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A Unified Definition of Electrostatic and Magnetic Fields

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Abstract

This article originates from the observation that field lines are drawn using distinctive rules in magnetic field and electrostatic fields. It aims at reconciliating the definitions of these fields and thus reaching a consensus on the interpretation of field lines. Our unified field definition combines three orthogonal vectors and a unique scalar value. Field lines are then defined as isovalue lines of the scalar value, rendering it simpler to interpret in both field types. Specific to our field definition is the use of square root of vector's cross product so that all vectors have the same physical unit. This enhanced field definition also enables a more efficient calculation of Biot-Savart law. This article is the first of a series allowing the drawing of isovalue contour lines.

Keywords

Magnetic Field, Electrostatic Field, Biot-Savart Law, Scalar Field Value, Equipotential Line, Square Root Cross Product

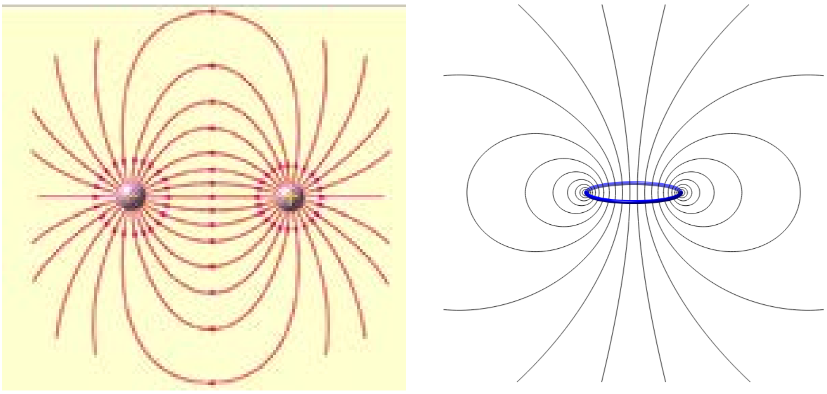
1. Introduction

In a field created by two punctual electrical charges, the electrostatic field lines connect the positive charge to the negative charge (**Figure 1(a)**) [1].

Strikingly, magnetic field lines however form circular shapes around an electrically conducting ring (**Figure 1(b)**) [2].

The common properties of the traits of these lines, clearly stated in physics literature, are that they never intersect or touch each other, and they present symmetrical patterns defined by the sources of the field.

The calculation of these lines uses two theoretical definitions of one vector for every physical point. These definitions have been proposed a long time ago by Faraday, Biot and Savart. Since that first proposition, no improvement of this theoretical definition has been found by the author.



(a) (b)

Figure 1. Electrostatic field lines (a) and magnetostatic field lines (b). Electrostatic field lines (a) go from a positive charge to a negative charge. In (b), magnetic field lines go around the side of an electrically active ring.

In the present article, we aim at giving a common definition of field lines, so that field lines are represented using a universal procedure. To attain this objective, we will propose a specific and unified definition of the field at every point around a physical source for both electrostatic and electromagnetic fields and derive a rule to draw field lines as isovalue curves across the field.

We start first by introducing our specific definition of a field, involving both a unique scalar value and related vectors. This specific definition is then applied in turn to electrostatic, then electro-magnetic contexts. In the discussion, an alternative calculation of Biot-Savart law is proposed, and compatibility of this new field definition with the superposition principle is considered.

2. Field Properties

2.1. Field Definition

A field is formed by a physical object which generates, a physical quantity represented by a scalar [3], a vector, or a tensor [4] in its surrounding. In the following, fields will consist, at any given point, of a numerical value bearing a physical unit, which will be named the "*field value*" and three vectors with the same physical units. These values can be continuously evaluated everywhere in the space around the object, independently of the choice of any reference frame.

Symbol "$\wedge$", a mathematical symbol is used for vector cross product.

To simplify physical formulas reported in this article, the permeability and the electric permittivity in vacuum are taken equal to one.

2.2. Values and Physical Units around an Infinite Line

Let us define the field value in "p" created by an infinite line with a given physical property U, as a continuous function of the field value equal to U/r, where r is the distance between p and the line.

$$Field = \frac{U}{r} \tag{1}$$

The value decreases when the distance "*r*" to the infinite line increases. Using this function, several curves of isovalue *U*/*r* can be represented in a plane perpendicular to the line. They form circles centered in the infinite line as shown in **Figure 2(a)**.

Additionally, let us define three vectors at point "*p*" as shown in **Figure 2(b)**. Their directions can be defined in relation to the wire using three directions: parallel, perpendicular, or orthogonal.

1) The projection of point "*p*" onto the line is point "*e*". Vector A is defined starting at "*p*" perpendicularly to the wire and pointing toward "*e*". The norm of vector *A* is equal to the physical field value at point "*p*", which is *U/r*. Note that the derivative of this vector can be named as gradient vector since the value of *U/r* continuously increases when *r* decreases.

2) Vector *C* is defined as originating at "*p*" and parallel to the line. The norm of vector *C* is also equal to *U/r*.

3) In the orthogonal direction to the plane formed by vectors *A* and *C*, vector *B* is drawn with a norm of *U/r*. This vector will be named potential vector. It is tangent to the circle centered in "*e*" and passing through "*p*".

With such a description, all three vectors have the same norm also equal to the field value and are orthogonal to the other two. These vectors which have arbitrary chosen orientations, can be created at any positions in the space around the infinite line.

Field value and vectors at any point in the field are now defined with a unique physical field value and three orthogonal directions originating from that point.

Figure 2 illustrates the case where *U* is positive. When the physical field source changes, all vector directions are reversed as the physical value at point *p*, and the square root of cross product is also reversed. Both cases are addressed in **Table 1**.

The following paragraph gives additional details about relationships between each vector and the others.

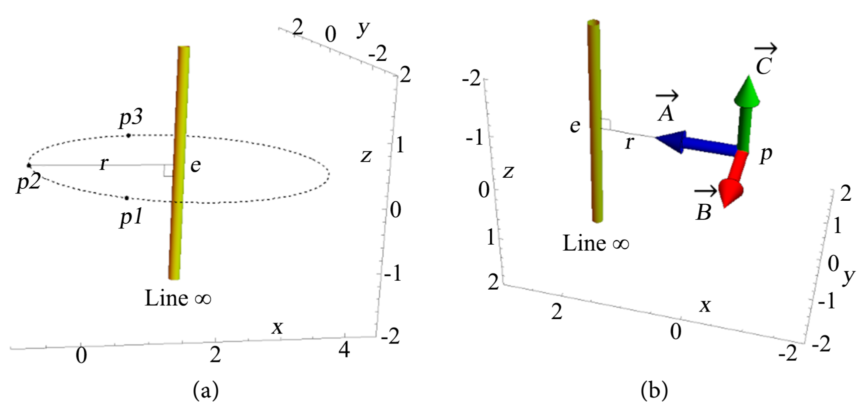


Figure 2. Two schematics of an infinite line and its surrounding field. In (a), a line with a given physical properties U, remotely creates a field at any given points "*p1*" that varies in U/r, where "*r*" is the distance from "*p1*" to the infinite line. The circle centered in "*e*" and passing through "*p1*, *p2* and *p3*" represents an isovalue of the field. In (b), vectors : *A*, *B* and *C*, are then defined at point "*p*" by three orthogonal vectors.

Table 1. Relations between the three orthogonal vectors originating at point "*p*": they are vectors *A*, *B* and *C* as drawn in **Figure 2(b)**. The square root of the cross product is proposed to respect the physical field unit of the proposed specific field.

Vector orientation versus infinite line	Perpendicular vector	Orthogonal vector	Parallel vector
Vector name	$\boldsymbol{A}$	$\boldsymbol{B}$	$\boldsymbol{C}$
Vector norm		$\|\boldsymbol{A}\| = \|\boldsymbol{B}\| = \|\boldsymbol{C}\| = \lvert U/r \rvert$	
		Square Root Cross product with positive source	
Physical field value (U/m)	U/r	U/r	U/r
Orthogonality When $U > 0$	$\boldsymbol{A} = (\boldsymbol{B} \wedge \boldsymbol{C})^{1/2}$	$\boldsymbol{B} = (\boldsymbol{C} \wedge \boldsymbol{A})^{1/2}$	$\boldsymbol{C} = (\boldsymbol{A} \wedge \boldsymbol{B})^{1/2}$
		Square Root Cross product with negative source	
Physical field value (U/m)	$-U/r$	$-U/r$	$-U/r$
Orthogonality When $U < 0$	$\boldsymbol{A} = -(\boldsymbol{B} \wedge \boldsymbol{C})^{1/2}$	$\boldsymbol{B} = -(\boldsymbol{C} \wedge \boldsymbol{A})^{1/2}$	$\boldsymbol{C} = -(\boldsymbol{A} \wedge \boldsymbol{B})^{1/2}$

2.3. Orthogonal Vectors with Physical Units and Cross Product

We have defined orthogonal vectors with identical norms (U/r), which means they have the same physical units. Yet, how is it possible to write a formal relationship between these three vectors and improve our definitions of the field described in the previous paragraph? To do so, we will use the square root of the cross product to link the three vectors (see also discussion for more details).

The corresponding mathematical relations between the three vectors are summarized in Table 1.

In Table 1, the field source is either positive (top of Table 1) or negative (bottom of Table 1). Then, when the physical field source changes, all vector directions are reversed as the physical value at point "*p*".

Moreover, at all points located at distance "*r*" from the infinite line, the field value remains constant. It is then possible to draw curves of iso-value of the field, curves that circular in the case of one infinite line. Along these lines, vectors *A* and *B* change directions gradually, while vector *C* remains in the same direction.

3. Applications to Infinite Lines with Static or Moving Charges

3.1. An Infinite Line with a Static Density of Electrical Charges

Let us explore the above definition when applied to a field created by an infinitely elongated line with a static density of electrical charge [5] [6] (in 2.3.3). We will draw parallels between our unified definition and the electrostatic field definition found in physics literature.

This theoretical line is very thin and not conducting. The line, which has a static density of charge U equal to Q/L in Coulomb by meter, and where Q is the charge on the length L, generates an electrostatic field in its surroundings (**Figure 3**), that can be described using our specific field definitions.

Figure 3 is a 3D drawing where the electrostatic field [3] is described using our unified field definition.

At any point "p", the physical field value, and hence the norms of the three vectors originating at that point, can be calculated. Vector A points away from the positive charge or toward the negative charge. The derivative of vector A is in usual terms called the gradient vector (where the field value increases most quickly and in $1/r^2$). Equipotential lines *i.e.*, curves along which the field value remains constant, are circles centered in point "e" and passing by point "p" [5]. These equipotential lines drawn here are the same trajectories found in literature. Vectors tangent to the equipotential lines could in this case be named S, for electrostatic potential vectors. The field value, which is also the norm of our potential vector is equal to $(Q/L)/r$, as defined in literature.

Our last vector, the parallel vector C, does not have an equivalent in classical electrostatic field description. However, it will prove useful in the case of a magnetic field.

Arbitrarily, we define that for a negative charge, the square root cross products of each pair of vectors are positive whereas for a positive charge, they are negative.

Here we have seen that our unified definition of electrical fields is compatible with the definition of electrostatic field found in physics literature.

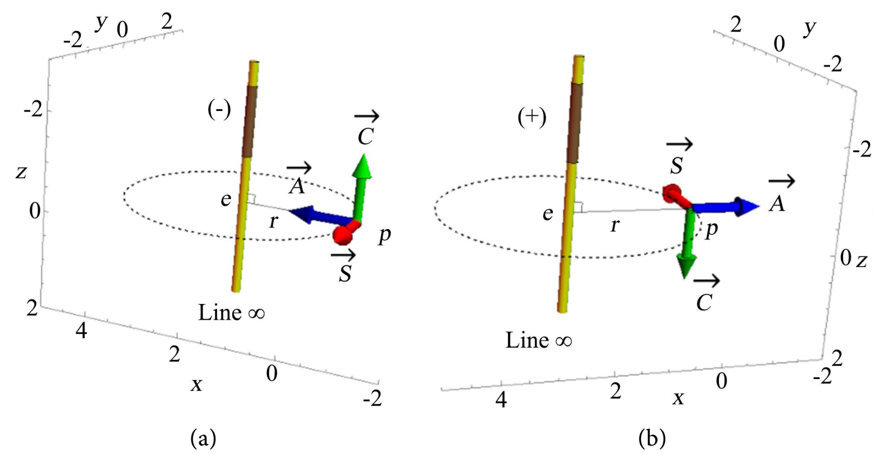


Figure 3. Drawing of two infinite electrostatic lines with a negative charge (a) and a positive charge (b) respectively. The generated electrostatic field values at point "p" are respectively $(-(Q/L)/r)$ (a) and $(+Q/L)/r$ (b). Vectors A perpendicular to the lines, are respectively oriented toward the negatively charged line (a) and away from the positively charged line (b). Vectors C are parallel to the line, pointing upwards and downwards depending on the charge. Vectors S are tangent to the circle centered in point "e" with radius "r". Their directions also depend on the sign of the static charge in the lines. Circles centered in point "e" and passing by point "p" are equipotential lines *i.e.*, lines of isovalue of the physical field.

In the following paragraph, we apply our model to magnetic fields induced by electrical current in a wire.

3.2. Infinite Wire with an Electrical Current

Around a conducting wire with a static current, Oersted [7] discovered in 1820 that the orientation of a compass changes, but surprisingly without any contact between the magnet and the wire [8]. Oersted and fellow scientists among which Biot, Savart and Ampere have shown that the field around an infinite electrical wire [9] [10] (in 5.2.2), varies in I/r, which is the ratio of the current by the distance from the wire.

In terms of our unified field definition, the parameter U is, in this case of an electromagnetic field, the current I that flows through the infinite line. The physical value at any point "p" is hence $-I/r$ (**Figure 4(a)**), and $+I/r$ when the electrical current is reversed (**Figure 4(b)**).

Vectors C are oriented by the direction of the electrical current. Vectors C depend on each sign of the field: going up in **Figure 4(a)** (left) and down **Figure 4(b)** (right). Vectors M are oriented with the right-hand screw rule, and its norm is I/r.

We can name our magnetic vector M, as magnetic field vector, as classically defined in literature. Isovalue of the field value, defined as the norm of M, are equidistant to the wire and draw circles to which vectors M are tangent. This is compatible with the definition of trajectories of magnetic field lines found in literature. Additionally, vectors M follow the right-hand rule with the two other vectors.

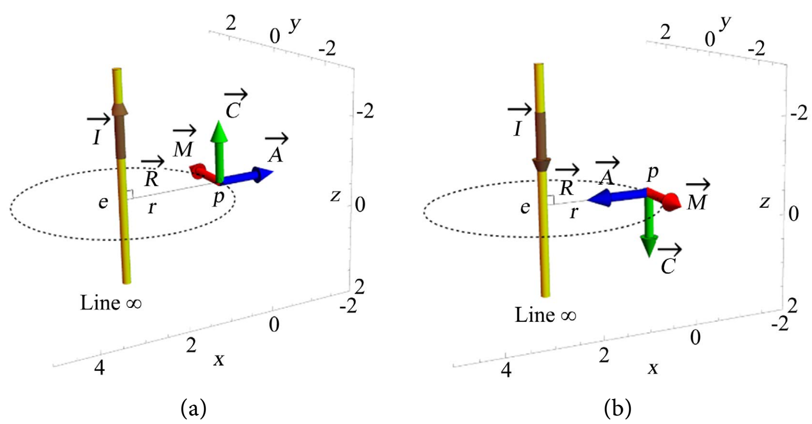


Figure 4. Opposite electrical current in infinite wires surrounded by opposite electromagnetic field [10]. Points "p" have three vectors with the same norm. Red vectors M are in the same direction than the classical right-hand side magnetic field vector. These vectors M are tangent to a circle centered in point "e" surrounding the electrical line and passing by point "p". Vectors C are parallel and oriented by the electrical current defined by vectors I. These vectors C are named current field vectors. Vectors A is oriented by the scalar cross product of the two other vectors, and oriented from "p" toward the wire or away from it depending on the sign of the field (I/r or $-I/r$). Their derivatives are usually named as gradient vectors.

Although, the classical theory and our definition of fields appears very close, but there is an important difference: in literature, the three vectors do not have identical physical expressions and not the same norm at point "*p*" [11]. This is the subject of the first paragraph in the discussion.

4. Discussion

4.1. Physical Field Values versus Vectors in Biot-Savart Law

The magnetic field vector introduced by Biot and Savart allows to calculate with an integration procedure the magnetic field around a conducting wire. The author has used this integration, but a difficulty has been found in the calculation used for the original field vector. This paragraph proposes a better calculation by exploring the parallel between the unified field definition in the case of an electromagnetic field and the Biot-Savart law which specifically defines the magnetic field around a small portion of an electrical wire [12]. In this comparison we calculate the magnetic field vectors d***B*** at points "p" located away from the conducting segment, in a direction that is not perpendicular to the current field vector (*i.e.*, to the wire).

In **Figure 5**, a segment of a wire of length "dl", through which flows a current "*I*" which generates a magnetic field at any point "p". In this figure, the magnetic vector is named "d***B***" [12]. Vector "*C*" is the current vector, parallel to the flowing current "*I*". Vector "*Ru*" in (a) et "*R*" in (b) are in the same direction of "d" between "m" and "p". The right-hand rule is applied to draw the three vectors in both drawings [15].

The resulting magnetic vector can be derived in both referential and using an angle θ which is necessary for the cross product of both first vectors.

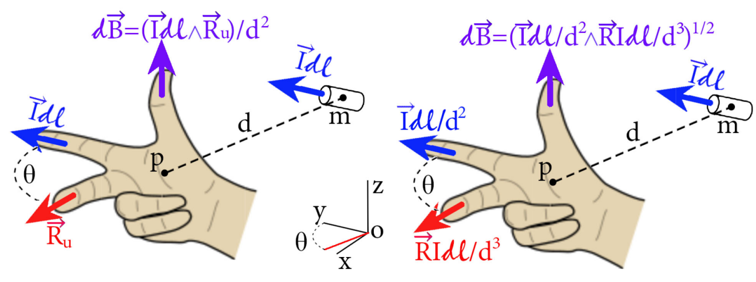


Figure 5. In (a) (left), a 3D view of right-hand rule [13] with Biot-Savart law [14] compared in (b) (right) to our unified field definitions. The cylinder is a small segment of length "dl" of an electrical wire with a current "I" and with center "m" (which is not the projection of point "*p*" on the segment). In both figure, the distance "d" is between points "m" and "p". In (a), vector "***Ru***" without physical unit, is the unit vector align with points "m" and "p". In (b) vector "***R***" has its norm equal to the distance between "m" and "p" (not perpendicular to both: the current vector *I* and the direction of the segment). In (a) and (b), fingers indicate how the direction of vector "d***B***" is found, as the cross product of the two other vectors. In (a), the classical cross product between "***Ru***" and "*I*dl" is used. In (b), the square root of the cross product allows the norm of "d***B***" and all three vectors are in the same physical space with the same physical value than in literature.

Classical theory (Biot-Savart law) shown in Figure 5(a) (left):

$$\mathrm{d}\boldsymbol{B} = \left(\boldsymbol{I}\mathrm{dl} \wedge \boldsymbol{Ru}\right)/\mathrm{d}^2 \tag{2}$$

$$\left\|\mathrm{d}\boldsymbol{B}\right\| = I\mathrm{dl}\sin\theta$$

Unified field definition shown in Figure 5(b) (right):

$$\mathrm{d}\boldsymbol{B} = \left(\boldsymbol{I}\mathrm{dl}/\mathrm{d}^2 \wedge \boldsymbol{R}I\mathrm{dl}/\mathrm{d}^3\right)^{1/2} \tag{3}$$

$$\left\|\mathrm{d}\boldsymbol{B}\right\| = I\mathrm{dl}\sin\theta$$

Although definitions of the base vectors are different in Biot-Savart commonly used law and our usage of field vectors, the resulting values of the norm of vectors $\mathrm{d}\boldsymbol{B}$ are the same. In Figure 5(a)) (left), as in scientific literature, the physical unit of the two vectors are not equal to the physical unit of vector $\mathrm{d}\boldsymbol{B}$. In Figure 5(b) (right), the physical units of the three vectors $\boldsymbol{I}\mathrm{dl}/\mathrm{d}^2$, $\boldsymbol{R}I\mathrm{dl}/\mathrm{d}^3$ and $\mathrm{d}\boldsymbol{B}$ are the same, allowing a better understanding of the calculation of the square root of the cross product.

4.2. Field Superposition

By superposing the two exposed cases of opposite charges in an electrostatic field or opposite flowing current in an electromagnetic field, vectors will be cancelled, and their norms will be naught. The field is cancelled. This confirms the additivity property of the wire (neutralized charges or neutralized currents) in the unified field model of this paper.

Since our proposed definition obeys the property of additivity of the wire, it specifically follows the superposition principle [16]. This has to be confirmed for the field additivity when several wires are used.

4.3. Square Root of the Cross Product

The cross product is a mathematical operator between two vectors giving a vector which cannot be represented in the same physical space. In the literature, many definitions are proposed for the resulting vector: pseudo vector, axial vector, bivector, external vector [11].

In this paper, we use the square root of the cross product to obtain a resulting vectors that belong to the same space, and then can be compared to each other.

This may be a strong proposition to guide the use of cross product in other physical domains.

5. Conclusion

A specific field model was exposed and applied to two different types of physical electrical sources with an empty environment. The field around a charged insulating line and the field around an electrically conducting wire are described. Parallels to field description in literature indicate good compatibility with existing theory, reinforcing the idea of a unified field description. A novel interpretation of the Biot-Savart law is possible thanks to this unified field definition, based on the use of vectors in the same physical space. In following articles, this field model will be put into use in more complex systems composed of several conducting wires.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Champ électrostatique: Surfaces équipotentielles et lignes de champ. https://fr.wikipedia.org/wiki/Champ_électrique

[2] Magnétostatique. Lignes de champ autour d'une spire de courant. https://fr.wikipedia.org/wiki/Magnétostatique

[3] Scalar field. https://en.wikipedia.org/wiki/Scalar_field

[4] Field. https://en.wikipedia.org/wiki/Field

[5] Champ créé par un fil infiniment long avec une charge statique. https://fr.wikipedia.org/wiki/Champ_électrique

[6] Field Due to an Infinitely Long Line Charge 2.3.3. Electromagnetic Field Theory: A Problem Solving Approach (Zahn)—Engineering LibreTexts.

[7] Equipotential. https://en.wikipedia.org/wiki/Equipotential

[8] Orsted Experiment. https://en.wikipedia.org/wiki/Hans_Christian_Ørsted

[9] Orsted's Law. https://en.wikipedia.org/wiki/Oersted%27s_law

[10] I/r https://en.wikipedia.org/wiki/Magnetic_field

[11] Cross Product of Two Vectors. https://en.wikipedia.org/wiki/Cross_product

[12] Field Due to an Infinitely Long Line Current 5.2.2. Electromagnetic Field Theory: A Problem Solving Approach (Zahn)—Engineering LibreTexts.

[13] "dB" https://fr.wikipedia.org/wiki/Loi_de_Biot_et_Savart

[14] Image Source No Image Name. https://upload.wikimedia.org/wikipedia/commons/d/d2/Right_hand_rule_cross_product.svg

[15] Biot-Savart Law https://en.wikipedia.org/wiki/Biot–Savart_law

[16] Superposition Process in Physique. https://en.wikipedia.org/wiki/Superposition_principle