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Applications of Non-Classical Equations and Their Approaches to the Solution of Some of Classes Equations Arise in the Kelvin-Helmholtz Mechanism and Instability

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Abstract

In the presented work, we consider applications of non-classical equations and their approaches to the solution of some classes of equations that arise in the Kelvin-Helmholtz Mechanism (KHM) and instability. In all areas where the Kelvin-Helmholtz instability (KHI) problem is investigated with the corresponding data unchanged, the solution can be taken directly in a specific form (for example, to determine the horizontal structure of a perturbation in a barotropic rotational flow, which is a boundary condition taken, as well as other types of Kelvin-Helmholtz instability problems). In another example, the shear flow along the magnetic field in the Z direction, which is the width of the contact layer between fast and slow flows, has a velocity gradient along the X axis with wind shear. The most difficult problems arise when the above unmentioned equation has singularities simultaneously at points and in this case, our results also remain valid. In the case of linear wave analysis of Kelvin-Helmholtz instability (KHI) at a tangential discontinuity (TD) of ideal magneto-hydro-dynamic (MHD) plasma, it can be attributed to the presented class, and in this case, as far as we know, solutions for eigen modes of instability KH in MHD plasma that satisfy suitable homogeneous boundary conditions. Based on the above mentioned area of application for degenerating ordinary differential equations in this work, the method of functional analysis in order to prove the generalized solution is used. The investigated equation covers a class of a number of difficult-to-solve problems, namely, generalized solutions are found for classes of problems that have analytical and mathematical descriptions. With the aid of lemmas and theorems, the existence and uniqueness of generalized solutions in the weight space are proved, and then general and particular exact solutions are found for the considered problems that are expressed analytically explicitly. Obtained our results may be used for all the difficult-to-solve processes of KHM and instabilities and instabilities, which cover widely studied areas like galaxies, Kelvin-Helmholtz instability in the atmospheres of planets, oceans, clouds and moons, for example, during the formation of the Earth or the Red Spot on Jupiter, as well as in the atmospheres of the Sun and other stars. In this paper, also, a fairly common class of equations and examples are indicated that can be used directly to enter data for the use of the studied suitable tasks.

Keywords

Kelvin-Helmholtz Mechanism and Instability, Ordinary Differential Equations, Weighted Space, Degenerating, Planetary, Jupiter, Non-Classical Approaches

1. Introduction

Many authors (e.g., [1] [2] [3] [4] [5], and the references given therein) investigated mixed type equations and equations of Keldys type (e.g., [6] [7] [8] [9], and the references given therein). Note that a parabolic equation and a mixed equation with changing time direction also have physical applications. The boundary value problems with such sewing conditions appear when modeling, for example, the process of interaction between two reciprocal flows with mutual permeating or when designing certain heat exchangers. Frankly speaking, forward-backward equations (equations of changing time direction) arise in supersonic dynamics, boundary layer theory, and plasma. Therefore, the boundary value problems for equations of mixed hyperbolic-elliptic type with changing time direction (or forward-backward equations) (e.g., [10] [11] and the references therein) present important objects for all investigators. Non-classical models are described in particular by equations of mixed types (for example, the Tricomi equation), degenerate equations (for example, the Keldysh equation), Sobolev type [12] equations (also the Barenblatt-Zsolt-Kachina equation), equations of mixed type with changing time direction, and forward-backward equations. At the same time, we include that the boundary value problems for equations with degeneracy belong to "non-classical" problems of mathematical physics. One of the main difficulties that arise in the theory of degenerate elliptic equations is related to the influence of lower (in the sense of the theory of regular elliptic operators) terms of the equation on the formulation of boundary value problems and their coercive solvability. In this case, degenerating ordinary differential equations (DODE) of second order have physical means as boundary layer as the same physical means with which such sewing conditions appear when modeling, for example, the process of interaction between two reciprocal flows with mutual permeating. These degenerating ordinary differential equations also do not fit into classes of ordinary differential equations (ODE), and may be called non-classical

ODE. It means that, in an analogical one-dimensional case, the non-classical equations having the same boundary layer as for ODE are said to be non-classical ordinary differential equations (NODE). This definition of a non-classical ODE can be justified below the presented physical and analytical descriptions at the first entry in this work. The investigators may carry out citation non-classical ordinary differential equations based on the results of the considered problems, as clearly justified in this paper and introduction of this paper subdivided the subsection, in order to show research background (in subsection 1.1), situation of the former researches analyze and the problems existed of works will be summarized (subsection 1.2) and problems we would like to solve in the paper will be listed (subsection 1.3).

1.1. Non-Classical Ordinary Differential Equations Second-Order and Applications Fields

In the mathematical analytical description process of investigation, first of all we need to give historical information for our research into matters. If the degeneration ordinary differential equations (DODEs) second order has physical meaning (for example, boundary layer), analogical to the one-dimensional case of the non-classical equation of mathematical physics is said to be non-classical ODE second order. This definition of non-classical ordinary differential equations of second order entered for the first time by author M.A. Nurmammadov, is proved in the following applications. The fundamental results for degenerate elliptic equations are due to M. V. Keldysh [6]. The results obtained by him were then developed and generalized by O.A Oleinik [13]. Generalized solutions of degenerate second-order elliptic equations were studied in the works of S. G. Mikhlin [14] and M. I. Vishik [15]. For this paper in order we use the main ideas in the derivation of the Friedrichs inequality with minor ones are transferred to the study of the positivity of operators, and obtain for NODEs the Fridric's-Pouncare inequality, in order to applying for our consideration boundary value problems of ordinary differential equations.

In 1985, the author [4] first considers a new equation and four well-posed boundary value problems, which leads to the equation presented in the form: as it is shown in the work [7]-[17], the theory of boundary value problems for degenerate equations and equations of mixed type, it is a well-known fact that the well-posedness and the class of its correctness essentially depend on the coefficient of the first order derivative (younger member) of equations. Therefore, non-classical ordinary differential equations (NODEs) with second order or for degenerating ODEs also satisfy these facts. In the case of well-posedness, the coefficient of the first order derivative (younger member) of the ODE with second order determines the class of correctness. Accordance to the work from degenerating nonlinear (of power type) mixed type equations considered by author formulated new nonlinear and non-classical ordinary differential equation (NODE) in this paper with physical means (for linear and nonlinear cases is shown in **Figure 1** and **Figure 2**) is expressed. Since the applications of the author of the works are first applied to the problems of astrophysics, planetary and solar systems in space, and a number of phenomena may not be sufficiently studied in works close to this one, there are a large number of open applications that can be an interesting direction for researchers on a global scale, such as new approaches indicated by the scientific field after the appearance of this article. In the Planetary boundary layer, considering object investigations, this work can include the following field applications. Within the lowest portion of the planetary boundary, a semi-empirical log wind profile is commonly used to describe the vertical distribution of horizontal mean wind speeds. The simplified equation that describes it could be from ordinary differential equations. Due to the limitation of observation instruments and the theory of mean values, the levels (z) should be chosen where there is enough difference between the measurement readings. If one has more than two readings, the measurements can be fit to the above equation to determine the shear velocity. The stellar atmosphere is the outer region of the volume of the star, lying above the stellar core, the radiation zone, and the convection zone. The stellar atmosphere is divided into several distinct regions: an unsolved problem in stellar astrophysics is how the corona can be heated to such high temperatures. The answer lies in the magnetic field, but the exact mechanism remains unclear. Another side is also the Kelvin-Helmholtz instability (KHI), which is the instability of a fluid that occurs when there is a velocity shear rate in one continuous fluid or a velocity difference at an interface between two fluids. Therefore, accounting for the above information, we say that Kelvin-Helmholtz instabilities are visible in the atmospheres of planets and moons, for example, in the formation of the Earth or the Red Spot on Jupiter, and in the atmospheres of the Sun and other stars (e.g., [18]-[24] and therein). Shear flows in plasmas are attracting a lot of interest because of their suppression effect on turbulence in magnetically confined fusion devices. The current naïve suggestion regarding this phenomenon holds that the stretching of modes in a shear flow brings about length scale reduction leading to the suppression of fluctuations. This argument for stability, however, ignores the fact that the available free energy associated with a shear flow may be a potent source for the destabilization of some other class of fluctuations (e.g., [18] [19] [20]). The Kelvin-Helmholtz (KH) instability, for instance, is a well-known example of an instability that feeds on the ambient flow-energy [23]. Linear wave patterns in Jupiter's clouds with wavelengths strongly clustered around 300 km are commonly observed in the planet's equatorial atmosphere (see [24]). In the work [25] proposed that the preferred wavelength is related to the thickness of an unstable shear layer within the clouds (see also [26]). If numerically analyze the linear stability of wavelike disturbances that have nonzero horizontal phase speeds in Jupiter's atmosphere and find that, if the static stability in the shear layer is very low (but still nonnegative), a deep vertical shear layer like the one measured by the Galileo probe can generate the instabilities [27] The fastest growing waves grow exponentially within an hour, and their wavelengths match the observations. Close to zero values of static stability that permit the growth of instabilities are within the range of values measured by the Galileo probe in a hot spot and the model probes Jupiter's equatorial atmosphere below the cloud deck and suggests that thick regions of wind shear and low static stability exist outside hot spots [28]. Note that all corresponding figures are relative to investigations of KHM and instabilities can be founded from the list of astrophysical and astronomical processes that arise from KHI problems (from Wikipedia). About KHI and ODE simultaneously, our ideas have the following forms.

Note that above unmentioned authors works arisen ordinary differential equations has been contained degenerated cases, but they do not obtained exactly solutions for its problems, because at the degenerated points to decomposing to the series impossible, since at these points the derivatives does not exists. In order to establish this situation all above considered problems which is analytical mathematical description of KHI beings to the so-called non-classical ordinary differential equations and its corresponding boundary conditions problems needed to prove solvability. After finding class of solvability we find exactly generalized solution, because generalized derivatives any order not dependent from existence of first order. It may be only applying non classical approaches methods. Therefore our method is new and equations also we used also belongs non-classical ordinary differential equations and having physical mean boundary layer, at the same time do not fit classes theory of ODEs. Therefore, our investigation basic objects is generalized theory than until exists theory of ODEs and its applications of physical problems. For this reason we introduce the following mathematical justification of this theory in subsection 1.2 and physical mean illustrated in subsection 1.3.

1.2. History of Non-Classical Equation which is Arises in KHI Problems

Equation of M. V. Keldysh (with strong degenerating at point $y=0$, elliptic equation) [8]:

$$y^{2m+1}u_{yy}+u_{xx}+a(x,y)u_x+b(x,y)u_y+c(x,y)u=f(x,y), y\geq 0 \tag{1}$$

Equation of M. V. Keldysh in model form (weak degenerating at point $y=0$, elliptic equation):

$$yu_{yy}+u_{xx}=f(x,y), y\geq 0 \tag{2}$$

Equation of F. Tricomi in model form:

$$yu_{xx}+u_{yy}=f(x,y) \tag{3}$$

In 1985 year, at first, obtained by author M.A. Nurmammadov (see [4] [29] in page 2 noted the citation of this situation) new equation and four new well-posed boundary value problems, which model of M. V. Keldysh type(degenerating elliptic, degenerating hyperbolic, equations of mixed type, equation of mixed type changing time of directions) (having perpendicular degenerating two lines(two sonic lines) (having physical means of aerodynamic, gas dynamic, fluid dynamics and so on) [4]:

$$K_1(t)u_{tt}+K_2(x)u_{xx}+a(x,t)u_t+b(x,t)u_x+c(x,t)u=f(x,t) \quad (4)$$

$tK_1(t)>0, t\neq 0, t\in(-T,T), xK_x(x)<0, x\neq 0, x\in(-L,L)$ (or $x\neq 0, x\in(a,b), a<0, b>0$)

$$\begin{aligned}&K_1(t)u_{tt}+K_2(x)u_{xx}+a(x,t)u_t+b(x,t)u_x+c(x,t)u+c_1(x)u|u|^\rho\\&=f(x,t,u),\ c_1(x)>0, x\neq 0, \rho>-1\end{aligned} \quad (5)$$

(where *l*, *a*, *b* are numbers). Nonlinear case:

$$Lu=k(x)u_{xx}+b(x)u_x+c(x)u+c_1(x)u|u|^\rho=f(x)$$

where,

$$c_1(x)>0, x\neq 0, \rho>-1. \quad (6)$$

$$K(x)u_{xx}+b(x)u_x+c(x)u=f(x), \quad (7)$$

$$K(t)u_{tt}+a(t)u_t+c(t)u=f(x), \quad (8)$$

$$\begin{aligned}&K_1(x_1)u_{x_1x_1}+K_2(x_2)u_{x_2x_2}+a(x_1,x_2)u_{x_1}+b(x_1,x_2)u_{x_2}+c(x_1,x_2)u\\&=f(x_1,x_2),\end{aligned} \quad (9)$$

$$x_1K_1(x_1)>0, x_1\neq 0, x_1\in(-T,T), x_2\in(-L,L) \quad (x\in(a,b), a<0, b>0)$$

(where *L*, *a*, *b* are numbers).

Note that the linear ordinary differential equations of (7) for case $xK(x)>0, x\neq 0$, and (8) for $tK(t)>0, t\neq 0$, are degenerates which is corresponds to non-classical equations of second order (4), (9) (for equation of (9) account into the spaces variables x_1, x_2), respectively. At the same time that the nonlinear ordinary differential equations of (5) in case of $c_1(x)>0, x\neq 0, \rho>-1$, $xK(x)>0, x\neq 0$, is degenerate which is corresponds to non-classical equations of second order (5). We will deal with equations of (6), (7), (8). While, in the world many author considered: i) case $p(x)>0, x>0$, ii)-case $p(x)\geq 0, x>0, p(0)=0, q(x)>0$

$$Lu=-\frac{\mathrm{d}}{\mathrm{d}x}\left(p(x)\frac{\mathrm{d}u}{\mathrm{d}x}\right)-q(x)u=f(x), a\leq x\leq b, \quad (10)$$

Or in terminology Equation (4):

$$Lu=-(K(x)u_x)_x+bu=f(x),\ K(x)>0, x>0,$$
$$\text{but } K(0)=0, b\geq 0, \text{ is number} \quad (11)$$

The mechanism was originally proposed by Kelvin and Helmholtz in the late nineteenth century to explain the source of energy of the Sun. By the mid-nineteenth century, conservation of energy conservation had been accepted, and one consequence of this law of physics is that the Sun must have some energy source to continue to shine. Because nuclear reactions were unknown, the main candidate for the source of solar energy was gravitational contraction problems almost equations being the following degenerating ordinary differential equations when $K(0)=0, K(-L)=K(L)=0$, also is $b(0)=0$,

$K(x)u_{xx} + b(x)u_x + c(x)u = f(x)$, $-L < x < L$, at the points:

$$x = L, x = -L, x = 0 \tag{12}$$

equation degenerate, which can be considered as a one dimensional alternative of a second order elliptic, mixed types equations degenerating (above written as (1), (4), (5), (6), (7), (8)) on the boundary of the domain (interval, plane suffuse, and so on). Finally, we note that in mathematically senses, this presented investigation equations are new ordinary differential equations and its physical means at first which is no considered with others authors and considered our ordinary differential equations include:

i) Presented difficult solvable of NODE is degenerating coefficient of first and second order derivatives;

ii) Nonlinearity of power type case—so called quasi-linear degenerating coefficient of first and second order derivatives;

iii) In given intervals coefficients of degenerating cases of NODEs are arbitrarily changes;

iv) Correctness of well-posed problems depends from coefficients higher and younger terms of derivatives;

v) Presented NODEs contains all possible cases classes of ordinary differential equations which arises KHI;

vi) Generalized solution at the same time is exact solution and common case can be used for all problems of KHI which being to ODEs;

vii) The main results of our investigations consist of finding generalized solutions to DODE or NODE and proving its solvability (*i.e.* existence and uniqueness of problems), after showing that our results are applicable to all problems of KHI which is being to the ODE;

viii) In spite of difficult problems that arise when the above aforementioned equation has singularities and degenerating cases simultaneously, at points and in this case, our results also remain valid, in the case of linear wave analysis of Kelvin-Helmholtz instability (KHI) at a tangential discontinuity (TD) of ideal magneto-hydro-dynamic (MHD) plasma, it can be attributed to the presented class, and in this case, as far as we know, solutions for eigen modes of instability KH in MHD plasma that satisfy suitable nonhomogeneous boundary conditions. Our results are establishing continuity at points of degenerating and singularity. Above listed i) - viii) are belongs to our objects investigations.

1.3. Illustration Hydrodynamic, Aerodynamic and Gas-Dynamic Physical Meaningful

In case of main non-classical Equations ((4), (6), (9)) (which is at first time obtained by author), also at first time in complex form transition of subsonic, sonic and supersonic for hydrodynamic, aerodynamic, gas-dynamic meaningful which for each equations using with the number Mache its corresponding physical process was presented (see, **Figure 1** is for Equations (4), (5), (9), but **Figure 2** for Equations (6), (12)) (also take nonhomogeneous B.V.C.):

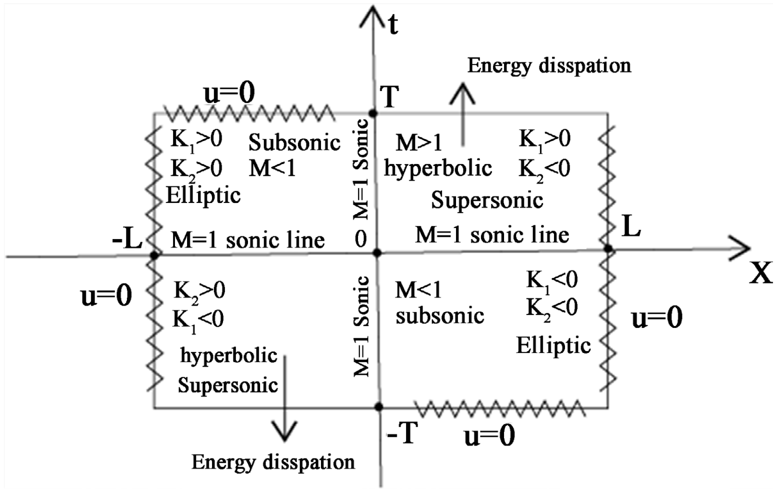

Figure 1. Depicts physical process transitions in gas dynamics, aerodynamics, and hydrodynamics for Equations (4), (5), and (9).

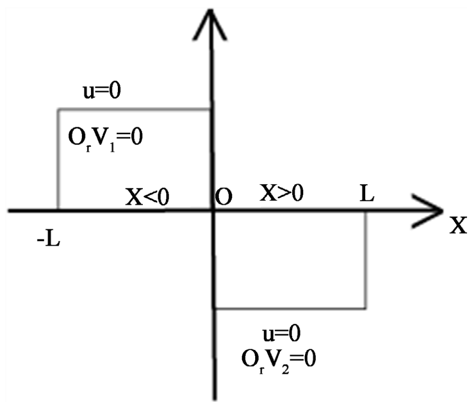

Figure 2. Depicts physical process transitions in gas dynamics, aerodynamics, and hydrodynamics for Equations (6) and (12).

2. Main Results: Solvability of Boundary Value of Problems for General Degenerates Ordinary Equations which Is Arise in KHM and Instabilities

2.1. Statements Well-Posed Problem and Auxiliary Lemmas

Boundary value problem. To find solution of Equation (12), such that satisfy the following boundary conditions

$$u(-L) = u(L) = 0 \tag{13}$$

By the symbol C_L, we denote a class of twice continuously differentiable functions in the closed domain $D = [-L, L]$ satisfying the boundary conditions (13), by $H_1(-L, L)$, Sobolev's space (see [10] [11]) with weighted spaces obtained by the class C_L, which is closed by the norm:

$$\|u\|_{H_1}^2 = \int_{-L}^{L} \left(|K(x)| u_x^2 + u^2 \right) \mathrm{d}x \tag{14}$$

Lemma 2.1. Let there exists $\delta > 0, \delta_0 > 0$ and $M > 0$ constants, such that

(i) $2b(x)-K_x(x)\ge\delta$, $x\in\bar{D}^-=\{x:x<0\}$; (ii) $b(x)\ge\delta_0$, $x\in\bar{D}^+=\{x:x>0\}$, (iii) $|K_x(x)|^2\le m|K(x)|, x\in\bar{D}$.

Then there exists constants λ_0,μ_0 are such that for any $u(x)\in C_L$ the inequalities are holds true:

$$\int_{-L}^{L}\mathrm{e}^{-\lambda_0 x}u_x Lu\mathrm{d}x+\mu_0\int_{-L}^{0}uLu\mathrm{d}x\ge m\|u\|_{H_1}^2,\ \|Lu\|_{L_2}^2\ge m_1\|u\|_{H_1}^2,\ \forall u(x)\in C_L. \quad (15)$$

As well as known that, ODEs of second order which is obtain from the elliptical types equation second order as one dimensional analogical sense, then in this case is true the inequality of Frederic's-Poincare, In order to apply non-classical approaches (method functional analyses) it is important prove it. Following as methods of work (e.g, [8] [9] [10] [11]) integrating by parts and account into boundary conditions, using inequality of Cauchy's we can prove inequalities (15).

Lemma 2.2. (For ODE, Frederic's-Poincare,). If the function $u(x)\in\overset{0}{H}_1$ then the following inequality $\|u\|_{L_2(D)}^2\le m(D)\|u\|_{\overset{0}{H}_1(D)_1}^2$ is holds true.

Proof. It is sufficient to take $u(x)$ continuously differential function vanishing on ∂D, and to take ∂D, to be the interval $x\in[-L,L]$. Then the boundary conditions on $u(x)$ allow us to write $u(x)=\int_{-L}^{x}u_x(t)\mathrm{d}t$. By the Schwarz inequality we have

$$u(x)\le\left[\int_{-L}^{x}u(t)\mathrm{d}t\right]^{\frac{1}{2}}\left[\int_{-L}^{x}u_x^2(t)\mathrm{d}t\right]^{\frac{1}{2}}\le m\left[\int_{-L}^{L}u_x^2(t)\mathrm{d}t\right]^{\frac{1}{2}} \quad (16)$$

where m is a positive constant which depends on interval $[-L,L]$, squaring both sides we obtain

$$u^2(x)\le m^2\left[\int_{-L}^{x}u_x^2(t)\mathrm{d}t\right]\le m^2(L)\int_{-L}^{x}u_x^2(t)\mathrm{d}t. \quad (17)$$

If we integrate both sides with repeat to x between $-L$ and L, we multiply the constant on the right by a new constant which also depends on interval, as $m(L)$, then we have

$$\begin{aligned}&\int_{-L}^{x}u^2(x)\mathrm{d}x\le m^\bullet(L)\int_{-L}^{L}u_x^2(x)\mathrm{d}x\\&\le m^\bullet(L)\int_{-L}^{L}\left[|K(x)|u_x^2(x)+u^2(x)\right]\mathrm{d}x=m^\bullet(L)\|u\|_{\overset{0}{H}_1([-L,L])_1}^2\end{aligned} \quad (18)$$

This completes the proof of Lemma 1.2, which has been proved for the first time for a non-classical ordinary differential Equation (12) (or degenerating ODE second order).

Thus, let us to introduce the following notations: $J_1=\int_0^L\frac{1}{K(x)}\mathrm{d}x$, $J_2=\int_{-L}^{0}\frac{1}{K(x)}\mathrm{d}x$, and preliminary functions

$$K_1(x)=\begin{cases}\int_0^x \frac{1}{K(x)}\mathrm{d}x, J_1<\infty\\ \int_x^L \frac{1}{K(x)}\mathrm{d}x, J_1=\infty\end{cases}, \quad K_2(x)=\begin{cases}\int_{-L}^x \frac{1}{K(x)}\mathrm{d}x, J_2<\infty\\ \int_x^0 \frac{1}{K(x)}\mathrm{d}x, J_2=\infty\end{cases} \tag{19}$$

Let $\overset{0}{W}{}_2^1(-L,L)$ is the closed classes functions of $C_0^\infty(-L,L)$ with the norm: $\|u\|_{\overset{0}{W}{}_2^1}=\int_{-L}^{L}\left[u_x^2+u^2\right]\mathrm{d}x$. Denote the new Hilbert space, which is completed by the norm $\|u\|_{\overset{0}{H}_1}=\int_{-L}^{L}K(x)u_x^2\mathrm{d}x$. Now, introducing the function (averaging)

$$\omega_1^\delta(x)=\begin{cases}0, 0\le x\le\delta\\ \left|\ln\left(K_1(\delta)\right)\right|^{\varepsilon_1}-\left|\ln\left(K_1(x)\right)\right|^{\varepsilon_1}, \delta\le x\le\delta_1\\ 1, \delta_1\le x\le L\end{cases} \tag{20}$$

$$\omega_2^\delta(x)=\begin{cases}0, -\delta_2\le x\le 0\\ \left|\ln\left(K_2(\delta)\right)\right|^{\varepsilon_2}-\left|\ln\left(K_2(x)\right)\right|^{\varepsilon_2}, -\delta_3\le x\le-\delta_2 \quad \text{where} \quad \varepsilon_2\le\varepsilon_1\le\frac{1}{2}\\ 1, -L\le x\le-\delta_2\end{cases} \tag{21}$$

and δ_1,δ_2 chosen from the relations: $\left|\ln\left(K_1(\delta)\right)\right|^{\varepsilon_1}-\left|\ln\left(K_1(x)\right)\right|^{\varepsilon_1}=1$ and $\left|\ln\left(K_2(\delta)\right)\right|^{\varepsilon_2}-\left|\ln\left(K_2(x)\right)\right|^{\varepsilon_2}=1$. Since, $\delta_1\le\delta$, $\delta_1\to 0$ for $\delta\to 0$ and

$$\int_\delta^{\delta_1}\omega_1^\delta(x)\mathrm{d}x=1;\quad \delta_3\le\delta_2,\quad \delta_2\to 0,\quad \int_{\delta_3}^{\delta_2}\omega_1^\delta(x)\mathrm{d}x=1; \tag{22}$$

then for any $u(x)\in\overset{0}{H}_1$ which is weighted space, $\omega_1^\delta(x)\in\overset{0}{W}{}_2^1(0,L)$, $\omega_2^\delta(x)\in\overset{0}{W}{}_2^1(-L,0)$, additionally, may satisfy: $\omega_1^\delta(x)\in\overset{0}{H}_1(0,L)$, $\omega_2^\delta(x)\in\overset{0}{H}_1(-L,0)$. For any function $\upsilon(x)\in\overset{0}{H}_1$ setting the new functions: $\upsilon_1^\delta(x)=\omega_1^\delta(x)\upsilon(x)$, $\upsilon_2^\delta(x)=\omega_2^\delta(x)\upsilon(x)$, additionally scalar products in the space L_2 the following:

$$\left(u,\upsilon_1^\delta(x)\right)_{L_2(D^+)}=\int_0^L K(x)u_x\upsilon_1^\delta(x)\mathrm{d}x,\ \left(u,\upsilon_2^\delta(x)\right)_{L_2(D^-)}=\int_{-L}^0 K(x)u_x\upsilon_2^\delta(x)\mathrm{d}x \tag{23}$$

Definition 2.1. The function $u(x)\in\overset{0}{H}_1$ is said to be generalized solution of the problem (12), (13) if for any $\upsilon(x)\in\overset{0}{H}_1$ and $\delta>0$, the following identity holds:

$$\begin{aligned}&\left(u,\upsilon_1^\delta(x)\right)_{L_2(D^+)}+b\left(u,\upsilon_2^\delta(x)\right)_{L_2(D^-)}-b\left[\left(u_x,\upsilon_1^\delta(x)\right)_{L_2(D^+)}+\left(u,\upsilon_2^\delta(x)\right)_{L_2(D^-)}\right]\\&=\left(f,\upsilon_1^\delta(x)\right)_{L_2(D^+)}+\left(f,\upsilon_2^\delta(x)\right)_{L_2(D^-)}.\end{aligned} \tag{24}$$

Lemma 2.3. If the function $u(x)\in\overset{0}{H}_1$ is generalized solution of the problem (12), (13), then there exist the following finite limits:

$\underset{\delta\to 0}{Lim}\left[\left(u_x,\upsilon_1^\delta(x)\right)_{L_2(D^+)}\right]=\underset{\delta\to 0}{Lim}\left[-\frac{1}{2}\int_0^\delta \omega_1^\delta(x)u^2(x)\mathrm{d}x\right]=-m_1^2\le 0$, hence also is $m_1=0$, for $J_1<\infty$,

$\underset{\delta\to 0}{Lim}\left[\left(u_x,\upsilon_1^\delta(x)\right)_{L_2(D^-)}\right]=\underset{\delta\to 0}{Lim}\left[-\frac{1}{2}\int_0^\delta \omega_2^\delta(x)u^2(x)\mathrm{d}x\right]=-m_2^2\le 0$, hence also is $m_2=0$, for $J_2<\infty$. Proof. From the equality of (24) we have

$$\begin{aligned}&\int_0^L\left[K(x)u_x^2\upsilon_1^\delta(x)+K(x)\upsilon_1^\delta(x)u-b\upsilon_1^\delta(x)u\right]\mathrm{d}x\\&+\int_{-L}^0\left[K(x)u_x^2\upsilon_2^\delta(x)+K(x)\upsilon_2^\delta(x)u-b\upsilon_2^\delta(x)u\right]\mathrm{d}x\\&=\int_0^L f(x)u(x)\mathrm{d}x+\int_{-L}^0 f(x)u(x)\mathrm{d}x\end{aligned}\tag{25}$$

Hence, for $\delta\to 0$ we get:

$$\int_0^L K(x)u_x^2\mathrm{d}x+bm_1^2+bm_2^2+\int_{-L}^0 K(x)u_x^2\mathrm{d}x=\int_0^{L_1} f(x)u(x)\mathrm{d}x+\int_{-L_2}^0 f(x)u(x)\mathrm{d}x\tag{26}$$

Since, for $\delta\to 0$ the following integrals are converges:

$\int_0^L K(x)\omega_1^\delta u_x^2\mathrm{d}x\to\int_0^L K(x)u_x^2\mathrm{d}x$. By virtue of condition, when $\delta\to 0$ the $\omega_1^\delta\to 1$ and hence, we get $\int_0^L\omega_1^\delta u_x u\mathrm{d}x\to 0$ when $\delta\to 0$.

$\int_{-L}^0 K(x)\omega_1^\delta u_x^2\mathrm{d}x\to\int_{-L}^0 K(x)u_x^2\mathrm{d}x$. Also, by virtue of condition, when $\delta\to 0$ the $\omega_2^\delta\to 1$ and hence, we get $\int_{-L}^0\omega_2^\delta u_x u\mathrm{d}x\to 0$ when $\delta\to 0$. Indeed in cases of when, $J_1<\infty$, $J_2<\infty$ the integrals $\int_0^L\omega_1^\delta u_x u\mathrm{d}x$ and $\int_{-L}^0\omega_2^\delta u_x u\mathrm{d}x$ can be estimates. Therefore, analogically, in cases $J_1=\infty$, $J_2=\infty$ the integrals are converges

$$\int_0^L f(x)\omega_1^\delta u\mathrm{d}x\to\int_0^L f(x)u\mathrm{d}x,\ \text{and}\ \int_{-L}^0 f(x)\omega_2^\delta u\mathrm{d}x\to\int_{-L}^0 f(x)u\mathrm{d}x.\tag{27}$$

From the lemma 2.1, 2.2, 2.3 it follows that for weakly degenerating case in equation of (12) the term of $K(x)u_x$ is small term and $u(-L)=u(L)=0$, keeps meaningful. But for strong degenerating case consider the following notations:

$$\text{Let } J_1=\infty,\ J_2=\infty \text{ and } J_1=\int_0^L\frac{x}{K(x)}\mathrm{d}x<\infty,\ J_2=\int_{-L}^0\frac{x}{K(x)}\mathrm{d}x<\infty.\tag{28}$$

Describe the common solution in case of Equation (11) in the form

$u(x)=C_1\left[\int_x^L\frac{\mathrm{e}^{-bK_1(t)}}{K(t)}\mathrm{d}t+\int_{-L}^x\frac{\mathrm{e}^{-bK_2(t)}}{K(t)}\mathrm{d}t\right]+C_2$ where C_1,C_2 are arbitrarily constants. Then we can write special solution of nonhomogeneous Equation (11) in the form

$$u_{specisl}(x)=\left[\int_0^L \frac{-1+e^{-b(K_1(x)-K_1(t))}}{b(t)}+\int_{-L}^0 \frac{-1+e^{-b(K_2(x)-K_1(t))}}{b(t)}\right] f(t)\,dt \tag{29}$$

Remark 2.1. If $b(x)=b$, $b>0$ is number, then

$$u_{specisl}(x)=\frac{1}{b}\left[\int_0^L\left[-1+e^{-b(K_1(x)-K_1(t))}\right]+\int_{-L}^0\left[-1+e^{-b(K_2(x)-K_1(t))}\right]\right] f(t)\,dt. \tag{30}$$

Lemma 2.4. Let fulfilled the Lemma 2.1 and Lemma 2.2 Then, for any $f(x)\in L_2(-L,L)$

$$\lim_{x\to 0}\left[\int_0^L \frac{-1+e^{-b(K_1(x)-K_1(t))}}{b(t)}+\int_{-L}^0 \frac{-1+e^{-b(K_2(x)-K_1(t))}}{b(t)}\right] f(t)\,dt=0. \tag{31}$$

Proof. Let's decompose (22) in the following

$$\begin{aligned} u_{specisl}(x)=&\left[-\int_0^L \frac{f(t)}{b(t)}dt-\int_{-L}^0 \frac{f(t)}{b(t)}dt\right]+\left[\int_{-L}^0 \frac{e^{-b(K_2(x)-K_1(t))}}{b(t)} f(t)\,dt\right. \\ &\left.+\int_0^L \frac{e^{-b(K_1(x)-K_1(t))}}{b(t)}dt\right] \end{aligned} \tag{32}$$

In equality of (24) the expression for the first bracket tends to zero and the second bracket, after estimation and applying the rule of L'Hopital, can be asserted by Lemma 2.4.

2.2. The Theorems of Existence and Uniqueness of Generalized Solution of Problem (12), (13)

Theorem 2.1. Let be satisfied Lemma 2.1, 2.2, 2.3 2.4 and i) $|2b(x)-K_x(x)|\ge\delta>0$, ii) $2c(x)-b_x(x)-K_{xx}(x)\ge\delta_0>\delta_1>0$, iii) $|K_{2x}(x)|^2\le m|K(x)|$, $x\in(-l,l)$ are satisfied. Then for any function from the class $f(x)\in L_2(-l,l)$ there exists the generalized solution of boundary value problem of (12), (13) from the space $u(x)\in \overset{0}{W}{}_2^1(-l,l)\cap \overset{0}{H}_1(-l,l)$.

Proof. Using definition of generalized solution $u(x)\in\overset{0}{H}_1$ and $\upsilon(x)\in\overset{0}{H}_1$ integrate the equality (24):

$$\begin{aligned} &\left(u,\upsilon_1^\delta(x)\right)_{L_2(D^+)}+b\left(u,\upsilon_2^\delta(x)\right)_{L_2(D^-)}-b\left[\left(u_x,\upsilon_1^\delta(x)\right)_{L_2(D^+)}+\left(u,\upsilon_2^\delta(x)\right)_{L_2(D^-)}\right] \\ &=\left(f,\upsilon_1^\delta(x)\right)_{L_2(D^+)}+\left(f,\upsilon_2^\delta(x)\right)_{L_2(D^-)}. \end{aligned} \tag{33}$$

and account into results of lemma 2.1, 2.2, 2.3, 2.4, and with aid a prior estimation and boundary condition (13) obtain that generalized solution is

$u(x)\in\overset{0}{W}{}_2^1(-l,l)\cap\overset{0}{H}_1(-l,l)$.

Theorem 2.2. Let be satisfied conditions of Theorem 2. Then, for any function of $f(x)\in L_2(-l,l)$. The generalized solution of problem (12), (13) from the class $u(x)\in\overset{0}{W}{}_2^1(-l,l)\cap\overset{0}{H}_1(-l,l)$ is unique.

Proof. **Assume that there are two distinct generalized solutions to problem (12).** (13) from the class $u_1(x), u_2(x) \in \overset{0}{W}{}_2^1(-l,l) \cap \overset{0}{H}_1(-l,l)$. Take the function $u(x) = u_1(x) - u_2(x)$ using inequality of (14) we can write the following inequality $\|Lu\|_{L_2}^2 \geq m_1 \|u\|_{H_1}^2$, hence we get $0 \geq m_1 \|u\|_{H_1}^2$, and it means that $u(x) = u_1(x) - u_2(x) = 0$, *i.e.* $u_1(x) = u_2(x)$.

Remark 2.2. Smoothness the generalized solution of problem (12), (13) from the space $u(x) \in \overset{0}{W}{}_2^1(-l,l) \cap \overset{0}{H}_1(-l,l)$ can be prove as it shown by author's in the works (e.g., [13] [14] [15] [16]).

3. Some Class Equation Solutions Arise in the KHM and Instability

3.1. Some of Solution of Class Equations which is Presented by Its Coefficients

Let's take for possible cases class equations for general non-classical ordinary equations (12), (in particular case of (11)) by selection the coefficients $K(x), b(x), c(x)$ which is corresponding arises in the different problems of KHM and instabilities

1) $K(x) = x(L-x)$ $x = 0, x = L$ $b(x) \geq 0, (b(x) > 0)$ or the number ($b > 0$)

$$Lu = x(L-x)u_{xx} + b(x)u_x + c(x)u = f(x) \tag{34}$$

2) $K(x) = (1-x)^2$ $x = -1, x = +1$, $b(x) \geq 0, (b(x) > 0)$ or the number ($b > 0$)

$$Lu = (1-x)^2 u_{xx} + b(x)u_x + c(x)u = f(x) \tag{35}$$

3) $K(x) = x(1-x^2)$ $x = -1, x = +1$ $x = 0$, and others singularities of $b(x)$,

$$Lu = (L-x)^2 u_{xx} + b(x)u_x + c(x)u = f(x) \tag{36}$$

4) $K(x) = x(L-x)^2$ $x = 0, x = L$ $b(x) \geq 0, (b(x) > 0)$ or the number ($b > 0$)

$$Lu = x(1-x)^2 u_{xx} + b(x)u_x + c(x)u = f(x) \tag{37}$$

5) $K(x) = L^2 - x^2$ $x = L, x = -L$ $b(x) \geq 0, (b(x) > 0)$ or the number ($b > 0$)

$$Lu = (L^2 - x^2)u_{xx} + b(x)u_x + c(x)u = f(x) \tag{38}$$

I case: $Lu = K(x)u_{xx} + b(x)u_x + c(x)u = f(x)$, $-L < x < L$, at points $x = L, x = -L, x = 0$ $u(-L) = 0$, $u(L) = 0$ and (dependence of problems sometimes $u(0) = 0$, $u_x(L) = 0$, in case of $K(x) = x(L-x)$).

Generalized solution of (8) is

$$u(x) = C_1 e^{P(x)} - \int_x^0 \frac{e^{K_2(x)-K_2(t)}}{b(t)} f(t)\,dt + C_2 e^{P_1(x)} - \int_x^L \frac{e^{K_1(x)-K_1(t)}}{b(t)} f(t)\,dt \tag{39}$$

where denoted by the

$$P(x)=\int_x^L \frac{c(t)}{b(t)}\mathrm{d}t,\ \ P_1(x)=\int_x^L \frac{c(t)}{t(L-t)}\mathrm{d}t \tag{40}$$

Hence, by the boundary conditions of (13) we have

$$C_1=-\left(\int_0^L \frac{\mathrm{e}^{-K_1(t)}}{t(L-t)}\right)\Big/\left(\int_0^L \frac{c(t)\mathrm{d}t}{t(L-t)}\right), \tag{41}$$

$$\begin{aligned} C_2=&\left[\int_{-L}^{L}\frac{\mathrm{e}^{-K_1(t)}}{t(L-t)}f(t)\mathrm{d}t\left(\int_{-L}^{0}\frac{\mathrm{e}^{-K_2(t)}}{t(L-t)}f(t)\mathrm{d}t\right)\right.\\ &\left.+\left[\int_0^L\frac{\mathrm{e}^{-K_1(t)}}{t(L-t)}f(t)\mathrm{d}t\left(\int_{-L}^{L}\frac{\mathrm{e}^{-K_1(t)}}{t(L-t)}f(t)\mathrm{d}t\right)\mathrm{e}^{\int_{-L}^{0}\frac{c(t)\mathrm{d}t}{b(t)}}\right]\Big/\left(-\int_0^L\frac{c(t)\mathrm{d}t}{b(t)}\right)\right]\Big/\int_{-L}^{L}\frac{c(t)}{t(L-t)} \end{aligned} \tag{42}$$

II case. when $b(x)=b>0$ is number then

$$u(x)=C_1\mathrm{e}^{P(x)}-\frac{1}{b}\int_x^0 \mathrm{e}^{K_2(x)-K_2(t)}f(t)\mathrm{d}t+C_2\mathrm{e}^{P_1(x)}-\frac{1}{b}\int_x^0 \mathrm{e}^{K_2(x)-K_2(t)}f(t)\mathrm{d}t \tag{43}$$

where denoted $P(x)=\frac{1}{b}\int_x^L c(t)\mathrm{d}t$, $P_1(x)=\int_x^L\frac{c(t)}{K(t)}\mathrm{d}t$,

$$C_1=-\left[\int_0^L\frac{\mathrm{e}^{-K_1(t)}}{K(t)}\right]\Big/\left[\int_0^L\frac{c(t)\mathrm{d}t}{K(t)}\right],$$

$$\begin{aligned} C_2=&\left[\int_{-L}^{L}\frac{\mathrm{e}^{-K_1(t)}}{K(t)}f(t)\mathrm{d}t\left(\int_{-L}^{0}\frac{\mathrm{e}^{-K_2(t)}}{K(t)}f(t)\mathrm{d}t\right)\right.\\ &\left.+\left[\int_0^L\frac{\mathrm{e}^{-K_1(t)}}{K(t)}f(t)\mathrm{d}t\left(\int_{-L}^{L}\frac{\mathrm{e}^{-K_1(t)}}{K(t)}f(t)\mathrm{d}t\right)\mathrm{e}^{\frac{1}{b}\int_{-L}^{0}c(t)\mathrm{d}t}\right]\Big/\mathrm{e}^{-\frac{1}{b}\int_0^L c(t)\mathrm{d}t}\right]\Big/\int_{-L}^{L}\frac{c(t)\mathrm{d}t}{K(t)} \end{aligned} \tag{44}$$

III case when

$$Lu=-\left(k(x)u_x\right)_x-b(x)u_x=g(x), \tag{45}$$

$$\begin{aligned} u(x)=&\int_x^L\frac{\left[\mathrm{e}^{-b\left[K_1(t)-K(t)\right]}-1\right]g(t)}{K(t)}\mathrm{d}t+C_1\int_x^L\frac{\mathrm{e}^{-b(t)K_1(t)}}{K(t)}\mathrm{d}t\\ &+\int_x^0\frac{\left[\mathrm{e}^{-b\left[K_2(t)-K_2(t)\right]}-1\right]g(t)}{K(t)}\mathrm{d}t+C_2\int_x^0\frac{\mathrm{e}^{-b(t)K_2(t)}}{K(t)}\mathrm{d}t \end{aligned}$$

Hence, by the boundary conditions of (13) we have

$$C_1=\left[\left[\int_{-L}^{0}\frac{[\mathrm{e}^{-b(t)K_2(t)}\mathrm{d}t}{K(t)}+\int_{-L}^{L}\frac{\left[\mathrm{e}^{-b(t)K_2(t)}-1\right]g(t)\mathrm{d}t}{K(t)}\right]\int_{-L}^{0}g(t)\mathrm{d}t\right]\Big/\int_0^L\frac{\mathrm{e}^{-b(t)K_2(t)}\mathrm{d}t}{K(t)}, \tag{46}$$

$$C_2=-\left[\int_0^L\frac{\left[\mathrm{e}^{-K_2(t)}-1\right]g(t)\mathrm{d}t}{K(t)}\right]\Big/\left[\int_0^L\frac{\mathrm{e}^{-b(t)K_2(t)}\mathrm{d}t}{K(t)}\right] \tag{47}$$

3.2. Applicable Cases (About 50 Problems Selected We Can Not Able Include Its)

Example 3.1. Comparison between barotropic and Kelvin-Helmholtz instability: the differential equation determining the horizontal structure of a perturbation in barotropic rotational flow has been stated in class. In the form which will be most evocative to us, it is: $(U-c)\left[\bar{\psi}_{yy}-k^2\bar{\psi}\right]+\left(\beta^2-U_y\right)\bar{\psi}=0$, $K(\xi)=U-c$, $b(\xi)=(U-c)k^2+\left(\beta^2-U_y\right)$ at the point $U=c$ is degenerate. In the analysis of barotropic instability, this is labeled the Rayleigh equation, and its similarity to the Taylor-Goldstein equation determining *vertical* structure of irrational flows is striking. In point of fact, they are actually the same equation with different antecedents.

Example 3.2.

$$(U-c)^2\left[\frac{\mathrm{d}^2\bar{\Phi}}{\mathrm{d}^2 z}-\alpha^2\bar{\Phi}\right]+\left(N^2-(U-c)\frac{\mathrm{d}^2\bar{U}}{\mathrm{d}^2 z}\right)\bar{\Phi}=0, \tag{48}$$

$(U-c)^2\dfrac{\mathrm{d}^2\bar{\Phi}}{\mathrm{d}^2 z}-(U-c)^2\alpha^2\bar{\Phi}+\left(N^2-(U-c)\dfrac{\mathrm{d}^2\bar{U}}{\mathrm{d}^2 z}\right)\bar{\Phi}=0$, $K(\xi)=(U-c)^2$, $b(\xi)=N^2-(U-c)$ where $N^2=\dfrac{g}{L_p}$ denotes by the Brunt-Vaisala frequency. The eigenvalue parameter of the problem is c. If the imaginary part of the wave speed wave speed C is positive, then the flow is unstable, and the small perturbation introduced to the system is amplified in time (the coefficients of this equation put in corresponding formulas of solution directly problem is solvable).If top and bottom be rigid walls, then from linearized covering equations of continuity case boundary condition is $\bar{\Phi}(0)=0,\bar{\Phi}(d)=0$, and when $N^2=(U-c)$ then equation is no degenerating: $\dfrac{\mathrm{d}^2\bar{\Phi}}{\mathrm{d}^2 z}-\alpha^2\bar{\Phi}=0$, and may be solved as problem of Sturm-Lovell (in simple form). Note that the argument is unchanged, if the top and bottom are at $z=\infty, z=-\infty$. The Equation (48) constitute eigenvalue problem where $c=c_r+ic_i$ is the eigenvalue, if $c_i>0$ instability occurs. But the generalized solution of (48) can be expressed by means of formulas (39)-(42), if substitute the corresponding coefficients and boundary conditions.

Example 3.3. $Lu=k(x)u_{xx}+b(x)u_x+c(x)u+c_1(x)u|u|^{\rho}=f(x)$ case corresponding of equation

$$\begin{aligned}&\xi\left(1-\xi^2\right)R_{\xi\xi}-\left(1-4\xi^2\right)R_x-R\left(R^2-2\right)\xi\\&=\xi\left(1-\xi^2\right)R_{\xi\xi}-\left(1-4\xi^2\right)R_x-2\xi R+\xi R|R|^2=0\end{aligned} \tag{49}$$

In case of $K(\xi)=\xi\left(1-\xi^2\right)$, $b(\xi)=1-4\xi^2$, $C(\xi)=2\xi$, $C_1(\xi)=\xi$, $\rho=2$ are identically equations and at the points $\xi=0,\xi=1,\xi=-1$ is degenerating. Moreover, the coefficient $b(\xi)$, at point $\xi=\dfrac{1}{2}$ is also degenerates. In the case of finding the solution of a super linear ordinary differential equation with non-degenerating applications of form

$Lu = u_{xx} + a(x)u^m + \frac{1}{1+b(x)u} = 0, x \in [0,1], m \geq 2$ with boundary conditions, $u(0) = 0, u_x(1) = 0$ may be used as a theoretical sense of the work (see [30] and therein). Finally, our result for equation of (49) admissible to the Equations (6) and (12) are applicable and at the same time can be found generalized solution, if the coefficients of this equation we substitute in corresponding above solutions formulas and method of prove account into works (see [10] [11]), can asset the Equation (49) with corresponding boundary conditions is solvable.

Example 3.4. A KH instability on the planet Saturn, formed at the interaction of two bands of the planet; atmosphere (role of this example to illustrate boundary layer, see **Figure 3**).

Example 3.5. The Kelvin-Helmholtz can be seen in the bands of Jupiter (this example illustrates boundary layer, see **Figure 4**).

Example 3.6. The Kelvin-Helmholtz instability is commonly found in Earth's

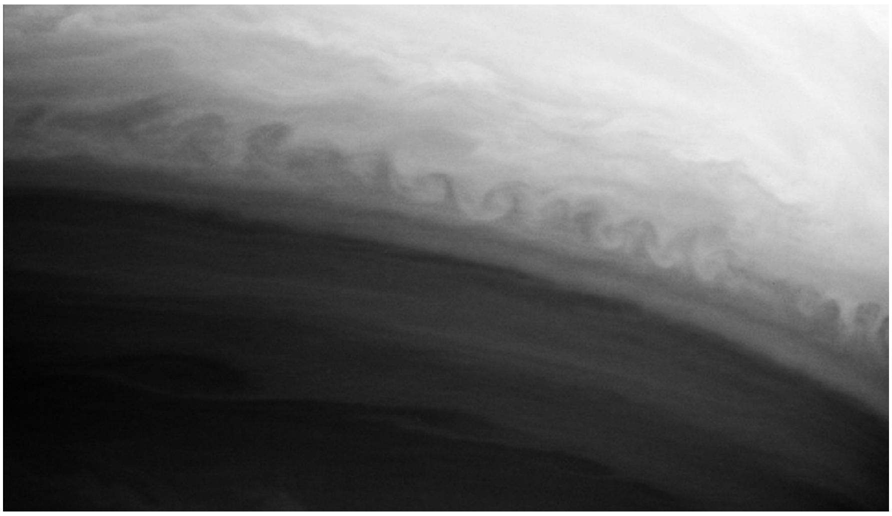

Figure 3. Saturn, for Example 3.4.

Figure 4. Jupiter, for Example 3.5.

atmosphere and ocean (this example indicate possibility of KHI, see list of KHI in Wikipedia).

Example 3.7. This example illustrates that Kelvin-Helmholtz instability is possible for ocean mixing (see Wikipedia's list of KHI).

4. Conclusions

In this work we obtain the following results:

Therefore, in the above information, we include that Kelvin-Helmholtz instabilities are visible in the atmospheres of planets and moons, for example, in the formation of the Earth or the Red Spot on Jupiter, and in the atmospheres of the Sun and other stars. For this reason, we investigate degenerate ordinary differential equations, which arise with very difficultly solvable problems of the Kelvin-Helmholtz mechanism and instabilities. In Figure 1 and Figure 2, we give illustrations of hydrodynamic, aerodynamic, and gas-dynamic physical meanings. The considered ODE of (12) is new (because it is degenerating) and corresponds in sense as physical aspects (see Figure 2) to a non-classical equation of mathematical physics (see the physical illustration in Figure 1) and is a one-dimensional analog of (4), as shown in the section introduction. Therefore it is called as non-classical ordinary differential equations (NODE). In Section 3, we proved the theorem of existence and uniqueness of common cases problems (8) and (13) in weighted spaces while also establishing generalized solutions. It was also demonstrated in this section that the results considered problems are applicable for such KHI families' problems that arise in a variety of fields, including processes in astrophysics, the Sun, galaxies, stars, and planets. Regardless of the difficulties that arise when the above-mentioned equation has singularities and degenerates cases at the same time, at certain points, and in this case, our findings are also applicable to the linear wave analysis of Kelvin-Helmholtz instability (KHI) at a tangential discontinuity (TD) of ideal magneto-hydrodynamics (MHD) plasma, and the obtained results are validity. Note that additionally, the continuities of solutions are at the points singularity and degenerates cases established in the weighted spaces.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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