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Reliability of Life Calculation Laws for Materials under Variable Amplitude Loading

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Abstract

For many years, researchers have been looking for a reliable law that will take into account the type of loading, the mechanical characteristics of the material, the geometric configuration in the determination of the service life of mechanical parts. The service life of structures at risk (automotive, aeronautics, among others.) in service, subjected to variable solicitations in time, are random for a same type of loading. This article proposes to highlight the influence of this variation in service life on the reliability of structures by a probabilistic approach. The characteristics of the proposed law are satisfactory compared to the classical laws because it takes into account the parameters of the classical laws (Weibull law) and the dispersions of the lifetimes of a same material.

Keywords

Design, Reliability, Durability, Variable Loading

1. Introduction

The safety of users must be prioritized over the load-bearing capacity of a structure. It is therefore a problem related more to its fitness for service and its structural safety. The notions of serviceability, structural safety and structural performance are nowadays grouped under the term reliability. Implicitly, society relies on designers and managers to ensure the reliability of equipment. The latter must ensure that the hazards associated with construction are controlled. In other words, that the risk is limited to an acceptable value. In general, the overall failure of a structure is rare except in the case of major natural disasters or obvious human errors.

Several researchers have proposed laws that allow for the analysis of the reliability of structural damage laws, including the exponential law, the Weibull law and the Normal law.

Considering the diversity of the laws and the randomness of the lifetimes of the materials, this article aims at proposing a new law allowing to take into account, in the elaboration of the new law, the two parameters of the Weibull law (form factor, the range of the distribution) as well as the average parameter of the lognormal law [1] [2].

2. Method and Materials

2.1. Method

1) Laws used for fatigue reliability analysis

The systems can be complex and the operating conditions variables. Several researchers have used laws such as: the exponential law, the Weibull law or the lognormal law to model the reliability of structures.

In the literature, the law that is most used in the field of reliability analysis of structures under variable amplitude loading is the Weibull law [3] [4].

2) Proposal of a reliability analysis law

a) Proposed new model

Reliability is defined as the probability that a system will perform its function during a given period of time and under given operating conditions. In practice, systems can be complex and operating conditions can vary. The exponential law, the Weibull law or the lognormal law to model the reliability of structures are the most used [5].

According to the experimental, the damage function $D(N)$ during the evolution of the crack, taking into account the initial crack $(\approx D_0 = D(1))$ can be modeled as follows:

$$D_c = D_0 N^{\beta} \tag{1}$$

where: $N \in IN$, D_0 and β are the parameters of the stochastic crack growth model. They are a function of the applied loading, material characteristics, and the geometric configuration of the part.

We can express the lifetime in terms of the critical damage D_c:

$$N = \left(\frac{D_c}{D_0}\right)^{\frac{1}{\beta}} \tag{2}$$

Cumulative distribution function is given by relation (3).

$$F(N) = \int_0^N f(N)\,\mathrm{d}N \tag{3}$$

where $D_0 = D(1)$ is a qualitative initial variable and has a fixed value or deterministic quantity and β is a normal random variable with mean $\mu(\beta)$ and variance $\sigma^2(\beta)$.

$$\frac{\mathrm{d}D(N)}{\mathrm{d}N} = \beta D_0 N^{\beta-1} \tag{4}$$

$$\frac{\mathrm{d}\left[\ln D(N)\right]}{\mathrm{d}(\ln N)}=\beta \tag{5}$$

$$\frac{\mathrm{d}E\left[D(N)\right]}{\mathrm{d}N}=\frac{D_0}{N}E\left[\beta N^{\beta}\right] \tag{6}$$

From Equation (3), we have:

$$\ln D(N)=\ln D_0+\beta\ln N \tag{7}$$

where $\ln D(N)$ follows the normal with mean $\ln\left[\ln D(N)\right]$ and variance $V\left[\ln D(N)\right]$.

For the failure criterion $D(N)\geq D_c$, the probability of survival of the component at a number of cycles N is given by:

$$P\left[IN\succ N\right]=P\left[D(N)\prec D_c\right] \tag{8}$$

Reliability, noted R, is defined as the probability that a system will perform its function during a given period and under given operating conditions. In mathematical terms, the reliability law is defined by:

$$R(N)=P\left[IN\succ N\right]=P\left[\ln N\succ\ln IN\right] \tag{9}$$

With $N\succ 0$, the random variable representing the lifetimes and D_c, the critical damage.

$$R(N)=P\left[D(N)\prec D_c\right]=P\left[\ln D(N)\prec\ln D_c\right] \tag{10}$$

$$R(N)=\Phi\left[\frac{\ln D_c-E\left[\ln D(N)\right]}{\sqrt{V\left[\ln D(N)\right]}}\right] \tag{11}$$

where $\Phi(z)=\frac{1}{\sqrt{2\pi}}\int_{-x}^{z}\mathrm{e}^{-\frac{z^2}{2}}\mathrm{d}z$

Φ is the distribution function for the distribution of the minimum extreme values.

From Equation (7), we obtain:

$$E\left[\ln D(N)\right]=\ln D_0+(\ln N)E[\beta] \tag{12}$$

$$V\left[\ln D(N)\right]=(\ln N)^2 E[\beta] \tag{13}$$

Replacing (12) and (13) in (11), we obtain:

$$R(N)=\Phi\left[\frac{\ln D_c-\ln D_0-(\ln D)E[\beta]}{\sqrt{(\ln N)^2 V[\beta]}}\right]$$

$$R(N)=\Phi\left[\frac{1}{\sqrt{\frac{V[\beta]}{E^2[\beta]}}}\left(\frac{\frac{\ln D_c-\ln D_0}{E[\beta]}-\ln N}{\ln N}\right)\right]$$

$$\text{By setting} \quad \frac{\ln D_c - \ln D_0}{E[\beta]} = \frac{\ln D_0 - \ln D_c}{\mu[\beta]} = C_t \tag{14}$$

$$\text{And} \quad \frac{V[\beta]}{E^2[\beta]} = \frac{\sigma^2[\beta]}{\mu^2[\beta]} = A_t \tag{15}$$

The expression is reduced to:

$$R(N) = \Phi\left[\frac{1}{\sqrt{A_t}}\left(\frac{C_t}{\ln N} - 1\right)\right]$$

Using the property of the Laplace function $\Phi(-z) = 1 - \Phi(z)$

The expression becomes:

$$R(N) = 1 - \Phi\left[\frac{1}{\sqrt{A_t}}\left(1 - \frac{C_t}{\ln N}\right)\right] \tag{16}$$

$$\text{So} \quad F(N) = \Phi\left[\frac{1}{\sqrt{A_t}}\left(1 - \frac{C_t}{\ln N}\right)\right] \tag{17}$$

The model presented in Equations (16) and (17) has been discussed in the general framework of the nonlinear damage process.

The application of this model to characterize the fatigue life is the subject of this work where the damage at each time is represented by the crack length.

b) Identification of model parameters

It is a common statistical method used to infer the parameters of the probability distribution of a given sample. It was developed by the statistician and geneticist Ronald Fisher between 1912 and 1922. In this paragraph, we present two methods used to estimate the parameters of the reliability functions: the maximum likelihood method and the regression method.

- *Maximum likelihood method*

The likelihood function is equal or proportional to the probability of observing the events (failures).

It allows to define the expressions of the constants C_t and A_t:

$$\hat{C}_t = \left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{\ln N_i}\right)^{-1} \tag{18}$$

$$\hat{A}_t = \frac{1}{n}\sum_{i=1}^{n}\left(1 - \frac{\hat{C}_t}{\ln N_i}\right)^2 \tag{19}$$

The empirical function, of distribution $F(N_i)$ is estimated using the mean degree formula.

$$\hat{F}(N_i) = \frac{i}{n+1} \tag{20}$$

- *Regression method*

The regression method consists of linearizing the reliability model and computing its parameters from the regression line that best fits the experimental points (N_i, σ_i).

$$R(N)=\Phi\left[\frac{1}{\sqrt{A_t}}\left(\frac{C_t}{\ln N}-1\right)\right] \quad (21)$$

$$R(N)=\Phi\left[\frac{\frac{1}{\ln N}-\frac{1}{C_t}}{\frac{\sqrt{A_t}}{C_t}}\right]=\Phi[U] \text{ With } U=\frac{\frac{1}{\ln N}-\frac{1}{C_t}}{\frac{\sqrt{A_t}}{C_t}} \quad (22)$$

$$U=\Phi^{-1}\left[R(N)\right] \quad (23)$$

The linear form of the distributions belonging to the family of log distributions as a function of a location parameter and a scale parameter is given by:

$$\frac{1}{\ln N}=\frac{1}{C_t}+\frac{\sqrt{A_t}}{C_t}\Phi^{-1}\left[R(N)\right] \quad (24)$$

This relationship(15) is a line equation:

$$Y=mX+C \quad (25)$$

where: the ordinate $Y=\frac{1}{\ln N}$, the abscissa $X=U=\Phi^{-1}\left[R(N)\right]$, C and m are the coordinates at the origin and the slope respectively.

Considering the value of the set of lifetimes $\{N_1, N_2, N_3, \cdots, N_n\}$:

$$\begin{cases} Y_i=\frac{1}{\ln N_i} \\ X_i=U_i=\Phi^{-1}\left(\frac{n+1-i}{n+1}\right) \end{cases} \quad (26)$$

2.2. Materials

Formulas (22) and (26) were used to plot the Y_i curves as a function of X_i.

The tests conducted by Morgenstern in 2006, based on the concept of localized stress, for the estimation of the life of aluminum welded joints of different types of alloys under constant amplitudes. For the same value of applied load, different lifetimes are obtained for a series of tests on a welded part.

In design, automotive, aerospace and other industries need reliable results for their applications.

We consider the results of force-controlled stump cutting tests on ALMgSi1 T6 (AW-6082 T6) parts welded by Morgenstern MIG to validate our model.

The representation of the values of the pairs (Y_i, X_i) in **Figure 1** gives a linear graph for the validation of the proposed model according to Equation (25).

Morgenstern's fatigue failure data is used to demonstrate this technique and the compliance of this model of the life distribution characterization [6].

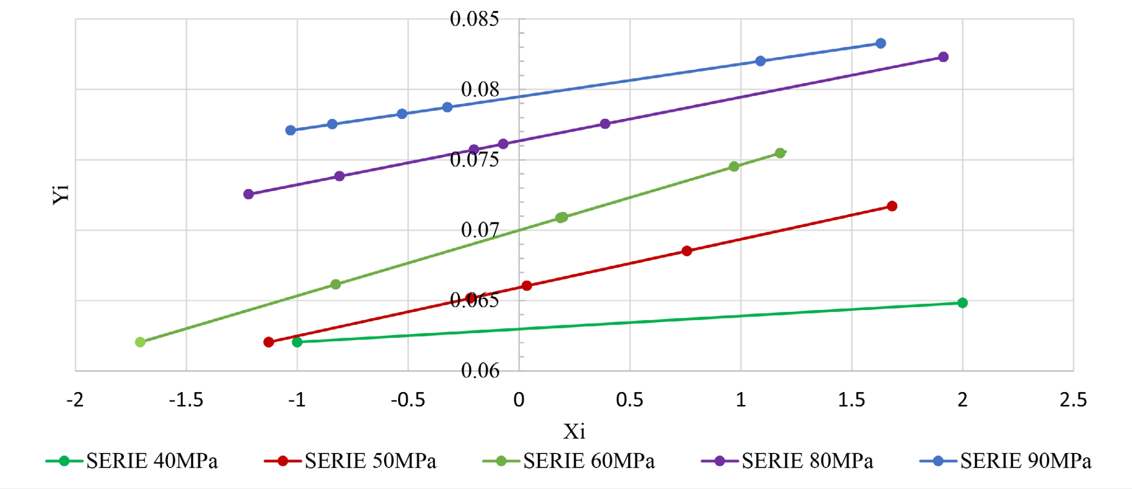

Figure 1. Evolution of the variable Y_i as a function of the X_i.

Table 1. Constants of the proposed law.

Series (MPa)	40	50	60	80	90
Ct	15.8802	15.1704	14.2882	13.0996	12.5826
At	0.000217795	0.002716759	0.00442141	0.001659127	0.000855543

Table 1 shows that these constants decrease with an increase in stress. This shows that the reliability of the structure depends on the stress level (see Table 1).

In all cases, the model gives satisfactory results. We can express some additional features of the model for fatigue life prediction.

- Failure probability density

It is defined by the relation: $f(N) = -\dfrac{\mathrm{d}F(N)}{\mathrm{d}N}$

$$f(N) = \frac{C_t}{\sqrt{2\pi A_t}\, N(\ln N))} \times \exp\left[-\frac{C_t}{2A_t}\left(1-\frac{C_t}{\ln N}\right)^2\right], N \succ 1 \tag{27}$$

- Damage or failure rate

It is defined by the relation: $\lambda(t) = -\dfrac{f(N)}{R(N)}$

$$\lambda(N) = \frac{C_t \exp\left[-\frac{C_t}{2A_t}\left(1-\frac{C_t}{\ln N}\right)^2\right]}{\sqrt{2\pi A_t}\, N(\ln N)^2\left[1-\Phi\left[\frac{1}{\sqrt{A_t}}\left(1-\frac{C_t}{\ln N}\right)\right]\right]}, N \succ 1 \tag{28}$$

3. Results and Discussion

The purpose of reliability analysis is to characterize the behavior of a mechanical structure during its lifetime, to quantify the impact of design changes on the integrity of the product and to improve its performance throughout its service life.

Not analyzing the reliability of a structure means increasing after-sales costs, *i.e.* warranty and/or legal fees. Building a more reliable structure means increasing design and production costs.

Through the probability density, we will draw in Figure 2, Figure 3, the curves comparing the Weibull distribution, the exponential distribution and the lognormal distribution to the proposed distribution and draw the conclusions.

3.1. Probability Density

The results show the variability of the failure density for different lifetimes. This difference may be due to the variable life span as well as the material's own characteristics

The Weibull probability density for beta = 0.5 and beta = 1 characterize respectively the period of youth and maturity of the welded parts. While for beta = 2, it characterizes the fatigue phenomena [1]. The curves are identical.

The probability density of Weibull for beta = 1 remains greater than those of Weibull for beta = 0.5 and beta = 2 which are almost identical for the same stress value. We can conclude that for this material, the behavior at the youth period is

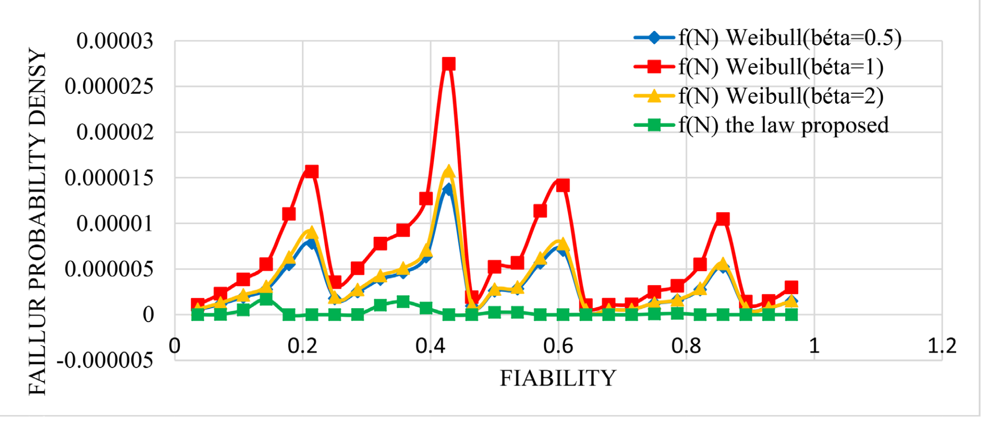

Figure 2. Comparisons of the failure probability density of the Weibull distribution with different parameters and that of the proposed distribution.

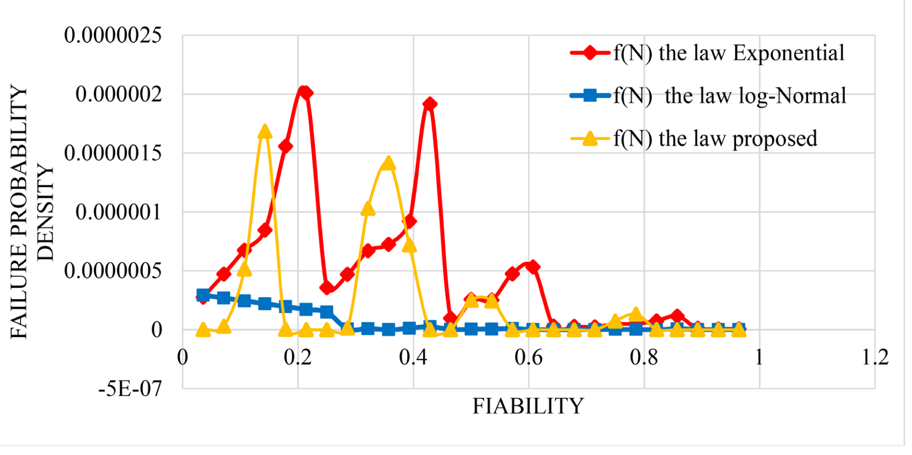

Figure 3. Comparisons of the failure probability densities of the exponential distribution, the lognormal distribution and the proposed distribution.

almost identical to that of fatigue in terms of failure density for the Weibull law.

The proposed model of the law is more interesting, since it has a low probability density of failure than the Weibull law.

The results show that the probability density of the exponential distribution and the proposed distribution are closer for the same value of constraint while they are low compared to the Weibull distribution.

Considering the previous results, we will compare the new law (low failure probability density) with the exponential law and the lognormal law for reliability analysis through the failure rate.

3.2. Failure Rate

In this subtitle we will present the failure rate of the material as a function of its lifetime (Figure 4 & Figure 5).

The results show that failure rates decrease with increasing service life:

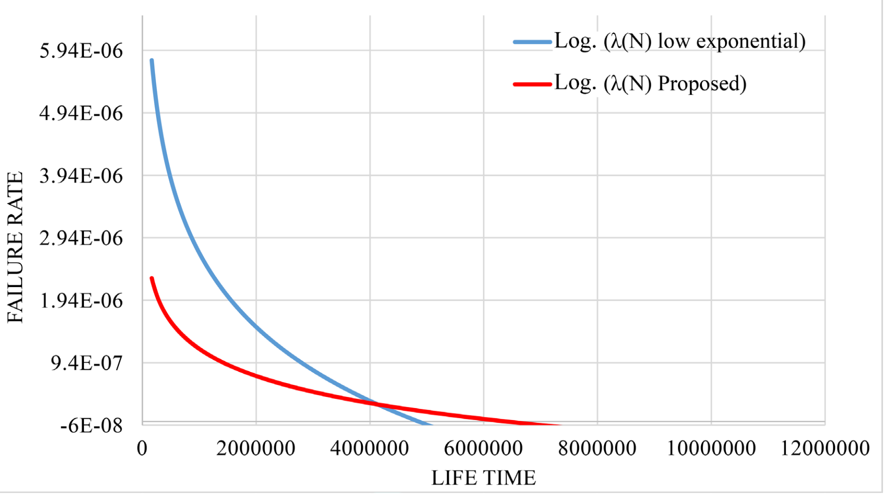

Figure 4. Comparisons of the failure rates of the exponential laws and the proposed law.

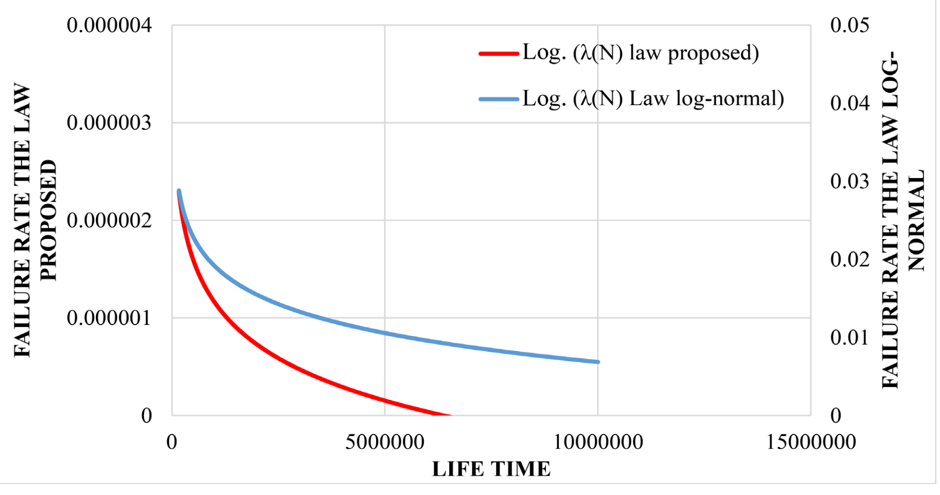

Figure 5. Comparisons of the failure rates of the lognormal distribution and the proposed distribution.

For the same lifetime, the failure rates of the model is very low than that of the exponential law for a cycle less than or equal to 4.3×10^6 on the logarithmic scale. Beyond the latter the failure rate is slightly above the large exponential law.

The results show that for the same lifetime, the failure rate of the model is very low compared to the lognormal distribution.

4. Conclusion

This article was about highlighting a law of reliability analysis of the laws of calculation of the lifetimes of structures under variable amplitude loading. The model is still satisfactory because it takes into account the parameters of the geometric configuration and the dispersion of service life which would be due to the problem of design, defect in the material or manufacture (presence of residual stresses for example) [7]. The advantage of this model is that the constants depend on the number of load repetitions and the lifetimes of the series of parts considered in the test campaign. The most reliable series of parts is the one with a stress amplitude of 60 MPa due to their high life span against a low failure rate.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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