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Study of Complex TOUGMA's Metric

Jean Luc Wendkouni Tougma

Burkina Institute of Technology, Koudougou, Burkina Faso
Email: jeanluctougma@outlook.fr

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Abstract

General relativity of Einstein's theory and Quantum physics theory are excellent pillars that explain much modern physics. Understanding the relation between these theories is still a theoretical physics central open question. Over last several decades, works in this direction have led to new physical ideas and mathematical tools broad range. In recent years TOUGMA's equation is established and solved, one of its solutions such as a real solution is studied in our last article. In this paper, complex TOUGMA's metric is studied, particularly the physics concepts that this metric implies such as light geodesic and metric's impacts at $r = 0$. The first time, we studied the fact of $r = 0$ and its limits, secondly, we consider a zero length light geodesic is a geodesic and ended by studying it mathematically. These underlying principles study, those various phenomena in universe are interconnected logic leading to develop new technologies for example: news engines, telecommunication networks. This study's applications are exceptionally wide such as Astrophysics, cosmology, Quantum gravity, Quantum Mechanics, Multiverse. Mostly this study lets us to know the quantum relativity universe behaviors.

Keywords

Complex Metric of TOUGMA, Quantum Relativity, Ricci Tensor

1. Introduction

In recent years new developments set has begun to give unexpected connections between some problems relating to gravity aspects, black holes, quantum information, and condensed matter systems. It's clear that quantum entanglement, quantum error correction, and computational complexity play a fundamental role in the spacetime geometry emergence such as we have demonstrated in our first article [1]. Moreover, these tools have given a substantial progress on black hole information problem, giving new avenues for searching for a tension resolution between black holes physics and quantum mechanics as TOUGMA's equation; and all its solutions study is important to understand Universe properties. Note that quantum physics theory and gravity theory interface is currently leading to generating new progress areas, and is expected to remain important in the coming decade.

TOUGMA's theory, first published in 2021, is a geometric theory [2],

$$\left[(\alpha-3)R_{uv}-\frac{1}{2}g_{uv}R\right]\left(1+\frac{2kL_m}{R}\right)-2k(\alpha-3)g_{uv}L_m=T_{uv} \tag{1}$$

The solution imposed spherically symmetric metric, caused by no electrical charge and non-rotating n-dimensions massive object in vacuum, to approach such a solution, the method used is calculating Ricci Tensor components for a metric general form for some conditions and give their equating to 0 [3]. In other works we solve this equation by finding to solution one is real and it was studied [4]:

$$\begin{aligned} ds^2 = & -\left[\tan\left(-\frac{kL_m}{4(\alpha-2)}\left[1-\ln(1-r)\right]\right)\right]^2 c^2 dt^2 \\ & + \frac{dr^2}{\left[\tan\left(-\frac{kL_m}{4(\alpha-2)}\left[1-\ln(1-r)\right]\right)\right]^2} \\ & + r^2\left(d\theta^2 + r^2\sin^2(\theta)d\varphi^2\right) + r^{\alpha-4}d\Omega_{\alpha-4} \end{aligned} \tag{2}$$

the other is complex

$$\begin{aligned} ds^2 = & -\exp\left(2i\left[\tan^{-1}\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)-\frac{\pi}{4}\right]\right)c^2dt^2 \\ & + \exp\left(-2i\left[\tan^{-1}\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)-\frac{\pi}{4}\right]\right)dr^2 \\ & + r^2\left(d\theta^2 + r^2\sin^2(\theta)d\varphi^2\right) + r^{\alpha-4}d\Omega_{\alpha-4} \end{aligned} \tag{3}$$

Exploring the possibility of showing the effects of complex TOUGMA's metric on systems is then necessary to know other universe properties that real solution does not show. In this paper, we are going to find Radial Light Geodesics by assuming that light geodesic is a geodesic of zero length with $d\theta = 0$ and $d\varphi = 0 = d\Omega_{n-4}$.

$$\left[(\alpha-3)R_{uv}-\frac{1}{2}g_{uv}R\right]\left(1+\frac{2kL_m}{R}\right)-2k(\alpha-3)g_{uv}L_m=T_{uv} \tag{4}$$

This metric is given by:

$$ds^2 = -\exp\left(2i\left[\tan^{-1}\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)-\frac{\pi}{4}\right]\right)c^2dt^2$$

$$+\exp\left(-2i\left[\tan^{-1}\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)-\frac{\pi}{4}\right]\right)dr^2 \\ +r^2\left(d\theta^2+r^2\sin^2(\theta)d\varphi^2\right)+r^{\alpha-4}d\Omega_{\alpha-4} \tag{5}$$

We are going to start with physics concepts at $r = 0$, and finish secondly we are going to determine the light geodesics with the material bodies' geodesics orbits.

2. Methods

A complex solution of TOUGMA's equation that can be defined by the existence of a coordinate system $(x^u)=(ct,r,\theta,\varphi,\Omega_{\alpha-4})$, such that the components g_{uv} of the metric tensor $\boldsymbol{g}$ are written there

$$ds^2=-\exp\left(2i\left[\tan^{-1}\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)-\frac{\pi}{4}\right]\right)c^2dt^2 \\ +\exp\left(-2i\left[\tan^{-1}\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)-\frac{\pi}{4}\right]\right)dr^2 \\ +r^2\left(d\theta^2+r^2\sin^2(\theta)d\varphi^2\right)+r^{\alpha-4}d\Omega_{\alpha-4} \tag{6}$$

firstly we can observe that we can make in view of (3) that the space-time (E, g) is spherically symmetric and static. The metric components are clearly independent of t and, $\vec{\partial}_t\vec{\partial}_t=-\exp\left(2i\left[\tan^{-1}\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)-\frac{\pi}{4}\right]\right)c^2<0$, then time-like is ∂_t, and spacetime is stationary. Moreover, space-time described by the TOUGMA metric is asymptotical [5] [6] [7]

We have in effect if $r = 0$

$$\exp\left(2i\left[\tan^{-1}\left(e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}\right)-\frac{\pi}{4}\right]\right) \tag{7}$$

physical phenomena of this function are going to be studied in the next article. as limites, we have if we take $u=\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}$:

$$\lim_{u\to\pm\frac{\pi}{2}}\exp\left(2i\left[\tan^{-1}(u)-\frac{\pi}{4}\right]\right)=+\exp\left(-i\frac{\pi}{2}\right)=-i \tag{8}$$

with

$$\lim_{r\to 1}u=\pm\frac{\pi}{2} \tag{9}$$

and

$$\lim_{r\to+\infty}\exp\left(2i\left[-\frac{\pi}{4}\right]\right)=+\exp\left(-i\frac{\pi}{2}\right)=-i \tag{10}$$

Now we place ourselves in the TOUGMA coordinates frame $\left(x^u\right)=\left(ct,r,\theta,\varphi,\Omega_{n-4}\right)$. A light geodesic is a geodesic of zero length:

$$\mathrm{d}s^2=g_{uv}\mathrm{d}x^u\mathrm{d}x^v=0 \tag{11}$$

we assume the radial geodesic, then $\mathrm{d}\theta=0$ and $\mathrm{d}\varphi=0=\mathrm{d}\Omega_{n-4}$ along it.

It happens:

$$-\exp\left(2i\left[\tan^{-1}\left(\frac{\mathrm{e}^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)-\frac{\pi}{4}\right]\right)c^2\mathrm{d}t^2+\exp\left(-2i\left[\tan^{-1}\left(\frac{\mathrm{e}^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)-\frac{\pi}{4}\right]\right)\mathrm{d}r^2=0 \tag{12}$$

$$-\exp\left(2i\left[\tan^{-1}\left(\frac{\mathrm{e}^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)-\frac{\pi}{4}\right]\right)c^2\mathrm{d}t^2=-\exp\left(-2i\left[\tan^{-1}\left(\frac{\mathrm{e}^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)-\frac{\pi}{4}\right]\right)\mathrm{d}r^2 \tag{13}$$

$$\exp\left(2i\left[\tan^{-1}\left(\frac{\mathrm{e}^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)-\frac{\pi}{4}\right]\right)c^2\mathrm{d}t^2=+\exp\left(-2i\left[\tan^{-1}\left(\frac{\mathrm{e}^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)-\frac{\pi}{4}\right]\right)\mathrm{d}r^2 \tag{14}$$

$$\exp\left(i\left[\tan^{-1}\left(\frac{\mathrm{e}^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)-\frac{\pi}{4}\right]\right)c\mathrm{d}t=\pm\frac{\mathrm{d}r}{\exp\left(i\left[\tan^{-1}\left(\frac{\mathrm{e}^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)-\frac{\pi}{4}\right]\right)} \tag{15}$$

$$c\mathrm{d}t=\pm\frac{\mathrm{d}r}{\exp\left(2i\left[\tan^{-1}\left(\frac{\mathrm{e}^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)-\frac{\pi}{4}\right]\right)} \tag{16}$$

3. Results

If we assume as dynamical time the proper time spent by the particle to reach the corresponding point on the $r=0$ disk from the outer horizon.

$$ct=\pm\int_{r_0}^{r}\frac{\mathrm{d}r}{\exp\left(2i\left[\tan^{-1}\left(\frac{\mathrm{e}^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)-\frac{\pi}{4}\right]\right)} \tag{17}$$

Let's:

$$ct=\pm\int\exp\left(-f\left(r\right)\right)\mathrm{d}r \tag{18}$$

then

$$ct = \pm\left[\frac{1}{-f'(r)\exp(-f(r))}\right]_{r_0}^{r} \tag{19}$$

which give:

$$ct = \pm\left[\frac{i\sqrt{(1-r)^3}\sin^2\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)e^{-i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\exp\left(2i\left[\tan^{-1}\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r_0}}\right)-\frac{\pi}{4}\right]\right)}\right]_{r_0}^{r} \tag{20}$$

$$ct = \pm\left[\frac{i\sqrt{(1-r)^3}\sin^2\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)e^{-i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\exp\left(2i\left[\tan^{-1}\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r_0}}\right)-\frac{\pi}{4}\right]\right)}-\frac{i\sqrt{(1-r_0)^3}\sin^2\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r_0}}\right)e^{-i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\exp\left(2i\left[\tan^{-1}\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r_0}}\right)-\frac{\pi}{4}\right]\right)}\right] \tag{21}$$

Due to the ±, we get two radial geodesics families, such as [8] [9]:

- the outgoing geodesics, for which $\mathrm{d}r/\mathrm{d}t > 0$; their equations are

$$ct = \frac{i\sqrt{(1-r)^3}\sin^2\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)e^{-i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\exp\left(2i\left[\tan^{-1}\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r_0}}\right)-\frac{\pi}{4}\right]\right)}-\frac{i\sqrt{(1-r_0)^3}\sin^2\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r_0}}\right)e^{-i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\exp\left(2i\left[\tan^{-1}\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r_0}}\right)-\frac{\pi}{4}\right]\right)} \tag{22}$$

- incoming geodesics, for which $\mathrm{d}r/\mathrm{d}t < 0$; their equations are

$$ct = -\left[\frac{i\sqrt{(1-r)^3}\sin^2\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r}}\right)e^{-i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\exp\left(2i\left[\tan^{-1}\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r_0}}\right)-\frac{\pi}{4}\right]\right)}-\frac{i\sqrt{(1-r_0)^3}\sin^2\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r_0}}\right)e^{-i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\exp\left(2i\left[\tan^{-1}\left(\frac{e^{i\frac{2(\alpha-2)-kL_m}{4(\alpha-2)}}}{\sqrt{1-r_0}}\right)-\frac{\pi}{4}\right]\right)}\right] \tag{23}$$

Our future work is to study a physical phenomena of these results and compare with telescope observations. The future works are going to be a simulation of incoming geodesics for which $\mathrm{d}r/\mathrm{d}t < 0$ and outgoing geodesics, for which $\mathrm{d}r/\mathrm{d}t > 0$ by coding in Python and compiling with two space dimensions suppressed, leaving just the time *t* and the distance from the center *r*. That are going to show, the null light cones on which borders light moves, while massive objects move inside the cones.

Scope of Future Work

The complex TOUGMA's solution is crucial to understand the universe's properties. This solution areas use is exceptionally wide:

1) Astrophysics;
2) cosmology;
3) Quantum gravity;
4) Quantum Mechanics;
5) Multiverse;

The present investigation will be very helpful to the researchers who are engaged in those areas of research work in unification of quantum mechanics and General Relativity. Our future work is to simulate these results and compare them with observations.

Conflicts of Interest

Funding and/or Conflicts of interests/Competing interests: The present study received no financial aid from any government/non-government agency. And there is no conflict of interest for this study.

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