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$\mathcal{H}_\infty$ Robust Control System of a High-Purity Distillation Column

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Abstract

This paper presents the design of a robust control system for a high-purity distillation column. It is concerned with the design of a two degree-of-freedom (2DOF) product-composition controller for a high-purity distillation column. The $\mathcal{H}_\infty$ optimization problem is set up to ensure a guaranteed level of robust stability, robust disturbance attenuation and robust reference tracking performance.

Keywords

H-Infinity, Robust Control, Distillation Column, Robust Stability, Robust Performance, Robustness, Uncertainty, Two Degree-of-Freedom (2DOF)

1. Introduction

Distillation is an important process in the separation and purification of chemicals that exploits the difference in the boiling points of multi-component liquids. It has some unique characteristics, such as complex dynamics, coupling amoung loops, time delay and various disturbances [1]. However, many problems in modeling and control of distillation processes arise from their strong nonlinear behavior. [2] proposed designed of linear controllers based on a linearized model of the plant, with reduction of non linearities by using logarithmic compositions. Linear robust controllers are widely seen as acceptable solutions when systems are controlled closed to known equilibrium points and a known trajectory [3]-[8]. Wiener model identification and predictive control, with approximated nonlinearities of the distillation column can be found in [9]. It is shown that an acceptable performance-robustness trade-off cannot be obtained by simple loop shaping [10] and a good understanding of the model uncertainty is essential for robust control system design in papers [11] [12] [13]. It has described several methods and techniques for controller design of high purity distillation column. The H-infinity technique is controlled of high purity two-degree of freedom controller. Physically motivated uncertainty description is translated into H-infinity loop shaping framework which is demonstrated to be a powerfull tools to analyze and understand complex phenomena. This paper presents a nonlinear version of robust H-infinity controller to satisfy the performance specifications based on 2-degree of freedom loop shaping-infinity controller for distillation. The aim of the design, is to find a controller that achieves robust stability and robust performance of the closed-loop control system of a high-purity distillation column.

2. Theory

As a consequence, the control engineer never completely knows the precise dynamics of the system. These systems often possess dynamics that are difficult to measure accurately such as friction, viscous drag, unknown torques and other dynamics. Thus there are a certain amount of uncertainty in the practical control problem. This problem of differences between the model and the physical system is called the robustness problem. A robust controller is one that operates well on the physical system despite the differences between the design model and the physical system. The key to robust designs is characterizing the uncertainty and adding it to the model in the appropriate ways. More about robust control, can be found in books e.g. [14] [15] [16] [17].

A typical control problem involves devising a controller that maximizes the performance (minimum tracking error) while providing a stable feedback system. The two main approaches are numerical and graphical. Numerical techniques such as $\mathcal{H}_2$ and $\mathcal{H}_\infty$ optimal control rely upon computers to find a controller to optimize a particular matrix norm of the system.

The approach adopted by robust control theory is to describe a physical system by means of an uncertain model. The latter is defined as a set of models and described in terms of a nominal plant together with bounded uncertainty.

An uncertain model with multiplicative dynamic uncertainty is shown in **Figure 1**. it describes a physical system as a set G of mathematical models, as follows:

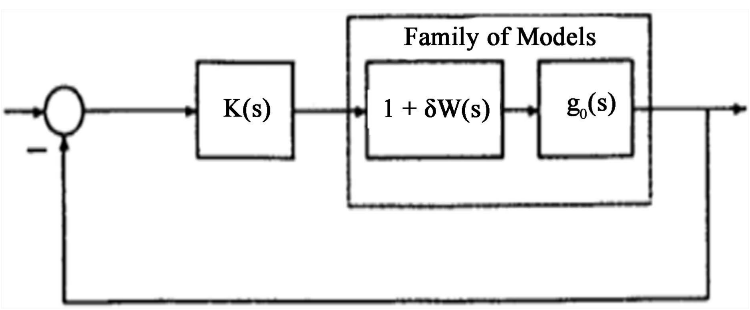

Figure 1. Family of models with dynamic multiplicative uncertainty.

$$G=\left\{g(s):g(s)=g_0(s)\left[1+\delta W(s)\right],\delta\in\mathbb{C},|\delta|<1\right\} \tag{1}$$

The set G is the family of models and is characterized by a nominal plant $g_0(s)$, a fixed weighting function $W(s)$, and a class of bounded uncertainty δ. The nominal model $g_0(s)$ corresponds to the case where there is no uncertainty, that is, $\delta=0$. Without loss of generality, the bound on the uncertainty δ can be taken to be one, because any other bound can be absorbed into the weight $W(s)$. the weighting function $W(s)$ represents the "dynamics" of the uncertainty, or in other words its "frequency distribution". An important restriction in the problem of robust stabilization is that each member of the family of models is required to have the same number of unstable poles.

A more general uncertainty description can be obtained by replacing $\delta\in\mathbb{C}$ by a real rational transfer function $\Delta(s)$, such that

$$\|\Delta(s)\|_\infty \triangleq \sup_{jw}|\Delta(jw)|<1 \tag{2}$$

Robust Performance

Robust performance refers to the ability of the system to maintain good performance, measured in terms of its tracking accuracy, given that modeling errors exist when designing the controller.

The final goal of robust control is to achieve the performance requirement on all members of the family of models (*i.e.*, robust performance).

Definition 1. The feedback loop of Figure 2 achieves robust performance if and only if $\|W_y(s)y(s)\|_2\le 1$, for all possible disturbances in the set $\left\{d\in\mathcal{L}_2,\|d\|_2\le 1\right\}$, for all inputs to the system equal to zero, and for all models in the set:

$$G=\left\{g(s):g(s)=\left[1+W_\delta(s)\delta\right]g_0(s),\delta\in\mathbb{C},|\delta|<1\right\}. \tag{3}$$

A necessary and sufficient condition for robust performance of the family of models in Figure 1 is

$$\|W_d(jw)S(jw)+T(jw)W_\delta(jw)\|_\infty\le 1,\ |\delta|<1 \tag{4}$$

It follows that robust performance can always be cast into robust stability of the set of models with structured uncertainty.

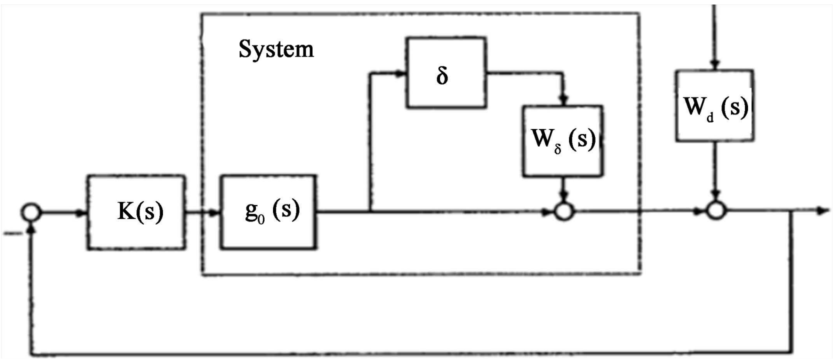

Figure 2. Disturbance rejection at the output for a family of model with multiplicative uncertainty.

Robust Stability

Robust stability refers to the ability of a closed loop stable system to remain stable in the presence of modeling errors, even though the model used for system design is very different from the plant model which exists in practice.

Consider the feedback loop of **Figure 3**, where $G(s)$ represent the nominal model of the plant, $K(s)$ a model of the controller, $y(s)$ the output, $r(s)$ a reference signal, $e(s) \triangleq r - y$ the racking error, $d(s)$ a disturbance, and $n(s)$ the measurement noise.

The relations among them are the following:

$$y(s) = G(s)K(s)\left[I + G(s)K(s)\right]^{-1}\left[r(s) - n(s)\right] + \left[I + G(s)K(s)\right]^{-1} d(s) \tag{5}$$

Define $S(s) = \left[I + G(s)K(s)\right]^{-1}$ is sensitivity function and $T(s) = G(s)K(s)\left[I + G(s)K(s)\right]^{-1}$ is the complementary sensitivity function.

It is easy to see that $S(s)$ and $T(s)$ satisfy the following equation:

$$T(s) + S(s) = I \tag{6}$$

where I is the identity matrix.

This equation places a serious constraint when designing a controller that should guarantee stability and performance, as well as robustness to model uncertainty.

3. Dynamic Model of the Distillation Column

A typical two-product distillation column is shown in **Figure 4**. The objective of the distillation column is to spilt the feed F, which is a mixture of a light and a heavy component with composition z_f, into a distillate product D with composition y_d, which contains most of the heavy component. For this aim, the column contains a series of trays that are located along its height. The liquid in the columns flows through the trays from top to bottom, while the vapour in the column rises from bottom to top. The constant contact between the vapour and liquid leads to increasing concentration of the more-volatile component in the vapour, while simultaneously increasing concentration of the less volatile component in the liquid. The operation of the column requires that some of the bottom product is re-boiled at a rate V to ensure the continuity of the vapor flow and some of the distillate is refluxed to the top tray at a rate L to ensure the

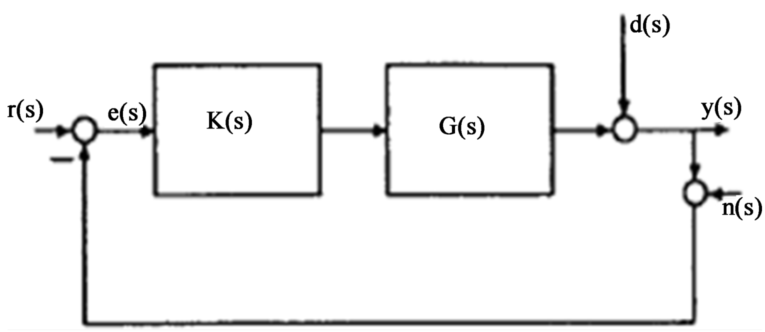

Figure 3. Multivariable Feedback loop control system.

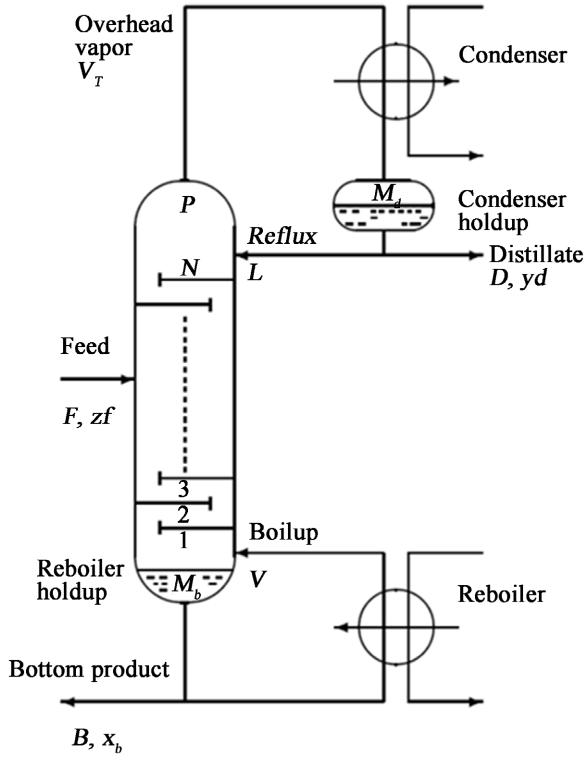

Figure 4. The distillation column system.

continuity of the liquid flow. The distillation column model used in this paper is a high –purity column, referred to as the "column at operating point A" by Skogestad and Morari (1988b), [18].

The notations used in the derivation of the column model are summarized in Table 1. This is a good example for design of controllers for ill-conditioned, high purity distillation column which is used by several papers e.g. [2] [19]. The index i denotes the stages numbered from the bottom ($i = 1$) to the top ($i = N_{tot}$) of the column. Index B denotes the bottom product and D the distillate product. A particular high-purity distillation column with 40 stages (39 trays and a reboiler) plus a total condenser is considered.

The nonlinear model equations are:

i-Total material balance on stage i

$$\mathrm{d}M_i/\mathrm{d}t = L_{i+1} - L_i + V_{i-1} - V_i \tag{7}$$

ii-Material balance for the ight component on each stage i

$$\mathrm{d}\left(M_i x_i\right)/\mathrm{d}t = L_{i+1}x_{i+1} + V_{i-1}y_{i-1} - L_i x_i - V_i y_i \tag{8}$$

This equation leads to the following expression for the derivative of the liquid mole fraction

$$\mathrm{d}x_i/\mathrm{d}t = \left(\mathrm{d}\left(M_i x_i\right)/\mathrm{d}t - x_i\left(\mathrm{d}M_i/\mathrm{d}t\right)\right)/M_i \tag{9}$$

iii-Algebraic equations

The vapor composition y_i is related to the liquid composition x_i on the same stage through the algebraic vapor-liquid equilibrium

Table 1. Column nomenclature.

Symbols	Description
F	Feed rate [Kmole/min]
z_f	Feed composition [mole fraction]
q_f	Fraction of liquid in feed
D and B	Distillate (top) and bottom product flow rate [Kmol/min]
y_d and x_b	Distillate and bottom product composition (usually of light component) [mole fraction]
L	Reflux flow [Kmole/min]
V	Boilup flow [Kmole/min]
N	Number of stages (including reboiler)
$N_{tot} = N$	Total number of stages (including condensor)
I	Stage number (1-bottom, N_F-feed stage, N_T-total condensor)
L_i and V_i	Liquid and vapour flow from stag i [Kmole/min]
x_i and y_i	Liquid and vapour composition of light component on i
M_i	Liquid holdup on stage i [Kmole] (M_B-reboiler, M_D-condenser holdup)
A	Relative volatility between light and heavy component
τ_L	Time constant for liquid flow dynamics on each stage [min]

$$y_i = \alpha x_i / \left(1 + (\alpha - 1) x_i\right) \tag{10}$$

From the assumption of constant molar flows and no vapour dynamics, one obtains the following expression for the vapour flows

$$V_i = V_{i-1} \tag{11}$$

The liquid flows depend on the liquid holdup on the stage above and the vapour flow as follows

$$L_i = LO_i + \left(M_i - MO_i\right)/\tau_L + \lambda\left(V_{i-1} - VO_{i-1}\right) \tag{12}$$

where LO_i [Kmol/min] and MO_i [Kmol/min] are the nominal values for the liquid flow and holdup on stage i and VO_i is the nominal boilup flow. If the vapor flow into the stage effects the holdup then the parameter λ is different from zero. For the column under investigation $\lambda = 0$.

The above equations apply at all stages except in the top(condenser), feed stage and bottom(reboiler).

a) Feed stage,

$i = N_F$ (we assume the feed is mixed directly into the liquid at the feed stage):

$$\mathrm{d}M_i/\mathrm{d}t = L_{i+1} - L_i + V_{i-1} - V_i + F \tag{13}$$

$$\mathrm{d}\left(M_i x_i\right)/\mathrm{d}t = L_{i+1}x_{i+1} + V_{i-1}y_{i-1} - L_i x_i - V_i y_i F z_f \tag{14}$$

b) Total condenser,

$$i = N_T \ \left(M_{NT} = M_D, L_{NT} = L_T\right)$$

$$\mathrm{d}M_i/\mathrm{d}t = V_{i-1} - L_i - D \tag{15}$$

$$\mathrm{d}(M_i x_i)/\mathrm{d}t = V_{i-1}y_{i-1} - L_i x_i - Dx_i \tag{16}$$

c) Reboiler,

$$i = 1, \ (M_i - M_B, V_i = V_B = V)$$

$$\mathrm{d}M_i/\mathrm{d}t = L_{i+1} - V_i - B \tag{17}$$

$$\mathrm{d}(M_i x_i)/\mathrm{d}t = L_{i+1}x_{i+1} - V_i y_i - Bx_i \tag{18}$$

4. Design Specification

The column operating point used for this design study is case 1 of operating point A in Morari and Zafiriou [20], page 440. The column data operating conditions are summarized in Table 2. A hydraulic time constant of $\tau = 0.063$ min was used and it was assumed that the feed was saturated liquid ($q_f = 1.0$)

The specifications for the control system design are as follows:

1) Closed-loop stability.

2) Disturbances of ±30% in the feed flow rate F, and changes of ±0.05 in the feed composition Z_f should be rejected to within 10% of steady-state within 30 minutes. In addition, the following frequency domain specification should be met:

3) $\bar{\sigma}(K_2 S)(j\omega) < 316 \ \forall\omega$ This specification is included mainly to avoid saturation of the plant inputs.

4) $\bar{\sigma}(K_2 S)(j\omega) < 316 \ \forall\omega$.

In the above, $\bar{\sigma}$ denotes the largest singular value, K_2 denotes the feedback part of the controller and $S = (1 + GK_2)^{-1}$ is the sensitivity function of G.

5. The Control System Model

All the simulation results were obtained using an 82-state nonlinear model which includes the liquid dynamics. There are two states per tray, one which represents the liquid composition and one which represents the liquid holdup. The nonlinear model was linearized about the operating point given in Table 2.

The plant model can be split into two 2-input 2-output sections. The first part G represents the transfer function matrix mapping the manipulated inputs u_c the output y, while the second part G_2 represents the transfer function matrix mapping the disturbances u_d to the output y. This gives

$$\dot{x} = Ax + \begin{bmatrix} B_c & B_d \end{bmatrix} \begin{bmatrix} u_c \\ u_d \end{bmatrix} \tag{19}$$

Table 2. Column data and nominal conditions.

N	N_{tot}	N_F	F	z_f	q_f	D
40	41	21	1	0.5	1	0.5
R	L	V	y_d	x_b	M_i	τ_L
0.5	2.706 29	3.206 29	3.0.99	0.01	0.5	0.063

$$\text{And}\quad y = Cx + \begin{bmatrix} D_c & D_d \end{bmatrix} \begin{bmatrix} u_c \\ u_d \end{bmatrix} \tag{20}$$

in which $u_c = \begin{bmatrix} L \\ V \end{bmatrix}$; $u_d = \begin{bmatrix} F \\ z_f \end{bmatrix}$; $y = \begin{bmatrix} y_d \\ y_b \end{bmatrix}$;

A state-space realization is

$$G(S) \triangleq \begin{bmatrix} A & B_c \\ C & D \end{bmatrix} \tag{21}$$

and

$$G_d(S) \triangleq \begin{bmatrix} A & B_d \\ C & D \end{bmatrix} \tag{22}$$

$$A = \begin{bmatrix} -5.131\text{e}-3 & 0 & 0 & 0 & 0 \\ 0 & 7.366\text{e}-2 & 0 & 0 & 0 \\ 0 & 0 & -1.829\text{e}-1 & 0 & 0 \\ 0 & 0 & 0 & -4.62\text{e}-1 & 9.895\text{e}-1 \\ 0 & 0 & 0 & 0 & -4.62\text{e}-1 \end{bmatrix}$$

$$B = [-0.629 \;\; 0.624; \;\; 0.055 \;\; -0.172; \;\; 0.03 \;\; -0.108; \;\; -0.186 \;\; -0.139; \;\; -1.23 \;\; -0.056]$$

$$C = \begin{bmatrix} -0.7223 & 0.517 & 0.3386 & 0.1633e-1 & 0.1121 \\ -0.8913 & 0.4758 & 0.9876 & 0.8425 & 0.2186 \end{bmatrix}$$

$$D = [0 \;\; 0; \;\; 0 \;\; 0]$$

$$B_d = [-0.062 \;\; -0.067; \;\; 0.131 \;\; 0.04; \;\; 0.022 \;\; -0.106; \;\; -0.188 \;\; 0.027; \;\; -0.045 \;\; 0.014]$$

6. Two-Degree-of-Freedom Controller Design

Since this problem has demanding time-response specifications, we make use of a two-degree-of-freedom (TDF) controller structure that can be designed within the generalized regulatory framework. An alternative TDF design procedure involves the separate optimization of the pre-filter and feedback controller, but requires the parameterization theory for all TDF controllers. If the controller in Figure 5 is partitioned to

$$K = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \tag{23}$$

it can be seen that the controller command is given by

$$u_c = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} \alpha \\ y \end{bmatrix} \tag{24}$$

where K_1 is the feedback part of the controller and K_2 is the pre-filter part.

7. The Optimization Problem

The configuration we will study is give in Figure 5, in which

$$M^{-1} \begin{bmatrix} N_c & N_d \end{bmatrix} = \begin{bmatrix} G_{nc} W_c & G_d W_d \end{bmatrix} \tag{25}$$

such that $M^{-1} N_c$ is a normalized left co-prime factorization. The model set we

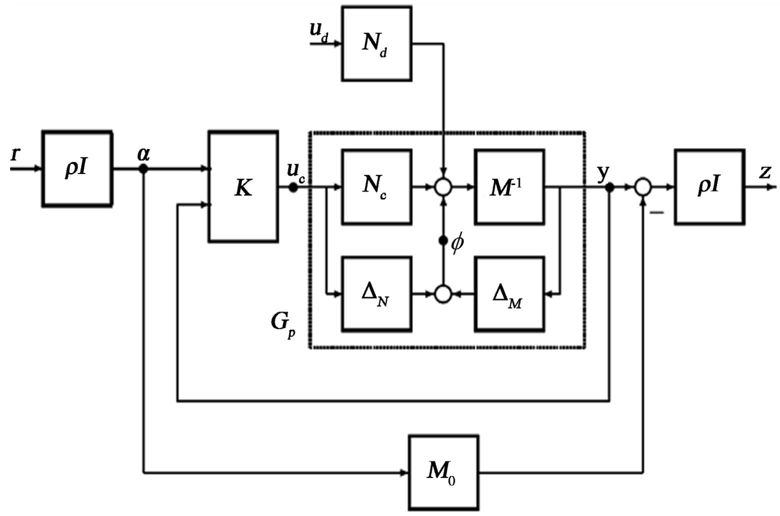

Figure 5. The design problem configuration.

consider is therefore

$$\varsigma_{\hat{\gamma}} = \left\{ M\left\{\left(M-\Delta_M\right)\right\}^{-1}\left(N_c+\Delta_N\right) : \begin{matrix} \left[\Delta_N \quad \Delta_M\right] \in H_\infty \\ \left\|\Delta_N \quad \Delta_M\right\| < \hat{\nu}^{-1} \end{matrix} \right\} \tag{26}$$

It is not difficult to modify the design to allow for perturbations to N_d, but we have not done this in the interests of simplifying the presentation. The weight W_c is used to shape the loop, while W_d contains spectral information about expected disturbances. The scaling factor ρ is used to weight the relative importance of robust stability as compared to robust model matching and robust disturbance rejection

It follows from **Figure 5** that the closed-loop system of interest is described by the linear fractional transformation

$$\begin{bmatrix} z \\ y \\ u_c \\ \alpha \\ y \end{bmatrix} = \begin{bmatrix} -\rho^2 M_0 & \rho G_d W_d & \rho M^{-1} & \rho G_{nc} W_c \\ 0 & G_d W & M^{-1} & \rho G_{nc} W_c \\ 0 & 0 & 0 & l \\ \rho l & 0 & 0 & 0 \\ 0 & G_d W & M^{-1} & G_{nc} W_c \end{bmatrix} \begin{bmatrix} \tau \\ u_d \\ \phi \\ u_c \end{bmatrix} \tag{27}$$

$$u_c = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} \alpha \\ y \end{bmatrix} \tag{28}$$

which we denote by R We are required to find an internally-stabilizing controller such that $\|R\|_\infty \leq \hat{\gamma}$ This is a generalized regulator problem, which may be solved using the theory described in [6]. Solving the loop equations shows that the closed-loop transfer function matrix R is given by

$$\begin{bmatrix} z \\ y \\ u_c \end{bmatrix} = \begin{bmatrix} \rho^2\left(SG_p - M_0\right) & \rho SG_d W_d & \rho SM^{-1} \\ \rho SG_p K_1 & SG_d W_d & SM^{-1} \\ \rho \tilde{S} K_1 & K_1 SG_d W_d & K_2 SM^{-1} \end{bmatrix} \begin{bmatrix} \tau \\ u_d \\ \phi \end{bmatrix} \tag{29}$$

In which $S = \left(1 - G_p K_2\right)^{-1}$ is the sensitivity operator and $\tilde{S} = \left(1 - K_2 G_p\right)^{-1}$.

Therefore, once a controller such that $\|R\|_\infty \le \hat{\gamma}$ has been found, we see that:

1) The loop will remain stable for all $G \in \varsigma_{\hat{\gamma}}$ This follows from the (2, 3) and (3, 3)-blocks of $\|R\|_\infty \le \hat{\gamma}$.

2) By considering the linear fractional transformation $F_l\left(R,\left[\Delta_N \quad \Delta_M\right]\right)$, we conclude from [3] that $\|SG_pK_1 - M_0\|_\infty \le \hat{\gamma}\rho^{-2}$ for all $G \in \varsigma_{\hat{\gamma}}$. This is a guaranteed robust performance property.

3) In the same way $\|SG_dW_d\|_\infty \le \hat{\gamma}$ for all $G \in \varsigma_{\hat{\gamma}}$ This is the robust disturbance attenuation property.

4) If ρ is set to zero, the TDF problem reduces to the normalized coprime factor robust stability problem.

8. Controller Design

Successful design of the Distillation Column control system may be obtained by using the $\mathcal{H}_\infty$ loop shaping design procedure (LSDP).

8.1. Loop Shaping Design Procedure

1) Select a loop-shaping weight W_c for $G(s)$. As with the earlier procedure, W_c is used to meet some of the closed-loop performance specifications.

2) Find the minimal value γ_{opt} in the pure robust stabilization problem; this may be calculated using the following equation [3]

$$\gamma_{opt} = \left(1 - \|M \quad N\|_H^2\right)^{-\frac{1}{2}} \tag{30}$$

3) A high value of γ_{opt} indicates that the specified loop shapes are inconsistent with robust stability requirements and that the feedback controller will significantly alter the loop shapes.

4) Select the weighting function W_d. This is used to shape the closed-loop disturbance rejection transfer functions.

5) Select a simple target model, M_0, for the closed-loop system. This is usually a diagonal matrix of first- or second-order lags that represent desired closed loop, time-domain command response properties. As with any other weight selection, the target model must be realistic, or the resulting closed-loop system will have poor robust stability properties and the controller will produce excessive control action.

6) Select a ρ value for the TDF configuration in **Figure 5**. In our experience, one obtains good results on process control problems when ρ is in the range $0.8 \le \rho \le 2$.

7) Find the optimal value of $\hat{\gamma}$. In distillation applications we found that $1.2 \times \gamma_{opt} \le \hat{\gamma} \le 3 \times \gamma_{opt}$ gave a good compromise between the robust stability and robust performance objectives.

8) Calculate the optimal controller. The final controller degree will be bounded above by $\deg(G) + \deg(W_d) + \deg(M_0) + 2 \times \deg(W_c)$.

8.2. Design Weight Selection

Following the prescriptive design method, the loop-shaping weight was selected to meet the robust stability and robust performance specification given in Section 4 After several design iterations we decided on

$$W_c = 1.7\frac{1.1s+1}{10s}I_2 \tag{31}$$

The choice of the gain equal to 1.7 is done to ensure a sufficiently small steady-state error. Larger gain leads to smaller steady-state errors but worse transient response. The integral action ensures zero steady-state error. The zero at −1 is used to reduce the roll-off rate to approximately 20 dB/dec in the unity gain range of frequencies. This has a beneficial effect on the closed-loop command and disturbance rejection response. The loop shaping function (Equation (30)) gives $\gamma_{opt} = 3.9224$ for the pure robustness problem associated with $G_s(s)$.

The disturbance weighting function W_d was chosen to be the identity matrix and the time-response model we selected was

$$M_0 = \frac{1}{Ts^2 3\zeta Ts+1}I_2 \tag{32}$$

The coefficients of the transfer functions ($T = 6$; $\zeta = 0.8$) in both channels of the model are chosen such that to ensure an over damped response with the settling time of about 30 min. The off-diagonal elements of the transfer matrix are set as zeros in order to minimize the interaction between the channels.

All that remains is for us to obtain an acceptable value of ρ. We discovered that $\rho = 2.0$ gave a good compromise between acceptable stability, disturbance attenuation and time-domain performance requirements. The loop shaping function in (30) gives $\gamma_{opt} = 3.4669$ for the pure robustness problem associated with G.

9. Simulation Results

The design of the Two degree of freedom $\mathcal{H}_\infty$ loop shaping design procedure is done by using the MATLAB M-file *program1.m*, that implement the functions *hinsyn*. The controller obtained is of order 14.

The structured singular value played an important role in the robustness analysis, the response of the original system (G) shaped plant (G_s), and disturbance (G_d), given in **Figure 6** and **Figure 7**.

The frequency response of the sensitivity function obtained by using the M-file (*program2.m*) is shown in **Figure 8** The maximum at largest singular value of this matrix does not exceed 1 for all values 1 the uncertain parameters.

Figure 9 shows the singular value plots of the closed loop transfer function effect of disturbances. The closed loop disturbance attenuation properties are also very good, each disturbance is attenuated to within ±10% within the required.

The singular value plots of K_2S and GK_2 are shown in **Figure 10** and **Figure 11**, respectively. The maximum of the largest singular value of K_2S is less than 316, and the maximum of the largest singular value of GK_2 is less than 1 for

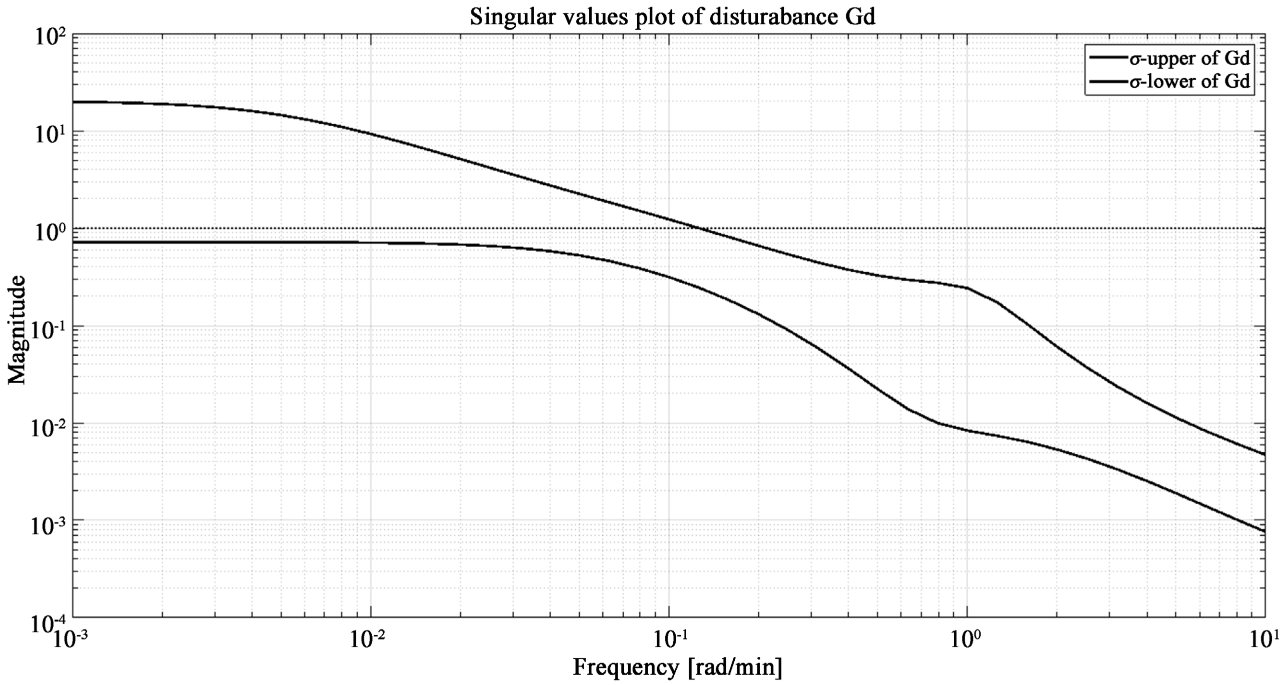

Figure 6. Frequency responses of the plant G_d.

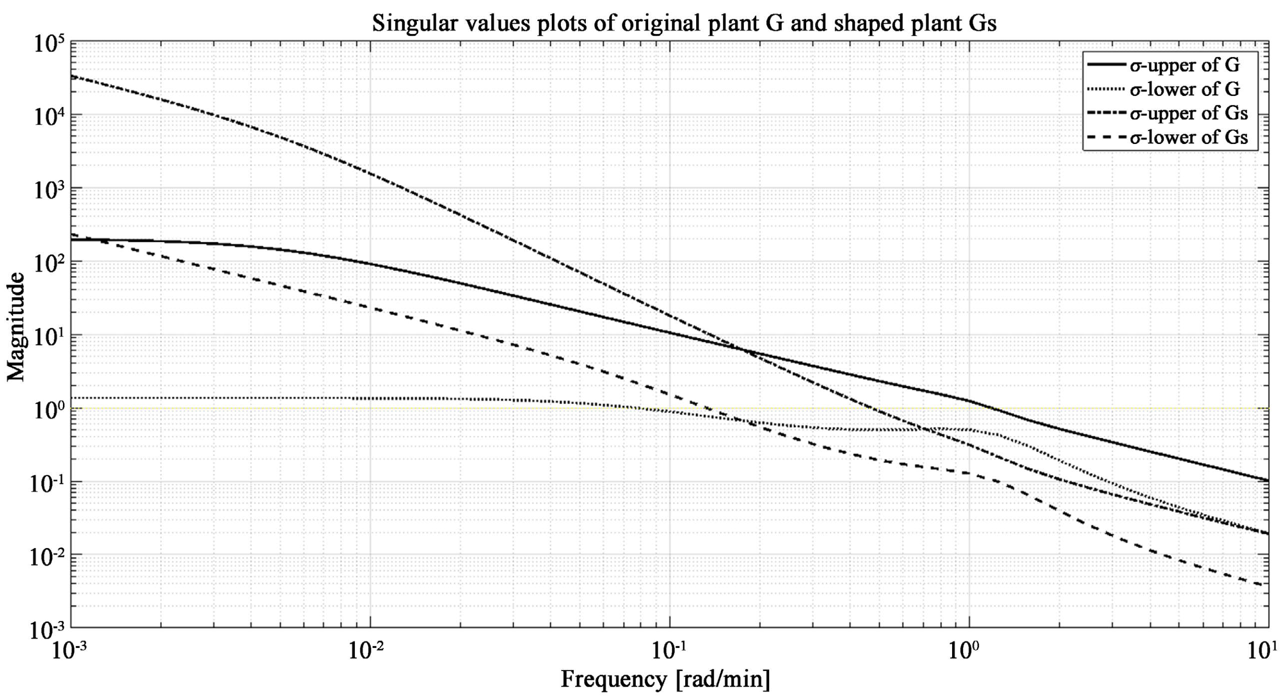

Figure 7. Frequency responses of the plant and shaped plant.

$w \geq 150$ thus the frequency domain specification being met.

Figure 12 and Figure 13, give the response change in the distillate composition demand, each response shows a zero steady-state offset and indicates that the robust stability, robust disturbance attenuation and robust performance specifications are met.

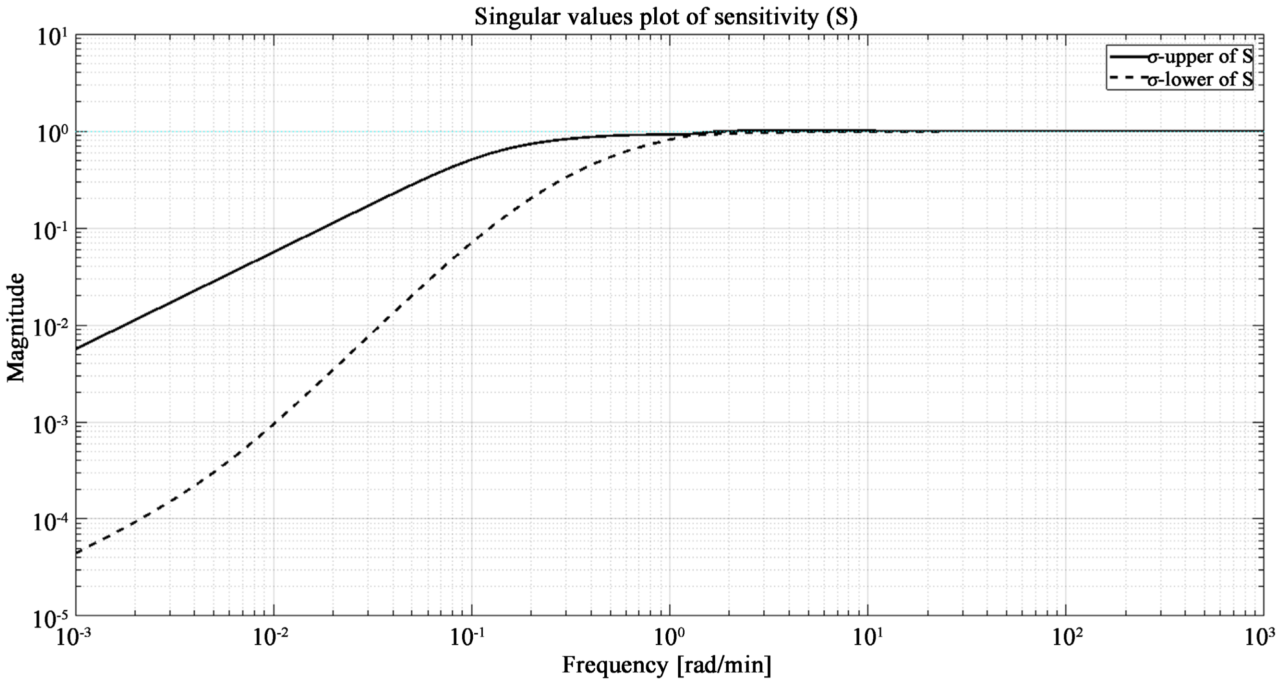

Figure 8. Frequency responses of closed loop effect of disturbance.

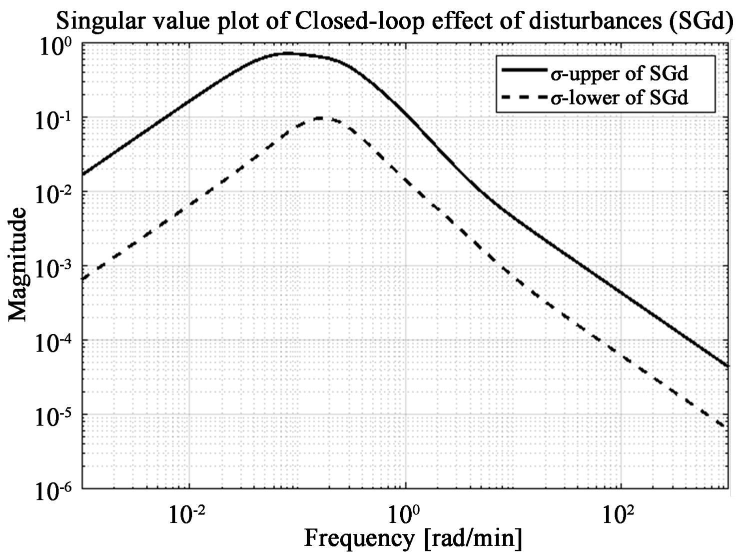

Figure 9. Frequency responses of the sensitivity (S).

MATLAB M-file *program3.m* analysis of the closed loop system, the robust stability and performance (with respect to the weighting performance function) is analyzed. Where system has very good robustness margins (robust stability easily satisfied objective in section 3 (RS = 0.6605)) and excellent nominal performance (NP = 0.9617), so the system is nominally stable we know from the simulation of Figure 13 that the robust performance is good (RP = 1.0052).

10. Conclusions and Future Work

The following conclusions can be drawn from this study:

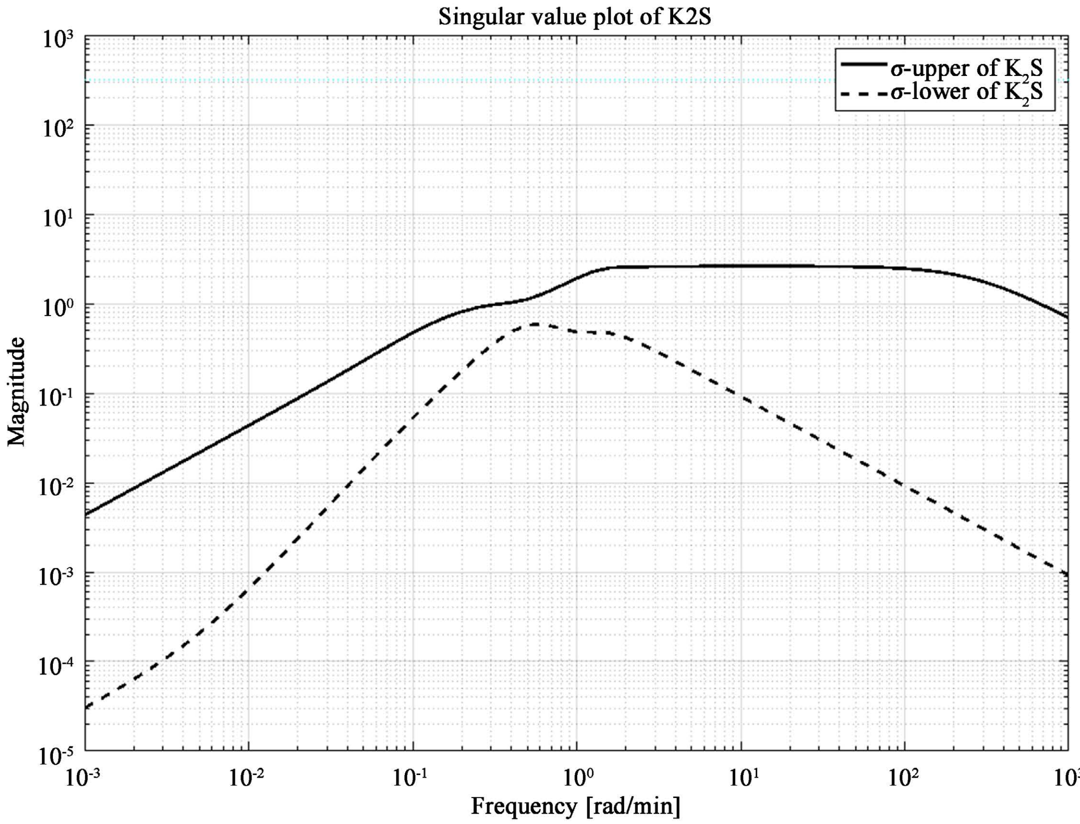

Figure 10. Frequency responses of K_2S.

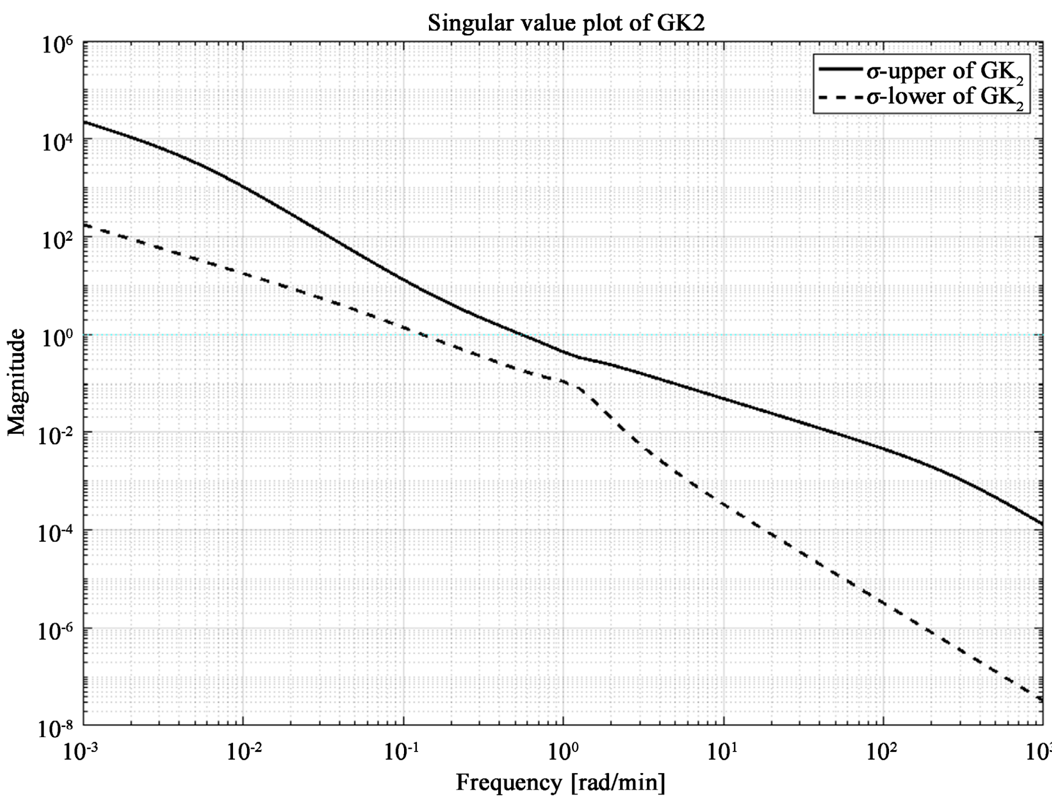

Figure 11. Frequency responses of GK_2.

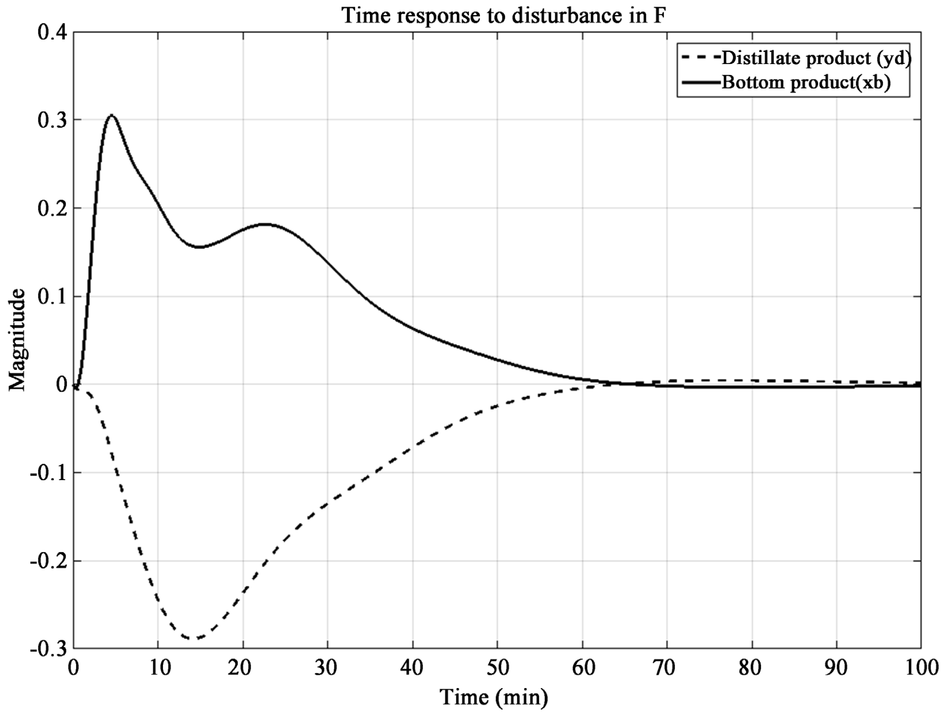

Figure 12. Response to a step change in the feed flow rate.

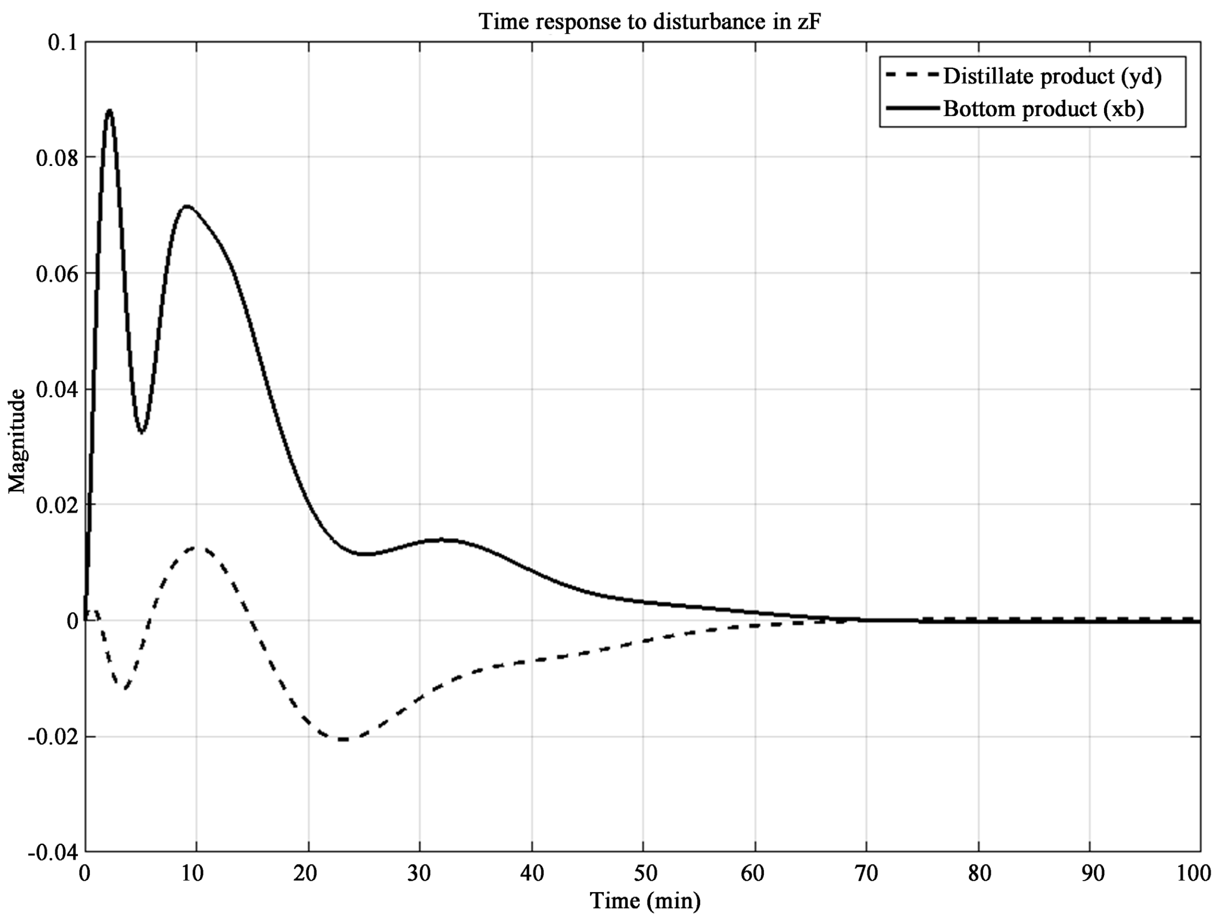

Figure 13. Response to a step change in feed composition.

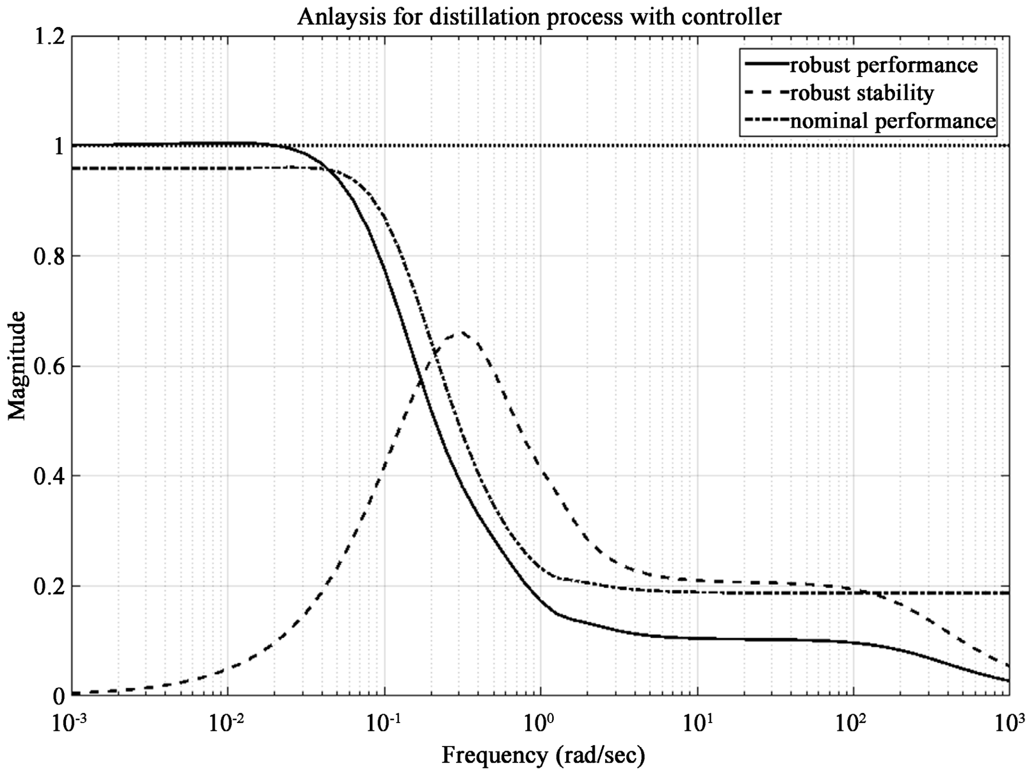

Figure 14. Robust performance, Nominal performance and Robuststability conditions for $\mathcal{H}_\infty$ loop shaping controller.

In this paper, the problems of optimal $\mathcal{H}_\infty$ controller design and robust performance for control systems are studied. A set of linear model of high purity distillation column is used for controller design. Controller design methods are studied, each with the aim of achieving sufficient robustness and performance by using the description of plant variations and uncertainties provided by the model set. The investigated methods comprise robust $\mathcal{H}_\infty$ optimal control with loop shaping design, which satisfies all control objectives. A two-degree of freedom controller was needed to satisfy the specifications. The results, in terms of meeting the specifications, satisfy the design specification and robustness required. The simulation results are reported and the different control methods are evaluated and analyzed using singular value decomposition (SVD). Design issues are briefly discussed.

Following the findings of this study, the following can be recommended to be future work:

- This research shows that robust control methodology can easily be applied to design real, industrial, robust control systems. Thus it is strongly recommended that designers approach their control systems with this methodology.
- The two-degree of freedom control law implementation requires good theoretical background and strong mathematical tools. Thus prior knowledge and excellent mathematical background must be prepared in advance.

- Robust control designs involve the choice of weighting variables. These variables are manipulated manually and thus require an engineering sense, experience, and sometimes trial and error technique to reach optimal values.
- The $\mathcal{H}_\infty$ loop-shaping method has been successfully used and tested in the design of high purity distillation column. The same scenario can be repeated for other similar real life industrial application systems such as Aircraft control.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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