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Fuzzy Soft Expert Matrix Theory and Its Application in Decision-Making

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Abstract

A unique mathematical strategy for dealing with uncertainty is fuzzy soft set theory. In this paper, we propose fuzzy soft expert matrices and describe numerous varieties of fuzzy soft expert matrices, as well as specific operations. Finally, by applying these matrices to decision-making scenarios, we widen our methodology.

Keywords

Fuzzy Soft Set, Fuzzy Softmatrix, Fuzzy Soft Expert Set, Fuzzy Soft Expert Matrix

1. Introduction

Most real-world problems in economics, social science, and the environment, for example, are fraught with uncertainty. Soft set theory [1] was initially given by Molodtsov in 1999 as a generic mathematical tool for dealing with ambiguous, fuzzily described, and uncertain things. Maji *et al.* [2] later investigated the notion of a fuzzy soft set. Fuzzy soft set theory can be widely used to solve decision-making problems [3] [4]. Majumdar *et al.* [5] extended the idea of fuzzy soft sets. Soft sets were extended to intuitionistic fuzzy soft sets by Maji *et al.* [6]. Matrices are useful in many fields of research and engineering. However, standard matrix theory does not always answer issues containing uncertainty. Yong *et al.* developed a matrix form of a fuzzy soft set and used it for particular decision-making issues [7]. Borah *et al.* expanded fuzzy soft matrix theory and its application in decision-making [8]. Cagman and Enginoglv [9] described fuzzy soft (fs) matrices and operations, as well as a fs-max-min decision-making approach.

Even though past models were effective, they frequently only engaged one expert. Many methods, such as joint and crossover, must be followed if you want to embrace the perspectives of many specialists. As a result, the user encounters challenges. To overcome this issue, Alkhazaleh and Salleh [10] [11] proposed the concepts of soft expert sets and fuzzy soft expert sets. The user may see all of the experts' opinions in one model without any alteration. Even after any changes, the user may still access all expert perspectives. Serdar and Hilal [12] altered fuzzy soft expert sets in ways that were critical for the expansion of the concept of soft sets by eliminating their contradictions.

By combining matrices with fuzzy soft expert sets, we presented fuzzy soft expert matrices and specified several forms of fuzzy soft expert matrices as well as certain operations in this paper. Finally, we broaden our approach by applying these matrices to decision-making situations.

2. Preliminaries

In this section, we will review several fundamental concepts from fuzzy soft set theory, fuzzy soft matrix theory, and fuzzy soft expert theory.

Definition 2.1 Let H be a universe set, V a parameter set, $P(H)$ denote the power set of H and $A \subseteq V$. A pair (f, A) is called a soft set over H, where f is a mapping

$$f : A \to P(H).$$

In other words, a soft set over H is a parameterized family of subsets of the universe H. For $\varepsilon \in A$, $f(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (f, A).

Definition 2.2 Let H be an initial universal set, and let V be a set of parameters. Let I^H denote the power set of all fuzzy subsets of H. Let $A \subseteq V$. A pair (f, V) is called a fuzzy soft set over H where f is a mapping given by

$$f : A \to I^H.$$

Definition 2.3 Let (f, V) be a fuzzy soft set over H, where $H = \{h_1, h_2, \cdots, h_m\}$ and $V = \{v_1, v_2, \cdots, v_n\}$, for $\forall v_i \in V$ and $\forall v_j \in V$, there exists the membership degree $a_{ij} = f_{v_j}(h_i)$, then we can present all membership degrees by a table as follows:

$f_{v_j}(h_i)$	v_1	v_2	$\cdots$	v_n
h_1	a_{11}	a_{12}	$\cdots$	a_{1n}
h_2	a_{21}	a_{22}	$\cdots$	a_{2n}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
h_m	a_{m1}	a_{m2}	$\cdots$	a_{mn}

The fuzzy matrix

$$A_{m\times n}=\left[a_{ij}\right]_{m\times n}=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

is said to be the fuzzy soft matrix of (f,V) over H. The set of all $m\times n$ soft matrices over H will be denoted by $FSM_{m\times n}$.

Example 2.1 Let $H=\{h_1,h_2,h_3,h_4\}$, $V=\{v_1,v_2,v_3\}$ and (f,V) be a fuzzy soft set over H given by

$$f(v_1)=\left\{\frac{h_1}{0.7},\frac{h_2}{0.5},\frac{h_3}{0.4},\frac{h_4}{0.8}\right\}, f(v_2)=\left\{\frac{h_1}{0.7},\frac{h_2}{0.3},\frac{h_3}{0.9},\frac{h_4}{0.4}\right\},$$
$$f(v_3)=\left\{\frac{h_1}{0.6},\frac{h_2}{0.4},\frac{h_3}{0.4},\frac{h_4}{0.7}\right\}.$$

Hence the corresponding fuzzy soft matrix $A_{3\times 4}$ is written by

$$A_{3\times 4}=\left[a_{ij}\right]_{3\times 4}=\begin{bmatrix} 0.7 & 0.5 & 0.4 & 0.8 \\ 0.7 & 0.3 & 0.9 & 0.4 \\ 0.6 & 0.4 & 0.4 & 0.7 \end{bmatrix}.$$

Definition 2.4 Let H be a universe, V a parameters set, E an experts set and O an opinions set. Let $U=V\times E\times O$ and $A\subseteq U$. Then (F,A) is said to be a fuzzy soft expert set over H, where F is a mapping given by

$$F:A\to I^H,$$

where I^H denotes the power set of H.

Example 2.2 Let $H=\{h_1,h_2,h_3,h_4\}$ be a universe set, $V=\{v_1,v_2,v_3\}$ a parameters set, $E=\{p,q,r\}$ be an experts set and $U=V\times E\times O$. Define the function $F:A\to I^H$ as follows:

$$F(v_1,p,1)=\left\{\frac{h_1}{0.5},\frac{h_2}{0.6},\frac{h_3}{0.4},\frac{h_4}{0.8}\right\},\quad F(v_1,q,1)=\left\{\frac{h_1}{0.7},\frac{h_2}{0.3},\frac{h_3}{0.5},\frac{h_4}{0.4}\right\},$$

$$F(v_1,r,1)=\left\{\frac{h_1}{0.6},\frac{h_2}{0.9},\frac{h_3}{0.4},\frac{h_4}{0.7}\right\},\quad F(v_2,p,1)=\left\{\frac{h_1}{0.7},\frac{h_2}{0.8},\frac{h_3}{0.4},\frac{h_4}{0.6}\right\},$$

$$F(v_2,q,1)=\left\{\frac{h_1}{0.4},\frac{h_2}{0.6},\frac{h_3}{0.5},\frac{h_4}{0.6}\right\},\quad F(v_2,r,1)=\left\{\frac{h_1}{0.7},\frac{h_2}{0.7},\frac{h_3}{0.8},\frac{h_4}{0.9}\right\},$$

$$F(v_3,p,1)=\left\{\frac{h_1}{0.2},\frac{h_2}{0.3},\frac{h_3}{0.4},\frac{h_4}{0.7}\right\},\quad F(v_3,q,1)=\left\{\frac{h_1}{0.6},\frac{h_2}{0.7},\frac{h_3}{0.3},\frac{h_4}{0.4}\right\},$$

$$F(v_3,r,1)=\left\{\frac{h_1}{0.4},\frac{h_2}{0.9},\frac{h_3}{0.3},\frac{h_4}{0.6}\right\},\quad F(v_1,p,0)=\left\{\frac{h_1}{0.5},\frac{h_2}{0.4},\frac{h_3}{0.6},\frac{h_4}{0.2}\right\},$$

$$F(v_1,q,0)=\left\{\frac{h_1}{0.3},\frac{h_2}{0.7},\frac{h_3}{0.5},\frac{h_4}{0.6}\right\},\quad F(v_1,r,0)=\left\{\frac{h_1}{0.4},\frac{h_2}{0.1},\frac{h_3}{0.6},\frac{h_4}{0.3}\right\},$$

$$F(v_2,p,0)=\left\{\frac{h_1}{0.3},\frac{h_2}{0.2},\frac{h_3}{0.6},\frac{h_4}{0.4}\right\},\quad F(v_2,q,0)=\left\{\frac{h_1}{0.6},\frac{h_2}{0.4},\frac{h_3}{0.5},\frac{h_4}{0.4}\right\},$$

$$F(v_2,r,0)=\left\{\frac{h_1}{0.3},\frac{h_2}{0.3},\frac{h_3}{0.2},\frac{h_4}{0.1}\right\},\quad F(v_3,p,0)=\left\{\frac{h_1}{0.8},\frac{h_2}{0.7},\frac{h_3}{0.6},\frac{h_4}{0.3}\right\},$$

$$F(v_3,q,0)=\left\{\frac{h_1}{0.4},\frac{h_2}{0.3},\frac{h_3}{0.7},\frac{h_4}{0.6}\right\},\quad F(v_3,r,0)=\left\{\frac{h_1}{0.6},\frac{h_2}{0.1},\frac{h_3}{0.7},\frac{h_4}{0.4}\right\}.$$

Then (F,U) is consists of the following approximate sets:

$$\begin{aligned}(F,U)=\Bigg\{&\left((v_1,p,1),\left\{\frac{h_1}{0.5},\frac{h_2}{0.6},\frac{h_3}{0.4},\frac{h_4}{0.8}\right\}\right),\left((v_1,q,1),\left\{\frac{h_1}{0.7},\frac{h_2}{0.3},\frac{h_3}{0.5},\frac{h_4}{0.4}\right\}\right),\\
&\left((v_1,r,1),\left\{\frac{h_1}{0.6},\frac{h_2}{0.9},\frac{h_3}{0.4},\frac{h_4}{0.7}\right\}\right),\left((v_2,p,1),\left\{\frac{h_1}{0.7},\frac{h_2}{0.8},\frac{h_3}{0.4},\frac{h_4}{0.6}\right\}\right),\\
&\left((v_2,q,1),\left\{\frac{h_1}{0.4},\frac{h_2}{0.6},\frac{h_3}{0.5},\frac{h_4}{0.6}\right\}\right),\left((v_2,r,1),\left\{\frac{h_1}{0.7},\frac{h_2}{0.7},\frac{h_3}{0.8},\frac{h_4}{0.9}\right\}\right),\\
&\left((v_3,p,1),\left\{\frac{h_1}{0.2},\frac{h_2}{0.3},\frac{h_3}{0.4},\frac{h_4}{0.7}\right\}\right),\left((v_3,q,1),\left\{\frac{h_1}{0.6},\frac{h_2}{0.7},\frac{h_3}{0.3},\frac{h_4}{0.4}\right\}\right),\\
&\left((v_3,r,1),\left\{\frac{h_1}{0.4},\frac{h_2}{0.9},\frac{h_3}{0.3},\frac{h_4}{0.6}\right\}\right),\left((v_1,p,0),\left\{\frac{h_1}{0.5},\frac{h_2}{0.4},\frac{h_3}{0.6},\frac{h_4}{0.2}\right\}\right),\\
&\left((v_1,q,0),\left\{\frac{h_1}{0.3},\frac{h_2}{0.7},\frac{h_3}{0.5},\frac{h_4}{0.6}\right\}\right),\left((v_1,r,0),\left\{\frac{h_1}{0.4},\frac{h_2}{0.1},\frac{h_3}{0.6},\frac{h_4}{0.3}\right\}\right),\\
&\left((v_2,p,0),\left\{\frac{h_1}{0.3},\frac{h_2}{0.2},\frac{h_3}{0.6},\frac{h_4}{0.4}\right\}\right),\left((v_2,q,0),\left\{\frac{h_1}{0.6},\frac{h_2}{0.4},\frac{h_3}{0.5},\frac{h_4}{0.4}\right\}\right),\\
&\left((v_2,r,0),\left\{\frac{h_1}{0.3},\frac{h_2}{0.3},\frac{h_3}{0.2},\frac{h_4}{0.1}\right\}\right),\left((v_3,p,0),\left\{\frac{h_1}{0.8},\frac{h_2}{0.7},\frac{h_3}{0.6},\frac{h_4}{0.3}\right\}\right),\\
&\left((v_3,q,0),\left\{\frac{h_1}{0.4},\frac{h_2}{0.3},\frac{h_3}{0.7},\frac{h_4}{0.6}\right\}\right),\left((v_3,r,0),\left\{\frac{h_1}{0.6},\frac{h_2}{0.1},\frac{h_3}{0.7},\frac{h_4}{0.4}\right\}\right)\Bigg\}.\end{aligned}$$

Definition 2.5 An agree-fuzzy soft expert set $(F,A)_1$ which is also a fuzzy soft expert subset of (F,A) over H is defined by in the following

$$(F,A)_1=\{(a,F(a)):a\in A_1\},$$

where $A_1\subseteq U_1$ such that $U_1=V\times E\times\{1\}$.

Definition 2.6 A disagree-fuzzy soft expert set $(F,A)_0$ which is a fuzzy soft expert subset of (F,A) over H is defined by in the following

$$(F,A)_0=\{(a,F(a)):a\in A_0\},$$

where $A_0\subseteq U_0$ such that $U_0=V\times E\times\{0\}$.

3. Several Matrix Types of Soft Expert Matrices

The definition of a fuzzy soft expert matrix is presented first in this section. Following that, many varieties of fuzzy soft expert matrices are introduced.

Definition 3.1 Let $H=\{h_1,h_2,\cdots,h_m\}$ be a universe, $V=\{v_1,v_2,\cdots,v_n\}$ a parameters set, E an experts set and O = {1 = agree, 0 = disagree} an opinions set. Let $U=V\times E\times O$ and $A\subseteq U$. (F,A) is a fuzzy soft expert set over H, where F is a mapping given by $F: A \to I^{H}$. Then the expert matrix of fuzzy soft expert set (F, A) is denoted as

$$X = \left[x_{ij}\right]_{m\times n} \text{ or } X = \left[x_{ij}\right]. \tag{1}$$

where $x_{ij} = \left(ag_{ij}, dg_{ij}\right)$, ag_{ij} represents the level of acceptance of h_i in the soft expert set $F\left(v_j\right)$, dg_{ij} represents the level non-acceptance of h_i in the soft expert set $F\left(v_j\right)$.

Example 3.1 Consider Example 2.2. With three experts making the decision, three fuzzy soft expert matrices can be obtained.

$$X = \begin{bmatrix} (0.5,0.5) & (0.6,0.4) & (0.4,0.6) & (0.8,0.2) \\ (0.7,0.3) & (0.8,0.2) & (0.4,0.6) & (0.6,0.4) \\ (0.2,0.8) & (0.3,0.7) & (0.4,0.6) & (0.7,0.3) \end{bmatrix},$$

$$Y = \begin{bmatrix} (0.7,0.3) & (0.3,0.7) & (0.5,0.5) & (0.4,0.6) \\ (0.4,0.6) & (0.6,0.4) & (0.5,0.5) & (0.6,0.4) \\ (0.6,0.4) & (0.7,0.3) & (0.3,0.7) & (0.4,0.6) \end{bmatrix},$$

$$Z = \begin{bmatrix} (0.6,0.4) & (0.9,0.1) & (0.4,0.6) & (0.7,0.3) \\ (0.7,0.3) & (0.7,0.3) & (0.8,0.2) & (0.9,0.1) \\ (0.4,0.6) & (0.9,0.1) & (0.3,0.7) & (0.6,0.4) \end{bmatrix}.$$

Definition 3.2 A fuzzy soft expert matrix ϕ_0 is called absolute disagree fuzzy soft expert matrix, if all of its elements are $(0,1)$.

Example 3.2 Consider

$$F(v_1, p, 1) = \left\{\frac{h_1}{0}, \frac{h_2}{0}, \frac{h_3}{0}, \frac{h_4}{0}\right\},\quad F(v_2, p, 1) = \left\{\frac{h_1}{0}, \frac{h_2}{0}, \frac{h_3}{0}, \frac{h_4}{0}\right\},$$

$$F(v_3, p, 1) = \left\{\frac{h_1}{0}, \frac{h_2}{0}, \frac{h_3}{0}, \frac{h_4}{0}\right\},\quad F(v_1, p, 0) = \left\{\frac{h_1}{1}, \frac{h_2}{1}, \frac{h_3}{1}, \frac{h_4}{1}\right\},$$

$$F(v_2, p, 0) = \left\{\frac{h_1}{1}, \frac{h_2}{1}, \frac{h_3}{1}, \frac{h_4}{1}\right\},\quad F(v_3, p, 0) = \left\{\frac{h_1}{1}, \frac{h_2}{1}, \frac{h_3}{1}, \frac{h_4}{1}\right\}.$$

Then the corresponding fuzzy soft expert matrix ϕ_0 is written by

$$\phi_0 = \begin{bmatrix} (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \end{bmatrix}.$$

Definition 3.3 A fuzzy soft expert matrix ϕ_1 is called absolute agree fuzzy soft expert matrix if all of its elements are $(1,0)$.

Example 3.3 Consider

$$F(v_1, p, 1) = \left\{\frac{h_1}{1}, \frac{h_2}{1}, \frac{h_3}{1}, \frac{h_4}{1}\right\},\quad F(v_2, p, 1) = \left\{\frac{h_1}{1}, \frac{h_2}{1}, \frac{h_3}{1}, \frac{h_4}{1}\right\},$$

$$F(v_3, p, 1) = \left\{\frac{h_1}{1}, \frac{h_2}{1}, \frac{h_3}{1}, \frac{h_4}{1}\right\},\quad F(v_1, p, 0) = \left\{\frac{h_1}{0}, \frac{h_2}{0}, \frac{h_3}{0}, \frac{h_4}{0}\right\},$$

$$F(v_2, p, 0) = \left\{\frac{h_1}{0}, \frac{h_2}{0}, \frac{h_3}{0}, \frac{h_4}{0}\right\}, \quad F(v_3, p, 0) = \left\{\frac{h_1}{0}, \frac{h_2}{0}, \frac{h_3}{0}, \frac{h_4}{0}\right\}.$$

Then the corresponding fuzzy soft expert matrix ϕ_1 is written by

$$\phi_1 = \begin{bmatrix} (1,0) & (1,0) & (1,0) & (1,0) \\ (1,0) & (1,0) & (1,0) & (1,0) \\ (1,0) & (1,0) & (1,0) & (1,0) \end{bmatrix}.$$

Definition 3.4 Let $X = [x_{ij}]_{m\times n}$ be a fuzzy soft expert matrix, then

(i) X is called a fuzzy soft expert rectangular matrix, if $m \neq n$.

(ii) X is called a fuzzy soft expert square matrix, if $m = n$.

(iii) X is called a fuzzy soft expert row matrix, if $m = 1$.

(iv) X is called a fuzzy soft expert column matrix, if $n = 1$.

(v) X is called a fuzzy soft expert diagonal matrix, if $m = n$ and $a_{ij} = (0,1)$, for all $i \neq j$.

(vi) X is called a fuzzy soft expert lower triangular matrix, if $m = n$, $a_{ij} = (0,1)$, for all $i < j$.

(vii) X is called a fuzzy soft expert upper triangular matrix, if $m = n$, $a_{ij} = (0,1)$, for all $i > j$.

4. Several Operations of the Soft Expert Matrices

In this section, operations such as addition, subtraction, product, complement, scalar multiple, transpose and trace of fuzzy soft expert matrices are given. Some related properties are also presented.

Definition 4.1 Let $X = [x_{ij}]_{m\times n}$ and $Y = [y_{ij}]_{m\times n}$ be two fuzzy soft expert matrices, then X is a fuzzy soft expert sub-matrix of Y, denoted by $X \subseteq Y$, if

$$ag_X \leq ag_Y, \ dg_X \geq dg_Y, \ \forall i, j. \tag{2}$$

Example 4.1 Let X and Y be two fuzzy soft expert matrices as follows:

$$X = \begin{bmatrix} (0.3, 0.7) & (0.2, 0.8) & (0.9, 0.1) \\ (0.7, 0.3) & (0.6, 0.4) & (0.5, 0.5) \end{bmatrix},$$

$$Y = \begin{bmatrix} (0.2, 0.8) & (0.1, 0.9) & (0.9, 0.1) \\ (0.6, 0.4) & (0.2, 0.8) & (0.5, 0.5) \end{bmatrix}.$$

Then $X \subseteq Y$.

Definition 4.2 Let $X = [x_{ij}]_{m\times n}$ and $Y = [y_{ij}]_{m\times n}$ be two fuzzy soft expert matrices, then X is a proper fuzzy soft expert sub-matrix of Y, denoted by $X \subset Y$, if $ag_X \leq ag_Y$ and $dg_X \geq dg_Y$, for at least one term $ag_X < ag_Y$, $dg_X > dg_Y$, $\forall i, j$.

Definition 4.3 Let $X = [x_{ij}]_{m\times n}$ and $Y = [y_{ij}]_{m\times n}$ be two fuzzy soft expert matrices, then X is equal to Y if $ag_X = ag_Y$, $dg_X = dg_Y$, $\forall i, j$.

Proposition 4.1 Let $X=\left[x_{ij}\right]_{m\times n}$, $Y=\left[y_{ij}\right]_{m\times n}$ and $Z=\left[z_{ij}\right]_{m\times n}$ be three fuzzy soft expert matrices, then

(i) $\phi_0\subseteq X$,

(ii) $X\subseteq\phi_1$,

(iii) $X\subseteq X$,

(iv) $X\subseteq Y, Y\subseteq Z\Rightarrow Z\subseteq Y$,

(v) $X=Y, Y=Z\Rightarrow X=Z$.

Definition 4.4 Let $X=\left[x_{ij}\right]_{m\times n}$ and $Y=\left[y_{ij}\right]_{m\times n}$ be two fuzzy soft expert matrices, then $X+Y=\left[c_{ij}\right]_{m\times n}$ is defined as the addition of X and Y, where

$$c_{ij}=\left(\max\left(ag_X,ag_Y\right),\min\left(dg_X,dg_Y\right)\right),\ \forall i,j. \tag{3}$$

Example 4.2 If X and Y are two fuzzy soft expert matrices as follows:

$$X=\begin{bmatrix}(0.5,0.5) & (0.6,0.4) & (0.4,0.6) & (0.8,0.2)\\ (0.7,0.3) & (0.8,0.2) & (0.4,0.6) & (0.6,0.4)\\ (0.2,0.8) & (0.3,0.7) & (0.4,0.6) & (0.7,0.3)\end{bmatrix}$$

$$Y=\begin{bmatrix}(0.7,0.3) & (0.3,0.7) & (0.5,0.5) & (0.4,0.6)\\ (0.4,0.6) & (0.6,0.4) & (0.5,0.5) & (0.6,0.4)\\ (0.6,0.4) & (0.7,0.3) & (0.3,0.7) & (0.4,0.6)\end{bmatrix}.$$

Then

$$X+Y=\begin{bmatrix}(0.7,0.3) & (0.6,0.4) & (0.5,0.5) & (0.8,0.2)\\ (0.7,0.3) & (0.8,0.2) & (0.5,0.5) & (0.6,0.4)\\ (0.6,0.4) & (0.7,0.3) & (0.4,0.6) & (0.7,0.3)\end{bmatrix}.$$

Proposition 4.2 Let $X=\left[x_{ij}\right]_{m\times n}$, $Y=\left[y_{ij}\right]_{m\times n}$ and $Z=\left[z_{ij}\right]_{m\times n}$ be three fuzzy soft expert matrices, then

(i) $X+\phi_0=X$,

(ii) $X+\phi_1=\phi_1$,

(iii) $X+X=X$,

(iv) $X+Y=Y+X$,

(v) $\left(X+Y\right)+Z=X+\left(Y+Z\right)$.

Definition 4.5 Let $X=\left[x_{ij}\right]_{m\times n}$ and $Y=\left[y_{ij}\right]_{m\times n}$ be two fuzzy soft expert matrices, then $X-Y=\left[c_{ij}\right]_{m\times n}$ is defined as the subtraction of X and Y, where

$$c_{ij}=\left(\min\left(ag_X,ag_Y\right),\max\left(dg_X,dg_Y\right)\right),\ \forall i,j. \tag{4}$$

Example 4.3 If X and Y are two fuzzy soft expert matrices as example 4.2, then

$$X-Y=\begin{bmatrix}(0.5,0.5) & (0.3,0.7) & (0.4,0.6) & (0.4,0.6)\\ (0.4,0.6) & (0.6,0.4) & (0.4,0.6) & (0.6,0.4)\\ (0.2,0.8) & (0.3,0.7) & (0.3,0.7) & (0.4,0.6)\end{bmatrix}.$$

Proposition 4.3 Let $X=\left[x_{ij}\right]_{m\times n}$, $Y=\left[y_{ij}\right]_{m\times n}$ and $Z=\left[z_{ij}\right]_{m\times n}$ be three fuzzy soft expert matrices, then

(i) $X-\phi_0=\phi_0$,

(ii) $X-\phi_1=X$,

(iii) $X-X=X$,

(iv) $X-Y=Y-X$,

(v) $\left(X-Y\right)-Z=X-\left(Y-Z\right)$,

(vi) $X-\left(Y+Z\right)=\left(X-Y\right)+\left(X-Z\right)$,

(vii) $X+\left(Y-Z\right)=\left(X+Y\right)-\left(X+Z\right)$.

Proof: Proof (vii) only, the others may be similarly certified.

Let $X=\left[x_{ij}\right]_{m\times n}=\left[ag_X,dg_X\right]$, $Y=\left[y_{ij}\right]_{m\times n}=\left[ag_Y,dg_Y\right]$, $Z=\left[z_{ij}\right]_{m\times n}=\left[ag_Z,dg_Z\right]$, then

$$\left(Y-Z\right)=\left[\min\left(ag_Y,ag_Z\right),\max\left(dg_Y,dg_Z\right)\right],$$

$$X+\left(Y-Z\right)=\left[\max\left(ag_X,\min\left(ag_Y,ag_Z\right)\right),\min\left(dg_X,\max\left(dg_Y,dg_Z\right)\right)\right],$$

$$X+Y=\left[\max\left(ag_X,ag_Y\right),\min\left(dg_X,dg_Y\right)\right],$$

$$X+Z=\left[\max\left(ag_X,ag_Z\right),\min\left(dg_X,dg_Z\right)\right],$$

$$\begin{aligned}&\left(X+Y\right)-\left(X+Z\right)\\&=\left[\min\left(\max\left(ag_X,ag_Y\right),\max\left(ag_X,ag_Z\right)\right),\max\left(\min\left(dg_X,dg_Y\right),\min\left(dg_X,dg_Z\right)\right)\right]\\&=\left[\max\left(ag_X,\min\left(ag_Y,ag_Z\right)\right),\min\left(dg_X,\max\left(dg_Y,dg_Z\right)\right)\right].\end{aligned}$$

Thus $X+\left(Y-Z\right)=\left(X+Y\right)-\left(X+Z\right)$.

Definition 4.6 Let $X=\left[x_{ij}\right]_{m\times n}$ and $Y=\left[y_{ij}\right]_{n\times p}$ be two fuzzy soft expert matrices, then we define the product of X and Y as $X*Y=\left[c_{ij}\right]_{m\times p}$, where

$$c_{ij}=\left(\max\min\left(ag_{X_i},ag_{Y_j}\right),\min\max\left(dg_{X_i},dg_{Y_j}\right)\right),\ \forall i,j. \tag{5}$$

Example 4.4 Let X and Y be two fuzzy soft expert matrices as follows:

$$X=\begin{bmatrix}(0.3,0.7) & (0.2,0.8) & (0.9,0.1)\\(0.7,0.3) & (0.6,0.4) & (0.5,0.5)\end{bmatrix},$$

$$Y=\begin{bmatrix}(0.7,0.3) & (0.5,0.5) & (0.4,0.6)\\(0.6,0.4) & (0.2,0.8) & (0.8,0.2)\\(0.3,0.7) & (0.9,0.1) & (0.4,0.6)\end{bmatrix}.$$

Then

$$X*Y=\begin{bmatrix}(0.3,0.7) & (0.9,0.1) & (0.4,0.6)\\(0.7,0.3) & (0.5,0.5) & (0.6,0.4)\end{bmatrix}.$$

Remark 4.1 When $X*Y$ exists, $Y*X$ does not necessarily exist. Even if both $X*Y$ and $Y*X$ exist, $X*Y$ and $Y*X$ may not be equal.

Proposition 4.4 Let $X=\left[x_{ij}\right]_{m\times n}$, $Y=\left[y_{ij}\right]_{n\times p}$ and $Z=\left[z_{ij}\right]_{p\times k}$ be three fuzzy soft expert matrices, then

$$(X*Y)*Z=X*(Y*Z). \tag{6}$$

Proposition 4.5 Let $X=\left[x_{ij}\right]_{m\times n}$, $Y=\left[y_{ij}\right]_{n\times p}$ and $Z=\left[z_{ij}\right]_{n\times p}$ be three fuzzy soft expert matrices, then

(i) $X*(Y-Z)=X*Y-X*Z$,

(ii) $X*(Y+Z)=X*Y+X*Z$.

Definition 4.7 Let $X=\left[x_{ij}\right]$ be a fuzzy soft expert matrix, where $x_{ij}=\left(ag_{ij},dg_{ij}\right)$. Then the complement of the matrix is denoted by $X^{o}=\left[x_{ij}^{o}\right]$, where

$$x_{ij}^{o}=\left(1-ag_{ij},1-dg_{ij}\right),\ \forall i,j. \tag{7}$$

Example 4.5 Let X be a fuzzy soft expert matrices as follows:

$$X=\begin{bmatrix}(0.7,0.3) & (0.5,0.5) & (0.4,0.6)\\ (0.6,0.4) & (0.2,0.8) & (0.8,0.2)\\ (0.3,0.7) & (0.9,0.1) & (0.4,0.6)\end{bmatrix}.$$

Then

$$X^{o}=\begin{bmatrix}(0.3,0.7) & (0.5,0.5) & (0.6,0.4)\\ (0.4,0.6) & (0.8,0.2) & (0.2,0.8)\\ (0.7,0.3) & (0.1,0.9) & (0.6,0.4)\end{bmatrix}.$$

Proposition 4.6 Let $X=\left[x_{ij}\right]_{m\times n}$ and $Y=\left[y_{ij}\right]_{m\times n}$ be two fuzzy soft expert matrices, then

(i) $\left(X^{o}\right)^{o}=X$,

(ii) $\left(\phi_{0}\right)^{o}=\phi_{1}$,

(iii) $\left(\phi_{1}\right)^{o}=\phi_{0}$,

(iv) $\left(X+\phi_{1}\right)^{o}=\phi_{0}$,

(v) $\left(X-\phi_{0}\right)^{o}=\phi_{1}$,

(vi) $\left(X+Y\right)^{o}=X^{o}-Y^{o}$,

(vii) $\left(X-Y\right)^{o}=X^{o}+Y^{o}$.

Definition 4.8 Let $X=\left[x_{ij}\right]$ be a fuzzy soft expert matrix, then the scalar multiple kX is defined by $kX=\left[kx_{ij}\right]$, where $0\le k\le 1$.

Example 4.6 Let X be a fuzzy soft expert matrix as follows:

$$X=\begin{bmatrix}(0.7,0.3) & (0.5,0.5) & (0.4,0.6)\\ (0.6,0.4) & (0.2,0.8) & (0.8,0.2)\\ (0.3,0.7) & (0.9,0.1) & (0.4,0.6)\end{bmatrix}.$$

Then the scalar multiple of this matrix by a scalar is

$$0.2X=\begin{bmatrix}(0.14,0.06) & (0.1,0.1) & (0.08,0.12)\\ (0.12,0.08) & (0.04,0.16) & (0.16,0.04)\\ (0.06,0.14) & (0.18,0.02) & (0.08,0.12)\end{bmatrix}.$$

Proposition 4.7 Let $X = [x_{ij}]_{m\times n}$ and $Y = [y_{ij}]_{m\times n}$ be two fuzzy soft expert matrices, if k_1, k_2 are two scalars such that $0 \le k_1, k_2 \le 1$, then

(i) $k_1(k_2 X) = (k_1 k_2) X$,

(ii) $X \subseteq Y \Rightarrow k_1 X \subseteq k_1 Y$.

Definition 4.9 Let $X = [x_{ij}]_{m\times n}$ be a fuzzy soft expert matrix, then the transpose matrix of X denoted by $X^{\mathrm{T}} = [x_{ij}]_{n\times m}$ is also a fuzzy soft expert matrix.

Example 4.7 Let X be a fuzzy soft expert matrix as follows:

$$X = \begin{bmatrix} (0.7,0.3) & (0.5,0.5) & (0.4,0.6) \\ (0.6,0.4) & (0.2,0.8) & (0.8,0.2) \end{bmatrix}.$$

Then the transpose matrix of X is

$$X^{\mathrm{T}} = \begin{bmatrix} (0.7,0.3) & (0.6,0.4) \\ (0.5,0.5) & (0.2,0.8) \\ (0.4,0.6) & (0.8,0.2) \end{bmatrix}.$$

Proposition 4.8 Let $X = [x_{ij}]_{m\times n}$ and $Y = [y_{ij}]_{m\times n}$ be two fuzzy soft expert matrices, if k is a scalar such that $0 \le k \le 1$, then

(i) $(kX)^{\mathrm{T}} = kX^{\mathrm{T}}$,

(ii) $(X^{\mathrm{T}})^{\mathrm{T}} = X$,

(iii) $(X^{o})^{\mathrm{T}} = (X^{\mathrm{T}})^{o}$,

(iv) $(X+Y)^{\mathrm{T}} = X^{\mathrm{T}} + Y^{\mathrm{T}}$,

(v) $(X-Y)^{\mathrm{T}} = X^{\mathrm{T}} - Y^{\mathrm{T}}$,

(vi) $X \subseteq Y \Rightarrow X^{\mathrm{T}} \subseteq Y^{\mathrm{T}}$.

Definition 4.10 Let $X = [x_{ij}]_{m\times n}$ be a fuzzy soft expert matrix, where $m = n$. Then the trace of X is defined as

$$trX = \sum_{i=1}^{m} x_{ii} = \sum_{i=1}^{m} (ag_{ii} - dg_{ii}). \tag{8}$$

Example 4.8 Let X be a fuzzy soft expert matrix as follows:

$$X = \begin{bmatrix} (0.7,0.3) & (0.5,0.5) & (0.4,0.6) \\ (0.6,0.4) & (0.2,0.8) & (0.8,0.2) \\ (0.3,0.7) & (0.9,0.1) & (0.4,0.6) \end{bmatrix}.$$

Then the $trX = \sum_{i=1}^{m} x_{ii} = (0.7-0.3)+(0.2-0.8)+(0.4-0.6) = -0.4$ trace of this matrix is

$$trX = \sum_{i=1}^{m} x_{ii} = (0.7-0.3)+(0.2-0.8)+(0.4-0.6) = -0.4.$$

Proposition 4.9 Let $X = [x_{ij}]_{n\times n}$ be a fuzzy soft expert matrix, k be a scalar such that $0 \le k \le 1$. Then

(i) $tr(kX) = k \cdot trX$,

(ii) $tr\left(\left(kX\right)^{\mathrm{T}}\right)=k\cdot tr\left(X^{\mathrm{T}}\right)$.

5. Fuzzy Soft Expert Matrix Theory in Decision-Making

In this part, we define the value matrix, the scoring matrix, and the total score in relation to the fuzzy soft expert matrix. The fuzzy soft expert matrix theory is then applied to a decision-making situation.

Definition 5.1 Let $X=\left[x_{ij}\right]_{m\times n}$ be a fuzzy soft expert matrix, then

$$V\left(X\right)=\left[ag_{ij}-dg_{ij}\right] \tag{9}$$

is defined as the value matrix of X, $i=1,2,\cdots,m$; $j=1,2,\cdots,n$.

Definition 5.2 Let $X=\left[x_{ij}\right]_{m\times n}$ and $Y=\left[y_{ij}\right]_{m\times n}$ be two fuzzy soft expert matrices, then

$$S_{(X,Y)}=\left[r_{ij}\right]_{n\times m}=V\left(X\right)-V\left(Y\right) \tag{10}$$

is defined as the score matrix of X and Y.

Definition 5.3 Let $X=\left[x_{ij}\right]_{m\times n}$ and $Y=\left[y_{ij}\right]_{m\times n}$ be two fuzzy soft expert matrices, $V\left(X\right)$, $V\left(Y\right)$ the corresponding value matrices, their score matrix is $S_{(X,Y)}=\left[r_{ij}\right]_{n\times m}$. Then the total score for each h_i in H is

$$S_i=\sum_{j=1}^{n}r_{ij}\ . \tag{11}$$

Example 5.1 Consider a business that is prepared to invest in a factory in another nation. There are six additional addresses $H=\left\{h_1,h_2,h_3,h_4,h_5,h_6\right\}$ available after the first discussions. Four decision factors $V=\left\{v_1,v_2,v_3,v_4\right\}$ are chosen after thorough analysis. The parameters $v_i\left(i=1,2,3,4\right)$ stand for reasonable prices, easy access to transportation, a healthy environment, and sound public policy, respectively. The business determined that three experts $E=\left\{p,q,r\right\}$ would make up a group of committee members to find the best candidate for the decision in order to make a fair selection. Let O = {1 = agree, 0 = disagree} an opinions set, $U=V\times E\times O$.

The committee members may use the following algorithm.

Step 1: input the fuzzy soft expert set $\left(F,U\right)$.

Following a thorough debate, the committee may develop the following fuzzy soft expert set $\left(F,U\right)$:

$$
\begin{aligned}
&\left(F,U\right)\\
&=\left\{\left(\left(v_1,p,1\right),\left\{\frac{h_1}{0.3},\frac{h_2}{0.6},\frac{h_3}{0.4},\frac{h_4}{0.2},\frac{h_5}{0.9},\frac{h_6}{0.8}\right\}\right),\left(\left(v_1,q,1\right),\left\{\frac{h_1}{0.7},\frac{h_2}{0.3},\frac{h_3}{0.5},\frac{h_4}{0.6},\frac{h_5}{0.8},\frac{h_6}{0.4}\right\}\right),\right.\\
&\quad\left(\left(v_1,r,1\right),\left\{\frac{h_1}{0.6},\frac{h_2}{0.9},\frac{h_3}{0.4},\frac{h_4}{0.7},\frac{h_5}{0.3},\frac{h_6}{0.8}\right\}\right),\left(\left(v_2,p,1\right),\left\{\frac{h_1}{0.7},\frac{h_2}{0.8},\frac{h_3}{0.4},\frac{h_4}{0.6},\frac{h_5}{0.3},\frac{h_6}{0.1}\right\}\right),\\
&\quad\left(\left(v_2,q,1\right),\left\{\frac{h_1}{0.4},\frac{h_2}{0.6},\frac{h_3}{0.8},\frac{h_4}{0.6},\frac{h_5}{0.1},\frac{h_6}{0.3}\right\}\right),\left(\left(v_2,r,1\right),\left\{\frac{h_1}{0.7},\frac{h_2}{0.7},\frac{h_3}{0.8},\frac{h_4}{0.9},\frac{h_5}{0.6},\frac{h_6}{0.4}\right\}\right),\\
&\quad\left(\left(v_3,p,1\right),\left\{\frac{h_1}{0.2},\frac{h_2}{0.3},\frac{h_3}{0.4},\frac{h_4}{0.7},\frac{h_5}{0.9},\frac{h_6}{0.2}\right\}\right),\left(\left(v_3,q,1\right),\left\{\frac{h_1}{0.6},\frac{h_2}{0.7},\frac{h_3}{0.3},\frac{h_4}{0.4},\frac{h_5}{0.3},\frac{h_6}{0.2}\right\}\right),
\end{aligned}
$$

$$\left((v_3,r,1),\left\{\frac{h_1}{0.4},\frac{h_2}{0.9},\frac{h_3}{0.3},\frac{h_4}{0.6},\frac{h_5}{0.7},\frac{h_6}{0.2}\right\}\right),\left((v_4,p,1),\left\{\frac{h_1}{0.5},\frac{h_2}{0.4},\frac{h_3}{0.6},\frac{h_4}{0.2},\frac{h_5}{0.3},\frac{h_6}{0.7}\right\}\right),$$
$$\left((v_4,q,1),\left\{\frac{h_1}{0.5},\frac{h_2}{0.9},\frac{h_3}{0.4},\frac{h_4}{0.8},\frac{h_5}{0.1},\frac{h_6}{0.6}\right\}\right),\left((v_4,r,1),\left\{\frac{h_1}{0.9},\frac{h_2}{0.7},\frac{h_3}{0.2},\frac{h_4}{0.4},\frac{h_5}{0.3},\frac{h_6}{0.6}\right\}\right),$$
$$\left((v_1,p,0),\left\{\frac{h_1}{0.7},\frac{h_2}{0.4},\frac{h_3}{0.6},\frac{h_4}{0.8},\frac{h_5}{0.1},\frac{h_6}{0.2}\right\}\right),\left((v_1,q,0),\left\{\frac{h_1}{0.3},\frac{h_2}{0.7},\frac{h_3}{0.5},\frac{h_4}{0.4},\frac{h_5}{0.2},\frac{h_6}{0.6}\right\}\right),$$
$$\left((v_1,r,0),\left\{\frac{h_1}{0.4},\frac{h_2}{0.1},\frac{h_3}{0.6},\frac{h_4}{0.3},\frac{h_5}{0.7},\frac{h_6}{0.2}\right\}\right),\left((v_2,p,0),\left\{\frac{h_1}{0.3},\frac{h_2}{0.2},\frac{h_3}{0.6},\frac{h_4}{0.4},\frac{h_5}{0.7},\frac{h_6}{0.9}\right\}\right),$$
$$\left((v_2,q,0),\left\{\frac{h_1}{0.6},\frac{h_2}{0.4},\frac{h_3}{0.2},\frac{h_4}{0.4},\frac{h_5}{0.9},\frac{h_6}{0.7}\right\}\right),\left((v_2,r,0),\left\{\frac{h_1}{0.3},\frac{h_2}{0.3},\frac{h_3}{0.2},\frac{h_4}{0.1},\frac{h_5}{0.4},\frac{h_6}{0.6}\right\}\right),$$
$$\left((v_3,p,0),\left\{\frac{h_1}{0.8},\frac{h_2}{0.7},\frac{h_3}{0.6},\frac{h_4}{0.3},\frac{h_5}{0.1},\frac{h_6}{0.8}\right\}\right),\left((v_3,q,0),\left\{\frac{h_1}{0.4},\frac{h_2}{0.3},\frac{h_3}{0.7},\frac{h_4}{0.6},\frac{h_5}{0.7},\frac{h_6}{0.8}\right\}\right),$$
$$\left((v_3,r,0),\left\{\frac{h_1}{0.6},\frac{h_2}{0.1},\frac{h_3}{0.7},\frac{h_4}{0.4},\frac{h_5}{0.3},\frac{h_6}{0.8}\right\}\right),\left((v_4,p,0),\left\{\frac{h_1}{0.5},\frac{h_2}{0.6},\frac{h_3}{0.4},\frac{h_4}{0.8},\frac{h_5}{0.7},\frac{h_6}{0.3}\right\}\right),$$
$$\left((v_4,q,0),\left\{\frac{h_1}{0.5},\frac{h_2}{0.1},\frac{h_3}{0.6},\frac{h_4}{0.2},\frac{h_5}{0.9},\frac{h_6}{0.4}\right\}\right),\left((v_4,r,0),\left\{\frac{h_1}{0.1},\frac{h_2}{0.3},\frac{h_3}{0.8},\frac{h_4}{0.6},\frac{h_5}{0.7},\frac{h_6}{0.4}\right\}\right)\Bigg\}.$$

Step 2: find the fuzzy soft expert matrices of (F,U) and find the complement of these matrices, respectively.

The fuzzy soft expert matrices of (F,U) are

$$X=\begin{bmatrix}(0.3,0.7) & (0.6,0.4) & (0.4,0.6) & (0.2,0.8) & (0.9,0.1) & (0.8,0.2)\\(0.7,0.3) & (0.8,0.2) & (0.4,0.6) & (0.6,0.4) & (0.3,0.7) & (0.1,0.9)\\(0.2,0.8) & (0.3,0.7) & (0.4,0.6) & (0.7,0.3) & (0.9,0.1) & (0.2,0.8)\\(0.5,0.5) & (0.4,0.6) & (0.6,0.4) & (0.2,0.8) & (0.3,0.7) & (0.7,0.3)\end{bmatrix},$$

$$Y=\begin{bmatrix}(0.7,0.3) & (0.3,0.7) & (0.5,0.5) & (0.6,0.4) & (0.8,0.2) & (0.4,0.6)\\(0.4,0.6) & (0.6,0.4) & (0.8,0.2) & (0.6,0.4) & (0.1,0.9) & (0.3,0.7)\\(0.6,0.4) & (0.7,0.3) & (0.3,0.7) & (0.4,0.6) & (0.3,0.7) & (0.2,0.8)\\(0.5,0.5) & (0.9,0.1) & (0.4,0.6) & (0.8,0.2) & (0.1,0.9) & (0.6,0.4)\end{bmatrix},$$

$$Z=\begin{bmatrix}(0.6,0.4) & (0.9,0.1) & (0.4,0.6) & (0.7,0.3) & (0.3,0.7) & (0.8,0.2)\\(0.7,0.3) & (0.7,0.3) & (0.8,0.2) & (0.9,0.1) & (0.6,0.4) & (0.4,0.6)\\(0.4,0.6) & (0.9,0.1) & (0.3,0.7) & (0.6,0.4) & (0.7,0.3) & (0.2,0.8)\\(0.9,0.1) & (0.7,0.3) & (0.2,0.8) & (0.4,0.6) & (0.3,0.7) & (0.6,0.4)\end{bmatrix}.$$

And the complement of these matrices are

$$X^o=\begin{bmatrix}(0.7,0.3) & (0.4,0.6) & (0.6,0.4) & (0.8,0.2) & (0.1,0.9) & (0.2,0.8)\\(0.3,0.7) & (0.2,0.8) & (0.6,0.4) & (0.4,0.6) & (0.7,0.3) & (0.9,0.1)\\(0.8,0.2) & (0.7,0.3) & (0.6,0.4) & (0.3,0.7) & (0.1,0.9) & (0.8,0.2)\\(0.5,0.5) & (0.6,0.4) & (0.4,0.6) & (0.8,0.2) & (0.7,0.3) & (0.3,0.7)\end{bmatrix},$$

$$Y^o = \begin{bmatrix} (0.3,0.7) & (0.7,0.3) & (0.5,0.5) & (0.4,0.6) & (0.2,0.8) & (0.6,0.4) \\ (0.6,0.4) & (0.4,0.6) & (0.2,0.8) & (0.4,0.6) & (0.9,0.1) & (0.7,0.3) \\ (0.4,0.6) & (0.3,0.7) & (0.7,0.3) & (0.6,0.4) & (0.7,0.3) & (0.8,0.2) \\ (0.5,0.5) & (0.1,0.9) & (0.6,0.4) & (0.2,0.8) & (0.9,0.1) & (0.4,0.6) \end{bmatrix},$$

$$Z^o = \begin{bmatrix} (0.4,0.6) & (0.1,0.9) & (0.6,0.4) & (0.3,0.7) & (0.7,0.3) & (0.2,0.8) \\ (0.3,0.7) & (0.3,0.7) & (0.2,0.8) & (0.1,0.9) & (0.4,0.6) & (0.6,0.4) \\ (0.6,0.4) & (0.1,0.9) & (0.7,0.3) & (0.4,0.6) & (0.3,0.7) & (0.8,0.2) \\ (0.1,0.9) & (0.3,0.7) & (0.8,0.2) & (0.6,0.4) & (0.7,0.3) & (0.4,0.6) \end{bmatrix}.$$

Step 3: compute the addition of the fuzzy soft expert matrices $X+Y+Z$ and $X^o+Y^o+Z^o$.

$$X+Y+Z = \begin{bmatrix} (0.7,0.3) & (0.9,0.1) & (0.5,0.5) & (0.7,0.3) & (0.9,0.1) & (0.8,0.2) \\ (0.7,0.3) & (0.8,0.2) & (0.8,0.2) & (0.9,0.1) & (0.6,0.4) & (0.4,0.6) \\ (0.6,0.4) & (0.9,0.1) & (0.4,0.6) & (0.7,0.3) & (0.9,0.1) & (0.2,0.8) \\ (0.9,0.1) & (0.9,0.1) & (0.6,0.4) & (0.8,0.2) & (0.3,0.7) & (0.7,0.3) \end{bmatrix},$$

$$X^o+Y^o+Z^o = \begin{bmatrix} (0.7,0.3) & (0.7,0.3) & (0.6,0.4) & (0.8,0.2) & (0.7,0.3) & (0.6,0.4) \\ (0.6,0.4) & (0.4,0.6) & (0.6,0.4) & (0.4,0.6) & (0.9,0.1) & (0.9,0.1) \\ (0.8,0.2) & (0.7,0.3) & (0.7,0.3) & (0.6,0.4) & (0.7,0.3) & (0.8,0.2) \\ (0.5,0.5) & (0.6,0.4) & (0.8,0.2) & (0.8,0.2) & (0.9,0.1) & (0.4,0.6) \end{bmatrix}.$$

Step 4: compute the value matrices and the score matrix.

$$V(X+Y+Z) = \begin{bmatrix} 0.4 & 0.8 & 0 & 0.4 & 0.8 & 0.6 \\ 0.4 & 0.6 & 0.6 & 0.8 & 0.2 & -0.2 \\ 0.2 & 0.8 & -0.2 & 0.4 & 0.8 & -0.6 \\ 0.8 & 0.8 & 0.2 & 0.6 & -0.4 & 0.4 \end{bmatrix},$$

$$V(X^o+Y^o+Z^o) = \begin{bmatrix} 0.4 & 0.4 & 0.2 & 0.6 & 0.4 & 0.2 \\ 0.2 & -0.2 & 0.2 & -0.2 & 0.8 & 0.8 \\ 0.6 & 0.4 & 0.4 & 0.2 & 0.4 & 0.6 \\ 0 & 0.2 & 0.6 & 0.6 & 0.8 & -0.2 \end{bmatrix},$$

$$V(X+Y+Z) - V(X^o+Y^o+Z^o) = \begin{bmatrix} 0 & 0.4 & -0.2 & -0.2 & 0.4 & 0.4 \\ 0.2 & 0.8 & 0.4 & 1 & -0.6 & -1 \\ -0.4 & 0.4 & -0.6 & 0.2 & 0.4 & -1.2 \\ 0.8 & 0.6 & -0.4 & 0 & -1.2 & 0.6 \end{bmatrix}.$$

Step 5: compute the total score S_i for each h_i in H. The decision will select with highest score. If more than one maximum value is present, these can be used as reference options.

Now, the score of u_i can be computed by using the score matrix in step 4:

$$score(h_1) = 0 + 0.2 + (-0.4) + 0.8 = 0.6,$$

$$score(h_2) = 0.4 + 0.8 + 0.4 + 0.6 = 2.2,$$

$$score(h_3) = -0.2 + 0.4 + (-0.6) + (-0.4) = -0.8,$$

$$score(h_4) = -0.2 + 1 + 0.2 + 0 = 1,$$

$$score(h_5) = 0.4 + (-0.6) + 0.4 + (-1.2) = -1,$$

$$score(h_6) = 0.4 + (-1) + (-1.2) + 0.6 = -1.2.$$

Because of

$$score(h_2) > score(h_4) > score(h_1) > score(h_3) > score(h_5) = score(h_6),$$

then h_2 can be used as optimal location.

6. Conclusion

The fuzzy soft expert theory is employed in a wide range of domains, from theory to practice. In this article, we define fuzzy soft expert matrices as matrix representations of fuzzy soft expert sets. Then, in order to obtain some fresh conclusions, many types of fuzzy soft expert matrices are introduced, and multiple processes are established. On this premise, a decision model is developed, and an example of its application is presented. This approach is based on the notion of fuzzy soft expert matrix operations, which are simple to implement. Each expert's opinion can be expressed very explicitly in a matrix, and the decision result can be reached through some calculations. Therefore, this decision-making method is simpler and more feasible than previous methods.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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