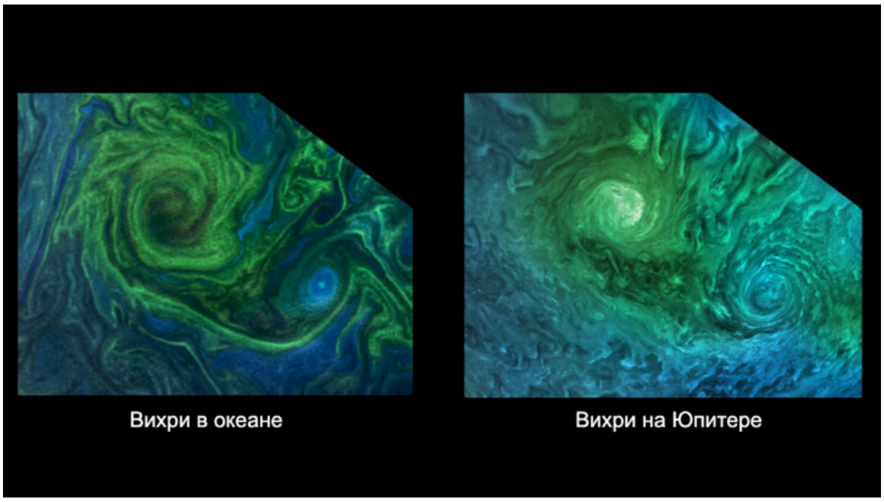

Figure 4. Phytoplankton eddies on Earth and cyclones on Jupiter. Translation Vesti.Ru. NASA OBPG illustration OB.DAAC/GSFC/Aqua/MODISImage/JPL/SwRI/MSSS/G. Eichstad NASA

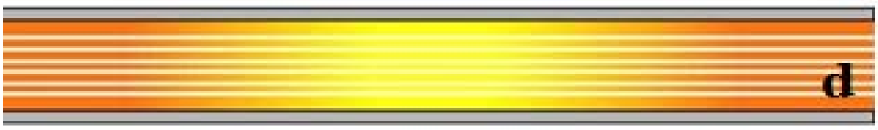

Figure 5. Illustration the Laminar rejime on Jupiter (see [6] [7] [8]).

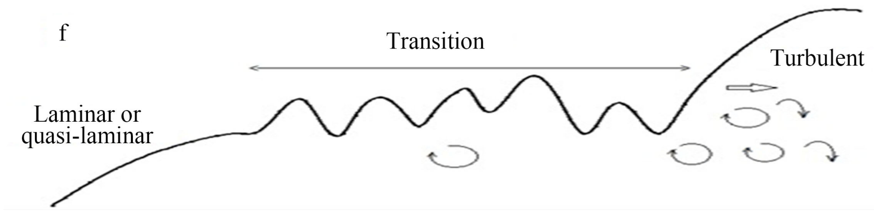

Figure 6. Illustration of scheme from laminar + quasi-laminar to transition turbulence regime. (see [7] [8]).

3.3. Equation of Wind and Transformation Energy for Supporting Cyclone and Anticyclone Long-Lives

In the field of the atmosphere, most of the methods used are uncertain. In [13], the solution of the wind speed equation was considered and the principles of a rotating storm system (see, **Figure 3**) such as a hurricane, typhoon, tropical storm, cyclonic storm, tropical depression, or simple cyclone. [13] [14] [15], in which our work also has applications of such standard methods as Boyle's law, Charles's law, and Newton's law [16]. The concepts of "point force" and "point mass" were already familiar to the engineering community, and the solution of the "point load" plays an important role in the analysis. For example, the solution of a force at a point inside an elastic space (the Kelvin solution) and the solution of a force at a point inside an elastic half-space (the Mindlin solution) form the basic solution for engineering analysis. Thus, the solution to Jupiter's atmosphere points must have application within a certain range. In the work [4] considered an equatorial thermal wind equation with applications to Jupiter, but we also consider the derivation of the wind equation and the energy of wind for supporting anticyclones and climate change in Jupiter's GRS. Now, based on the above, we will find the wind equations and find out that the wind speed and its dependence on the temperature and pressure of the gas at any point in the atmosphere of Jupiter For this purpose, we combine the Boyle-Mariotte and Charles laws and have $PV = RT$, where P is pressure, and $\dim P = [\mathrm{N{\cdot}m^{-2}}]$, V is volume $[\mathrm{cm^{-3}}]$, R is constant $\mathrm{N{\cdot}m{\cdot}degK^{-1}}$, T is temperature [K] [$T = t^{\circ}\mathrm{C} + T_{Kelvin}$], "$T = -273.15^{\circ}$" is absolute temperature.

Then, dimension of $PV = RT$ is $\dim(PV) = \mathrm{N.cm} = \dim(RT)$. Differentiating both sides of $PV = RT$ respect to (r, θ, z), where (r, θ, z) is the cylindrical coordinates and we have

$$\frac{\partial P}{\partial r}V = R\frac{\partial T}{\partial r}, \frac{\partial P}{\partial \theta}V = R\frac{\partial T}{\partial \theta}, \frac{\partial P}{\partial z}V = R\frac{\partial T}{\partial z} \tag{9}$$

The tangent of θ takes "cyclonic turn" as its direction. The z-axis sets on sea-level as $z = 0$, and up-ward for $z > 0$. Now we take any point of atmosphere, and suppose as volume of $V = V(x, y, z)$, $V = \Delta x \Delta y \Delta z$ and we consider the box reduced to a point, *i.e.* $Lim V = V_0$. when $\Delta x \to 0$, $\Delta y \to 0$, $\Delta z \to 0$. In Cartesians coordinates system, if pressure P take as matrix, then we have:

$$\nabla(PV) = R\nabla T,\ PV = \left[P_x \boldsymbol{i} + P_y \boldsymbol{j} + P_z \boldsymbol{k}\right](\mathrm{d}x\mathrm{d}y\mathrm{d}z) = F_x \boldsymbol{i} + F_x \boldsymbol{j} + F_x \boldsymbol{k} = \boldsymbol{F}\mathrm{d}s \tag{10}$$

where $\boldsymbol{F}$ is the applied force and s is the displacement. Hence by the Newton's second law, we have $\boldsymbol{F} = m\frac{\mathrm{d}v}{\mathrm{d}t} = m\left(\frac{\mathrm{d}v_x}{\mathrm{d}t}\boldsymbol{i} + \frac{\mathrm{d}v_y}{\mathrm{d}t}\boldsymbol{j} + \frac{\mathrm{d}v_z}{\mathrm{d}t}\boldsymbol{k}\right)$. Hence, we get: $m\frac{\mathrm{d}v}{\mathrm{d}t} = R\nabla T$. It is easy to see that $m\frac{\mathrm{d}v}{\mathrm{d}t}$ is proportion to gradient temperature is the locomotive driven wind speed acceleration along a straight line. If taking $P = P(x,y,z,t)$, $V = V_0$, $T = T(x,y,z,t)$ are pressure, volume and temperature, respectively. R is constant, $\frac{\partial V}{\partial x} = 0$, $\frac{\partial V}{\partial y} = 0$, $\frac{\partial V}{\partial z} = 0$. As known that P and T are continues function and have continues derivatives, then from the equality of (1) there exists respect to x the equation of partial derivative: $\frac{\partial P}{\partial x}V = R\frac{\partial T}{\partial x}$. Hence accordance to $P_x = P_x(x,y,z,t)$, as component of force applied on box(part of point of atmosphere Jupiter's), along the axis, which is obtained by the equilibrium $\sum F_x = 0$ (or $\sum x = 0$) and by the Newton's second law: $P_x = m\frac{\partial v_x}{\partial t}$, where the mass of box of atmosphere is $\frac{w}{g} = \rho\frac{V}{g} = const$. Where w is the weight, g is the acceleration due to gravity, ρ is the unit weight, s is the displacement, $v = \frac{\mathrm{d}s_x}{\mathrm{d}t}$ is the velocity, (the wind speed), s_x is the component of s along the x-axis, $v_x = \frac{\mathrm{d}s_x}{\mathrm{d}t}$ the component of v along the x-axis (wind speed component). Hence follows

$$m\frac{\partial v_x}{\partial x} = R\frac{\partial T}{\partial x} \tag{11}$$

and analogically we can write:

$$m\frac{\partial v_y}{\partial y} = R\frac{\partial T}{\partial y} \quad \text{and} \quad m\frac{\partial v_z}{\partial z} + mg = R\frac{\partial T}{\partial z} \tag{12}$$

which (11), (12) are partial differential equations and so called "wind speed equation" any point of atmosphere Jupiter's. The resultant force P of pressure applied on a point and by Newton's second law, we have

$$P = \frac{\partial v}{\partial t} \quad \text{or} \quad mg = \frac{\partial v}{\partial t}, \quad m\frac{\partial v}{\partial t} + mg = R\frac{\partial T}{\partial s}. \tag{13}$$

Hence,

$$\frac{\partial v}{\partial t} + g = k\frac{\partial T}{\partial s}, \quad k = \frac{R}{m}, \quad \frac{\partial v}{\partial t} = k\frac{\partial T}{\partial s}, \quad P = R\frac{\partial T}{\partial s} \tag{14}$$

Note that the direction of the positive vertical pressure gradient

$$\frac{\partial P}{\partial z}V = R\frac{\partial T}{\partial z} \tag{15}$$

Note that above obtained (11)-(15) are partial differential equation first order is solvable, if take initial value of conditions:

$$v(x,y,z,t)\big|_{t=0} = v_0, \quad P(x,y,z,t)\big|_{t=0} = P_0, \quad T(x,y,z,t)\big|_{t=0} = T_0 \qquad (16).$$

Since (10)-(15) is a set of differential equations in partial (uncertain, but the solution is available as a numerical, also analytical) derivative and at the same time, shows the relationship between the main component of wind speed v and pressure p, shows the relationship between v and T Also along the axis there is a relationship between P (pressure) and T (temperature). Then it can be justified that since the vertical movement is directed upwards, since the direction of the positive vertical pressure gradient (caused by the descending air density) coincides with the z axis, then it is confirmed by the recorded video as in a tornado [14] and the description (the main vertical upward movement [15]) therefore the temperature gradient T with respect to z decreases with increasing z. This result is consistent with the fact that the temperature decreases with increasing altitude, that the mass of air approaches zero in an isothermal layer, where the temperature remains the same for any layer.

If taken instead of $v(x,y,z,t)=v(s,t)$ is the wind speed, $T(x,y,z,t)=T(s,t)$ is the temperature, s is the trace of point, $k=\frac{R}{m}$ is a constant, then multiplying ds to both sides of $\frac{\partial v}{\partial t}+g=k\frac{\partial T}{\partial s}$ and integrating from 0 to s, we have:

$$v^2(s,t)=\frac{2}{m}v_0^2-2g[h-h_0]+\frac{2R}{m}[T(s,t)-T(s_0,0)] \tag{17}$$

$$\frac{m}{2}\left\{[v(s,t)]^2-[v(0,0)]^2\right\}+mg[h-h_0]=R[T(s,t)-T(0,0)] \tag{18}$$

The left hand side of (18) represents the work done by force P moving ds and by mg moving $[h-h_0]$ and finally we get:

$$W(s,t)-W(0,0)=\frac{m}{2}\left\{v^2-v_0^2\right\}+mg[h-h_0]=R[T(s,t)-T(s_0,0)] \tag{19}$$

So, the internal wind energy of Jupiter's can be spent on doing work in order to constantly provide a long-lived anticyclone.

$$\frac{\partial u}{\partial t}+g\frac{\partial \eta}{\partial x}=0,\quad \frac{\partial \eta}{\partial t}+\frac{\partial Du}{\partial x}=0 \tag{20}$$

Here u is the horizontal velocity of the water flow over all layers to the bottom, is the mixing of the surface wave relative to the zero level, g is the free fall acceleration, D is the depth. And the speed itself is found by the formula. Therefore, the kinetic and potential energy will be distributed evenly. A similar picture also takes place for a storm, a hurricane on Jupiter's, since the Equation (20) for the tsunami and wind of Jupiter's (see, Equations (11-(15)) operate almost similar principles.

3.4. Instability Kelvin-Helmholtz in Case of Brunta Vaisala Frequency for Storm of Jupiter's GRS

$$(U-c)^2\left[\frac{\mathrm{d}^2\overline{\Phi}}{\mathrm{d}^2 z}-\alpha^2\overline{\Phi}\right]+\left(N^2-(U-c)\frac{\mathrm{d}^2\overline{U}}{\mathrm{d}^2 z}\right)\overline{\Phi}=0, \tag{21}$$

(see, [17], formula (48)

$$(U-c)^2 \frac{d^2\bar{\Phi}}{d^2 z} - (U-c)^2 \alpha^2 \bar{\Phi} + \left(N^2 - (U-c) \frac{d^2 \bar{U}}{d^2 z} \right) \bar{\Phi} = 0, \tag{22}$$
$$K(\xi) = (U-c)^2, b(\xi) = N^2 - (U-c)$$

where $N^2 = \frac{g}{L_p}$ denotes by the Brunt-Vaisala frequency. The eigenvalue parameter of the problem is *c*. If the imaginary part of the wave speed C is positive, then the flow is unstable, and the small perturbation introduced to the system is amplified in time (the coefficients of this equation put in corresponding formulas of solution directly problem is solvable). If top and bottom be rigid walls, then from linearized covering equations of continuity case boundary condition is $\bar{\Phi}(0) = 0$, $\bar{\Phi}(d) = 0$, and when $N^2 = U - c$ then equation is no degenerating:

$$\frac{d^2\bar{\Phi}}{d^2 z} - \alpha^2 \bar{\Phi} = 0, \tag{23}$$

and may be solved as problem of Sturm-Lovell (in simple form). Note that the argument is unchanged, if the top and bottom are at $z = \infty$, $z = -\infty$. The equation (48) constitute eigenvalue problem where $c = c_r + ic_i$ is the eigenvalue, if $c_i > 0$ instability occurs. But the generalized solution of (48) (see, [17]) can be expressed by means of formulas (as it shown in (39)-(42) see [17]), if substitute the corresponding coefficients with $K(\xi) = (U-c)^2$, $b(\xi) = N^2 - (U-c)$ and boundary conditions. By virtue of nonclassical ordinary differential equation and its of generalized solutions which is at first constructed by Mahammad Nurmammadov [17] at least one the considered Equation (21) is solvable, for this reason only need substitute the coefficients of (21) in formula of generalized solution [17]. When $U = c$ it is resonance case, *i.e.* $N^2 = (U-c) = 0$ the equation of (21) is degenerated is also account into an example 3.2 (as it shown in (39)-(42) see [17]).

Note that, at first, author M.A. Nurmammadov in the work [17] entered the definition of nonclassical ordinary differential equation and its physical means (hydro dynamical, aerodynamically gas dynamical means in case of degenerating cases) and founded a generalized analytical solution for some of these equations, which were applied in field astrophysical problems, space sciences, ocean problems, and buoyancy frequency.

Brunt-Väisälä frequency (*N*), also called buoyancy frequency, can be used as a measure of stability for a fluid column and is written as $N = \sqrt{\frac{g}{\rho} \frac{\partial \rho}{\partial z}}$ where g is the gravitational acceleration, ρ density and z depth. N describes how a fluid parcel would oscillate after being vertically displaced. In the case of stable stratification N is positive and larger positive values indicate stronger stratification, leading to higher frequency of oscillation and therefore results in lower oscillatory period, while weaker stratification results in longer oscillatory period. In a case with neutral stratification N has the value of zero, and the displacement doesn't lead to any oscillatory motion. Finally, in a case of unstable stratification N has complex values and there is no oscillation, as the parcel doesn't return to its original position, but is instead accelerated away. Unstable conditions tend to thus break and if observed they indicate active overturning process and convective mixing. Finally, if $N^2 > 0$ then storm frequency is stable, $N^2 < 0$ then storm frequency is unstable, $N^2 = 0$ then storm frequency is neutral. For example, Conrath et al. (see, [18]) hypothesized that the jet widths in Jupiter's atmosphere, at least 3 - 10 times the tropospheric deformation radius, would enable transfer of energy from eddies to jets. Instabilities are baroclinic within this range. The atmospheric radius of deformation is defined as $L = NH/|f|$ where N is the Brunt-Vaisala frequency, H is the scale height, and f is the Coriolis parameter. The Coriolis parameter, accordance by the formula $f = 2\Omega(\sin(\varphi)) = 35.2 \times 10^{-5} \times 0.25882 = 9.1 \times 10^{-5}\ \mathrm{s}^{-1}$ at 15° N pantographic latitude.

4. The Mathematical Justification for GRS Summation Energy of Motion for All Small Vortices and Ovals

Some researchers argue that the Red Spot is energized by absorbing smaller eddies. "Some computer models show that large eddies are able to live longer if they merge with smaller eddies. But this does not happen often enough to explain the Red Spot's longevity," says P. Markus.

Remark 1. Our presented mathematical model is explaining the longevity of the Great Red Spot and corresponding and asserts as real justifications of the hypotheses of P. Marcus, Hasanzadeh and others investigators about longevity of GRS.

First, let's start with the fact that White Ovals, small vortices (see **Figure 3**) transfer their energies to a large vortex, including the GRS, as result of which is provided with constant kinetic energy. For the purpose we will try to build a visual description of this process by a mathematical formula, the justification of which be concrete and clear. Consider a number of isolated free vortices of force $F_i\left(i = 1,2,3,\cdots,n\right)$ at points $M_i\left(x,y,z\right)\left(i = 1,2,3,\cdots,n\right)$ of an incompressible fluid moving rotationally in region D. These boundaries $\gamma_k\left(k = 1,2,3,\cdots,m\right)$ that the belt of Jupiter near of the GRS. IF we denote the usual flow function of the fluid motion as

$$\Psi = \Psi\left(x, y; x_i, y_i, z_i\right) = \Psi\left(\left(x_1, y_1\right), \cdots, \left(x_n, y_n\right), h\right), z_i = h = const, \tag{24}$$

which is independents of time t, then the components of the i-th vortex ($i = 1,2,3,\cdots,n$) (for example $n = 100$ ovals on Jupiter) have the following form:

$$\begin{gathered} \frac{\mathrm{d}x_i}{\mathrm{d}t} = u_i = -\left.\frac{\partial \Psi^{(i)}}{\partial y}\right|_{M_i}, \frac{\mathrm{d}y_i}{\mathrm{d}t} = \vartheta_i = \left.\frac{\partial \Psi^{(i)}}{\partial x}\right|_{M_i}, \\ \Psi^{(i)} = \Psi - \frac{F_i}{2\pi}\ln r_i, r_i = \sqrt{\left(x - x_i\right)^2 + \left(y - y_i\right)^2} \end{gathered} \tag{25}$$

Since the boundary (surface) on Jupiter is not solid, there is no stable flow in the region D. Therefore, in the hydrodynamic process, the flow of the function exists in the motion of *i*-th vortex and will have the following form:

$$F_i \frac{\mathrm{d}x_i}{\mathrm{d}t} = F_i u_i = -\frac{\partial E}{\partial y_i}, F_i \frac{\mathrm{d}y_i}{\mathrm{d}t} = F_i \upsilon_i = -\frac{\partial E}{\partial x_i} \tag{26}$$

Theorem 4. For the motion of vortices with force ($n = 100$, for Jupiter) in general domain which contains all ovals and vortices' having boundaries $\gamma_k\left(k = 1,2,3,\cdots,m\right)$, there exists a function $E_k = E\left(\left(x_1, y_1\right),\cdots,\left(x_n, y_n\right), h\right)$ such that

$$F_i u_i = -\frac{\partial E}{\partial y_i}, F_i \upsilon_i = -\frac{\partial E}{\partial x_i}, \tag{27}$$

where $M_i\left(x, y, z\right)\left(i = 1,2,3,\cdots,n\right)$ are the instantaneous of the vortices. The function defined in the following indicated immediately as transformed later to kinetic energy:

$$\begin{aligned} E_k = &\sum_{i=1}^{n} F_i \Psi\left(x_i, y_i; x_0, y_0\right) + \sum_{\substack{i,j=1 \\ (i>J)}}^{n} F_i F_j G\left(x_i, y_j; x_j, y_j\right) \\ &+ \sum_{i=1}^{n} F_i^2 G\left(x_i, y; x_i, y_i\right) \end{aligned} \tag{28}$$

Remark 2 It seems to us after few (example four years) years this energy by the theory of modeling tornado or tsunami may be transform to potential energy (see, **Figure 4**).

Proof. This can be immediately seen by comparing the results obtained similarly to the results (see [6]. Note that the system of Equations (26) (or (27)) is a Hamiltonian system of differential equations in the system of variables $\sqrt{F_i x_i}$ and $\sqrt{F_i y_i}$ in case of $n = 100$ (for Jupiter, including all vortices and ovals). Equality (28) is the kinetic energy of Jupiter's liquid motion for all vortices, so Equation (28) leads to the energy conservation laws $E_k = const$.

Note 2. "This is difficult to diagnose because Hubble cannot see the bottom of the storm very well. Anything below the top of the clouds is invisible in the data," Wong said. "But it's interesting data that could help us understand what fuels the Great Red Spot and how it keeps the energy going." According to NASA, there is still a lot of work to be done to fully understand this phenomenon. New NASA-funded research suggests that Jupiter's Great Red Spot may be the mysterious heat source behind Jupiter's surprisingly high upper atmospheric temperatures. Researchers from the US and Italy have figured out what force drives the huge Jupiterian cyclones. Earlier there was news that the Great Red Spot on Jupiter will soon disintegrate. Now scientists have refuted this fact: the storm is likely to exist for an indefinite amount of time.

5. Conclusions

In the introduction, the hypotheses of various major experts about the non-disappearance of the GRS and their recovery are deduced from an almost magmatic description and in Section 2, investigates about conditions of applicability and mathematical substantiation of the hypothesis about the longevity of the Great Red Spot, some clarity, essentially questions; in Section 3 given mathematical substantiation of the hypothesis about the longevity of the Great Red Spot by means of subsections which is presented about the new Jupiter GRS models, existence of circulation on this circuit, vortex lines for long-live of Jupiter's GRS, derivation of equation of wind and transformation energy for supporting cyclone and anticyclone long-lives and comparisons with equations of tsunami mathematical models; finally, investigating the instability of Kelvin-Helmholtz in the case of Brunta Vaisala frequency for the storm of Jupiter's GRS and indicating frequency stabilities. In Section 4, new approaches are given for proof of justification, which is considered the mathematical justification for GRS summation energy of motion for all small vortices and ovals (see, **Figure 7**). Accounting into Theorem 1, 2, 3, 4, all the investigations agreed with the previous hypotheses and results. It proves that GRS has constant interior energy and which proves that the anticyclone, GRS, will live longer. Finally, note that at least one will be satisfied the following: the movement of gas and liquid on the GRS is divided into three processes that combine laminar (or approximate, so-called quasi-laminar) and transitional flow along ovals with turbulent flow (See **Figure 8**).

In cyclones, the Coriolis force is directed from the center of the vortex. Therefore, a decrease is formed in it, and in anticyclones, on the contrary, an increase in the gas density;

Anticyclones are much longer-lived than cyclones, which is associated with the increased density inside them and, therefore, other things being equal, the total angular momentum of the anticyclone turns out to be higher than that of the cyclone (see **Figure 4**), so it is more difficult for it to disintegrate;

Rossby vortices slowly drift along the parallel to the west at speed not exceeding the phase velocity. The equations of motion admit a solution under the form of a compact ellipsoidal vortex of constant vertical vorticity, we assert that the corresponding Rossby number of the vortex as $R_o < 0$ for anticyclones, but $R_o > 0$ for cyclones provide as longer cycle.

By means of stream function and Green's function, we construct energy for one vortex motion and then summarize all ovals and energy for (for Jupiter n case, ovals and vortex) vortex motion. By the energy conservation laws, this summarized energy is constant. It means that the total motion of Jupiter's rotation under the indicated assumption will always be stability and, therefore, the hydro-dynamical equilibrium and stability of Jupiter and GRS will be accepted any time, as longer years.

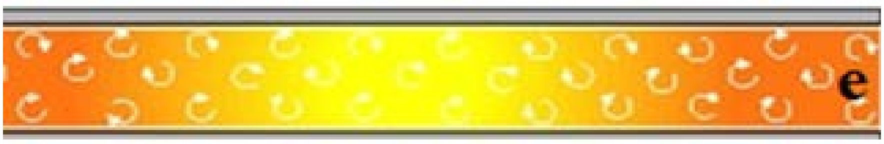

Figure 7. The formation Ovals and turbulence formation (see [6] [7] [8]).

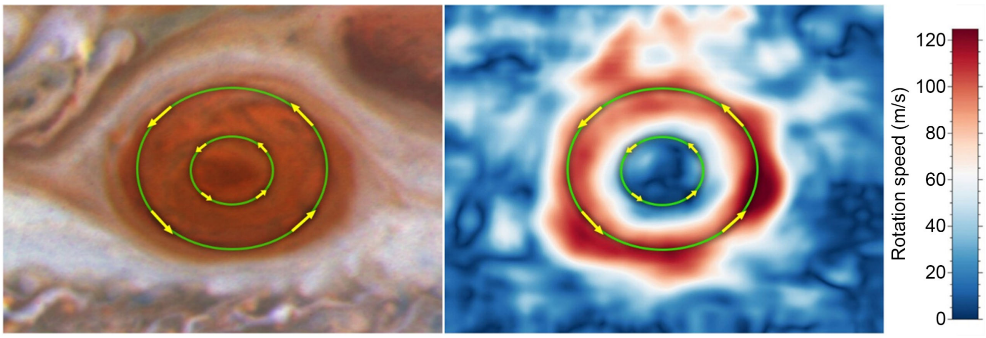

Figure 8. Winds in the Great Red Spot as analyzed from Hubble's data. Red means faster wind, blue means slower wind.

However, the thermodynamically and magnetic-dynamical equilibrium and steady rotation of Jupiter mathematically are almost finished.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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