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On Almost Type α-F-Z-Weak Contraction in Metric Spaces

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Abstract

In this article, we introduce a new type of contraction named almost type α-F-Z-weak contraction, which comes from a combination of F-contraction, Z-contraction, and almost contraction, and then we provide sufficient conditions for the existence and uniqueness of fixed point of such contractions in complete metric spaces and give some related fixed point results. In addition, some related fixed point results can derive from our main results.

Keywords

Fixed Point, α-Admissible, α-F-Z-Weak Contraction, Almost Contraction

1. Introduction

In 1922, Banach [1] proved a famous theorem named the Banach contraction principle. Many scholars developed it. In 2012, Wardowski [2] introduced F-contraction which is an amusing development of Banach contraction. In 2014, Wardowski and Dung [3] extended F-contraction to F-weak contraction. In the same year, several fixed point results of the F-Suzuki contraction were got by Piri and Kumam [4], and an F-contraction of Hardy-Rogers type was raised by Cosentino and Vetro [5]. In 2018, Ali *et al.* [6] presented an (α, F)-contraction which is a generalization of the Wardowski type contraction. Qawaqneh *et al.* [7] posed (α-β-F)-Geraghty contraction. Several authors gained some interesting extensions and generalizations of the F-contraction (see [8] [9] [10] [11] and references therein). In 2015, Khojasteh *et al.* [12] projected the Z-contraction via simulation function, this kind of contraction was generalized to the Banach contraction and several known types of nonlinear contractions. Recently, a number of researchers have studied these contractive conditions (see [13]-[18] and references therein). For more recent results related to fixed point theory, refer to [19] [20] [21] [22]. In 2018, Isik *et al.* [23] acquired the existence and uniqueness of fixed point of almost contraction via simulation functions in metric spaces.

Inspired by the researches of F-contractions, Z-contractions, and almost contraction, in this article, by combining the ideas of these contractions, we propose a new almost type α-F-Z-weak contraction in complete metric spaces. Some sufficient conditions for the existence and uniqueness of fixed points in complete metric spaces were provided. Furthermore, some related fixed point results can derive from our main results.

2. Preliminaries

In the section, we firstly list some useful definitions and results. we denote by R the set of all real numbers, by R^+ the set of all non-negative real numbers, by N the set of all non-negative integers and by X the set of all nonempty.

In 2012, α-admissible mapping was firstly introduced by Samet *et al.* [24].

Definition 1. [24] If there exists a function $\alpha : X \times X \to R^+$ such that a mapping $T : X \to X$ satisfies

$$\alpha(s,t) \geq 1 \Rightarrow \alpha(Ts,Tt) \geq 1 \text{, for all } s,t \in X .$$

Then mapping T is an α-admissible mapping.

Example 1. Let $X = [0,\infty)$, $T : X \to X$ and $\alpha : X \times X \to R^+$ by $Ts = \frac{5s}{2}$, $s \in X$, and

$$\alpha(s,t) = \begin{cases} \frac{5}{4}, & s \geq t \\ \frac{1}{2}, & s < t \end{cases}$$

Then T is an α-admissible mapping.

In 2013, Karapınar *et al.* [25] presented triangular α-admissible mapping.

Definition 2. [25] If there exists a function $\alpha : X \times X \to R^+$ such that a mapping $T : X \to X$ satisfies

1) $\alpha(s,t) \geq 1 \Rightarrow \alpha(Ts,Tt) \geq 1$, for all $s,t \in X$;

2) $\alpha(s,t) \geq 1$ and $\alpha(t,z) \Rightarrow \alpha(x,z)$, for all $s,t,z \in X$.

Then mapping T is a triangular α-admissible mapping.

Definition 3. [2] If $F : (0,+\infty) \to R$ satisfies the following conditions:

(F1) F is strictly increasing, *i.e.*, $s < t \Rightarrow F(s) < F(t)$, for all $s,t \in (0,+\infty)$;

(F2) for each sequence $\{a_m\}$ of positive numbers, $\lim_{m\to\infty} a_m = 0$ if and only if $\lim_{m\to\infty} F(a_m) = -\infty$;

(F3) there exists $l \in (0,1)$ such that $\lim_{a\to 0^+} a^l F(a) = 0$.

Then we say that F is a F-function.

We denote the set of all functions by *F*.

Example 2. The following functions $F : (0,+\infty) \to R$ are elements of $\mathbb{F}$.

1) $F(s)=\lambda_1 s$, where $\lambda_1>0$;

2) $F(s)=-\frac{1}{s^{\lambda_2}}$, where $s,\lambda_2>0$;

3) $F(s)=\frac{s^{\lambda_3}}{1-e^s}$, where $s,\lambda_3>0$.

In 2012, Wardowski [2] advanced F-contraction and obtained the existence and uniqueness of the fixed point of F-contraction in complete metric spaces. Without special explanation, F later in the article belongs to $\mathbb{F}$.

Definition 4. [2] Let (X,d) be a metric space. If there exists a $\tau>0$ such that a mapping $T:X\to X$ satisfies

$$d(Ts,Tt)>0\Rightarrow \tau+F(d(Ts,Tt))\le F(d(s,t)),\ \text{for all}\ s,t\in X.$$

Then T is said to be an F-contraction.

In 2014, Wardowski and Dung [3] extended F-contraction to F-weak contraction.

Definition 5. [3] Let (X,d) be a metric space. If there exists a $\tau>0$ such that a mapping $T:X\to X$ satisfies

$$d(Ts,Tt)>0\Rightarrow \tau+F(d(Ts,Tt))\le F(M(s,t)),\ \text{for all}\ s,t\in X,$$

where

$$M(s,t)=\max\left\{d(s,t),d(s,Ts),d(t,Tt),\frac{d(s,Tt)+d(t,Ts)}{2}\right\}$$

Then T is said to be an F-weak contraction.

Remark 1. If $M(s,t)=d(s,t)$, then F-weak contraction becomes F-contraction. It indicated that F-contraction is a special form of F-weak contraction.

In 2018, Ali *et al.* [6] projected (α, F)-contraction. When $\alpha(s,t)=1$ for all $s,t\in X$, then (α, F)-contraction reduces to F-contraction.

Definition 6. [6] Let (X,d) be a metric space. If there exists a $\tau>0$ and $\alpha:X\times X\to R^+$ such that a mapping $T:X\to X$ satisfies

$$d(Ts,Tt)>0\Rightarrow \tau+F(\alpha(s,t)d(Ts,Tt))\le F(d(s,t)),\ \text{for all}\ s,t\in X,$$

Then T is said to be an (α, F)-contraction.

In 2015, Khojasteh *et al.* [12] defined Z-contraction and gained the existence and uniqueness of the fixed point.

Definition 7. [12] Let (X,d) be a metric space. If there exists a $\zeta:R^+\times R^+\to R$ such that a mapping $T:X\to X$ satisfies

$$\zeta(d(Ts,Tt),d(s,t))\ge 0,\ \text{for all}\ s,t\in X.$$

Then T is said to be a Z-contraction, where $\zeta:R^+\times R^+\to R$ is a mapping satisfying the following conditions:

(ζ1) $\zeta(0,0)=0$;

(ζ2) $\zeta(s,t)<t-s$, for all $s,t>0$;

(ζ3) if $\{s_m\},\{t_m\}$ are sequences with $s_m,t_m\in(0,\infty)$ such that $\lim_{m\to\infty}s_m=\lim_{m\to\infty}t_m=0$, then $\limsup_{m\to\infty}\zeta(s_m,t_m)<0$.

Now we take some examples.

Example 3. [12] (1) $\zeta(s,t) = \lambda s - t$, $\lambda \in [0,1)$;

(2) $\zeta(s,t) = \psi(t) - \phi(t)$, where ψ, ϕ are self-mappings on $[0,\infty)$ such that $\psi(t) = \phi(t) = 0$ if and only if $t = 0$ and $\psi(t) < t \le \phi(t)$, for all $t > 0$;

(3) $\zeta(s,t) = t - s - \varphi(t)$, where φ is a self-mapping on $[0,\infty)$ with $\varphi^{-1}(0) = 0$ and $s,t \ge 0$.

Theorem 1. [12] Every Z-contraction in complete metric spaces has a unique fixed point.

In 2018, Isik and Gungor *et al.* presented almost Z-contraction and obtained the following fixed point theorem.

Definition 8. [23] Let (X,d) be a metric space. We say that $T: X \to X$ is an almost Z-contraction, if there exists a constant $L \ge 0$ such that

$$\zeta\left(d(Ts,Tt), d(s,t) + LR(s,t)\right) \ge 0, \text{ for all } s,t \in X.$$

where

$$R(s,t) = \min\left\{d(s,Ts), d(t,Tt), d(s,Tt), d(t,Ts)\right\}.$$

Theorem 2. [23] Let (X,d) be a complete metric space and $T: X \to X$ be an almost Z-contraction. Then, T has a unique fixed point, for arbitrary initial point $x_0 \in X$, the Picard sequence $\{Tx_0\}$ converges to the fixed point.

3. Main Results

Firstly, we put forward almost type α-F-Z-weak contraction in metric spaces.

Definition 9. Let (X,d) be a metric space and $T: X \to X$ be a given mapping. We say that T is said to be an almost type α-F-Z-weak contraction if there exist $\alpha: X \times X \to R^+$, $L \ge 0$, $\tau > 0$, $F \in \mathbb{F}$ and $\zeta \in Z$ such that

$$d(Ts,Tt) > 0 \Rightarrow \zeta\left(\tau + F\left(\alpha(s,t)d(Ts,Tt)\right), F\left(M(s,t) + LN(s,t)\right)\right) \ge 0, \quad (1)$$

for all $s,t \in X$, where

$$M(s,t) = \max\left\{d(s,t), d(s,Ts), d(t,Tt), \frac{d(s,Tt) + d(t,Ts)}{2}\right\},$$

$$N(s,t) = \min\left\{d(s,Ts), d(t,Ts)\right\}.$$

Remark 2. If T is an almost type α-F-Z-weak contraction, then

$$\tau + F\left(\alpha(s,t)d(Ts,Tt)\right) \le F\left(M(s,t) + LN(s,t)\right), \text{ for all } d(Ts,Tt) > 0. \quad (2)$$

Example 4. Let (X,d) and d be the usual metric on X. Define a mapping $T: X \to X$ by

$$Ts = \begin{cases} \frac{5}{2}, & s \in \left(\frac{2}{5}, \frac{5}{2}\right], \\ 0, & \text{otherwise.} \end{cases}$$

Also define $F(s) = \ln s$ and $\zeta(s,t) = \frac{1}{2}t - s$.

Then T is an almost type α-F-Z-weak contraction with

$$\alpha(s,t)=\begin{cases}1, & s,t\in\left[0,\frac{2}{5}\right] \text{ or } s,t\in\left(\frac{2}{5},\frac{5}{2}\right]\\ \frac{\sqrt{5}}{5e}, & \text{otherwise}\end{cases}$$

and $0<\tau\le-\frac{5}{\ln 2e}$, $L=2$. But T is not an F-weak contraction. Indeed, when $s=\frac{5}{2}$, $t=0$, $\tau+\ln\frac{5}{2}\le\ln\frac{5}{2}$, this is a contradiction.

Remark 3. By the definition of T and Remark 2, we notice that an (α, F)-contraction must be an almost type α-F-Z-weak contraction, but the converse is not true (see Example 4). The converse holds only if $L=0$.

Example 5. Let $X=[0,1]$ and $d(s,t)=|s-t|$. Define $T:X\to X$ by

$$Ts=\begin{cases}\frac{1}{2}, & s\in[0,1)\\ 0, & \text{otherwise}\end{cases}$$

Also define $F(s)=\ln s$, and $\zeta(s,t)=\frac{1}{2}t-s$

$$\alpha(s,t)=\begin{cases}6, & s,t\in[0,1)\\ \frac{1}{4}, & \text{otherwise}\end{cases}$$

Then T is an almost type α-F-Z-weak contraction for all $s,t\in X$ and $\tau=\ln 8$, $L=2$.

Now we prove our main results.

Theorem 3. Let (X,d) be a complete metric space. Suppose that T is an almost type α-F-Z-weak contraction. If T satisfies the following conditions:

1) There exists $s_0\in X$ such that $\alpha(s_0,Ts_0)\ge 1$;

2) T is triangular α-admissible;

3) T or $\{s_n=T^n s_0\}$ satisfy one of the following conditions:

a) T is continuous;

b) T^2 is continuous and if $s_n\to s$, such that $\alpha(Ts,s)\ge 1$;

c) If $\{s_n\}$ is a sequence in X such that $\alpha(s_n,s_{n+1})\ge 1$ for all n and $s_n\to s\in X$, then $\alpha(s_n,s)\ge 1$, for all $n\in N$, then T has at least a fixed point.

Proof. Define a sequence $\{s_n\}$ by $s_{n+1}=Ts_n=T^n s_0$, $n\in N$. By (1), (2) and Mathematical induction, it easily follows that

$$\alpha(s_n,s_{n+1})\ge 1,\ \text{for all } n\in N, \tag{3}$$

and

$$\alpha(s_m,s_n)\ge 1,\ \text{for all } m,n\in N \text{ with } n>m. \tag{4}$$

If there exists $n_0\in N$ such that $s_{n_0}=s_{n_0+1}$, then $Ts_{n_0}=s_{n_0}$, so s_{n_0} is a fixed point of T, the proof is completed. If $s_n\ne s_{n+1}$ for all $n\in N$, *i.e.*, $d(s_n,s_{n+1})>0$ for all $n\in N$. Set $s=s_n$, $t=s_{n+1}$ in (1), by (2) and (3), then

$$\begin{aligned}\tau+F\left(d\left(s_n,s_{n+1}\right)\right)&\le\tau+F\left(\alpha\left(s_{n-1},s_n\right)d\left(s_n,s_{n+1}\right)\right)\\&\le F\left(M\left(s_{n-1},s_n\right)\right)+LN\left(s_{n-1},s_n\right)\end{aligned}\tag{5}$$

where

$$\begin{aligned}&M\left(s_{n-1},s_n\right)\\&=\max\left\{d\left(s_{n-1},s_n\right),d\left(s_{n-1},Ts_{n-1}\right),d\left(s_n,Ts_n\right),\frac{d\left(s_{n-1},Ts_n\right)+d\left(s_n,Ts_{n-1}\right)}{2}\right\}\\&=\max\left\{d\left(s_{n-1},s_n\right),d\left(s_n,s_{n+1}\right)\right\},\end{aligned}$$

$$N\left(s_{n-1},s_n\right)=\min\left\{d\left(s_{n-1},Ts_{n-1}\right),d\left(s_n,Ts_{n-1}\right)\right\}=0.$$

Now if $M\left(s_{n-1},s_n\right)=d\left(s_n,s_{n+1}\right)$, by (5), it deduce that

$$\tau+F\left(d\left(s_n,s_{n+1}\right)\right)\le F\left(M\left(s_{n-1},s_n\right)+LN\left(s_{n-1},s_n\right)\right)=F\left(d\left(s_n,s_{n+1}\right)\right),\tag{6}$$

this is a contradiction. So $M\left(s_{n-1},s_n\right)=d\left(s_{n-1},s_n\right)$, we get

$$\tau+F\left(d\left(s_n,s_{n+1}\right)\right)\le F\left(M\left(s_{n-1},s_n\right)+LN\left(s_{n-1},s_n\right)\right)=F\left(d\left(s_{n-1},s_n\right)\right),$$

So

$$F\left(d\left(s_n,s_{n+1}\right)\right)<F\left(d\left(s_{n-1},s_n\right)\right),$$

By (F1), we have $d\left(s_n,s_{n+1}\right)<d\left(s_{n-1},s_n\right)$. Thus $\left\{d\left(s_n,s_{n+1}\right)\right\}$ is a strictly non-increasing sequence with $d\left(s_n,s_{n+1}\right)\ge 0$. so assume that $\lim_{n\to\infty}d\left(s_n,s_{n+1}\right)=a$. If $a>0$, take the right limits on the both sides of (6), it follows that

$$\tau+F\left(a+0\right)\le F\left(a+0\right),$$

this is a contradiction. So $a=0$, that is

$$\lim_{n\to\infty}d\left(s_n,s_{n+1}\right)=0.\tag{7}$$

Now we claim that $\left\{s_n\right\}$ is a Cauchy sequence. If $\left\{s_n\right\}$ is not a Cauchy sequence, then there exist $\varepsilon>0$, and two sequences $\left\{s_{n(p)}\right\}$, $\left\{s_{m(p)}\right\}$, where $n(p)$, $m(p)$ are two positive integers and $n(p)>m(p)$ such that $d\left(s_{m(p)},s_{n(p)}\right)\ge\varepsilon$, $d\left(s_{m(p)},s_{n(p)-1}\right)<\varepsilon$. By the triangle inequality, it follows that

$$\begin{aligned}\varepsilon&\le d\left(s_{m(p)},s_{n(p)}\right)\\&\le d\left(s_{m(p)},s_{n(p)-1}\right)+d\left(s_{n(p)-1},s_{n(p)}\right)\\&<\varepsilon+d\left(s_{n(p)-1},s_{n(p)}\right).\end{aligned}\tag{8}$$

Take the limits on the both sides of (8), we obtain

$$\lim_{p\to\infty}d\left(s_{m(p)},s_{n(p)}\right)=\varepsilon.\tag{9}$$

By the triangle inequality, we have

$$d\left(s_{n(p)},s_{m(p)+1}\right)\le d\left(s_{n(p)},s_{m(p)}\right)+d\left(s_{m(p)},s_{m(p)+1}\right),\tag{10}$$

and

$$d\left(s_{n(p)},s_{m(p)}\right)\le d\left(s_{n(p)},s_{m(p)+1}\right)+d\left(s_{m(p)+1},s_{m(p)}\right).\tag{11}$$

Let $p \to \infty$ in (10) and (11), hence

$$\lim_{p\to\infty} d\left(s_{m(p)+1}, s_{n(p)}\right) = \varepsilon. \tag{12}$$

In the similar way, therefore

$$\lim_{p\to\infty} d\left(s_{m(p)}, s_{n(p)+1}\right) = \varepsilon, \tag{13}$$

$$\lim_{p\to\infty} d\left(s_{m(p)+1}, s_{n(p)+1}\right) = \varepsilon. \tag{14}$$

Set $s = s_{m(p)}$, $t = s_{n(p)}$ in (1), by (2) and (4), it follows that

$$\begin{aligned} \tau + F\left(d\left(s_{m(p)+1}, s_{n(p)+1}\right)\right) &\le \tau + F\left(\alpha\left(s_{m(p)}, s_{n(p)}\right) d\left(s_{m(p)+1}, s_{n(p)+1}\right)\right) \\ &\le F\left(M\left(s_{m(p)}, s_{n(p)}\right) + LN\left(s_{m(p)}, s_{n(p)}\right)\right) \end{aligned} \tag{15}$$

where

$$\begin{aligned} M\left(s_{m(p)}, s_{n(p)}\right) &= \max\left\{ d\left(s_{m(p)}, s_{n(p)}\right), d\left(s_{m(p)}, Ts_{m(p)}\right), d\left(s_{n(p)}, Ts_{n(p)}\right), \right. \\ &\qquad \left. \frac{d\left(s_{m(p)}, Ts_{n(p)}\right) + d\left(s_{n(p)}, Ts_{m(p)}\right)}{2} \right\} \\ &= \max\left\{ d\left(s_{m(p)}, s_{n(p)}\right), d\left(s_{m(p)}, s_{m(p)+1}\right), d\left(s_{n(p)}, s_{n(p)+1}\right), \right. \\ &\qquad \left. \frac{d\left(s_{m(p)}, s_{n(p)+1}\right) + d\left(s_{n(p)}, s_{m(p)+1}\right)}{2} \right\}, \end{aligned} \tag{16}$$

and

$$\begin{aligned} N\left(s_{m(p)}, s_{n(p)}\right) &= \min\left\{ d\left(s_{m(p)}, Ts_{m(p)}\right), d\left(s_{m(p)}, Ts_{n(p)}\right) \right\} \\ &= \min\left\{ d\left(s_{m(p)}, s_{m(p)+1}\right), d\left(s_{m(p)}, s_{n(p)+1}\right) \right\}. \end{aligned} \tag{17}$$

Let $p \to \infty$ in (16), (17) and by (9), (12), (13), (14), it shows

$$\lim_{p\to\infty} M\left(s_{m(p)}, s_{n(p)}\right) = \varepsilon, \ \lim_{p\to\infty} N\left(s_{m(p)}, s_{n(p)}\right) = \varepsilon.$$

So

$$\lim_{p\to\infty} M\left(s_{m(p)}, s_{n(p)}\right) + LN\left(s_{m(p)}, s_{n(p)}\right) = \varepsilon. \tag{18}$$

Take the right limits on the both sides of (15) and by (14) and (18),

$$\tau + F(\varepsilon + 0) \le F(\varepsilon + 0),$$

this is a contradiction. So $\{s_n\}$ is a Cauchy sequence in complete metric space (X, d). Thus there exists a $s^* \in X$ such that $s_n \to s^*$, as $n \to \infty$. Furthermore

Case I: (1) holds;

Then we obtain

$$s^* = \lim_{n\to\infty} s_{n+1} = \lim_{n\to\infty} Ts_n = T\left(\lim_{n\to\infty} s_n\right) = Ts^*,$$

that is $s^* = Ts^*$.

Case II: (2) holds;

We have

$$s^* = \lim_{n\to\infty} s_{n+2} = \lim_{n\to\infty} T^2 s_n = T^2\left(\lim_{n\to\infty} s_n\right) = T^2 s^*,$$

that is

$$s^* = T^2 s^*. \tag{19}$$

If $s^* \neq Ts^*$, set $s = Ts^*$, $t = s^*$ in (1), by (2) and (19), it follows that

$$\begin{aligned} \tau + F\left(d\left(T^2 s^*, Ts^*\right)\right) &\le \tau + F\left(\alpha\left(T^2 s^*, Ts^*\right) d\left(T^2 s^*, Ts^*\right)\right) \\ &\le F\left(M\left(Ts^*, s^*\right) + LN\left(Ts^*, s^*\right)\right) \end{aligned} \tag{20}$$

where

$$\begin{aligned} &M\left(Ts^*, s^*\right) \\ &= \max\left\{d\left(Ts^*, s^*\right), d\left(T^2 s^*, Ts^*\right), d\left(s^*, Ts^*\right), \frac{d\left(T^2 s^*, s^*\right) + d\left(Ts^*, Ts^*\right)}{2}\right\} \\ &= d\left(Ts^*, s^*\right), \end{aligned}$$

$$\begin{aligned} N\left(Ts^*, s^*\right) &= \min\left\{d\left(Ts^*, T^2 s^*\right), d\left(s^*, T^2 s^*\right)\right\} \\ &= \min\left\{d\left(Ts^*, s^*\right), d\left(s^*, s^*\right)\right\} \\ &= 0. \end{aligned}$$

So (20) can be simplified to $\tau + F\left(d\left(Ts^*, s^*\right)\right) \le F\left(d\left(Ts^*, s^*\right)\right)$, this is a contradiction. So $s^* = Ts^*$.

Case III: (3) holds;

If there exists n_0 such that $s_n = Ts^*$, $n \ge n_0$, then $s^* = \lim_{n\to\infty} s_n = Ts^*$, that is $s^* = Ts^*$. On the contrary, set $s = s_n$, $t = s^*$ in (1), by (2) and $\alpha\left(s_n, s^*\right) \ge 1$, then

$$\begin{aligned} \tau + F\left(d\left(s_{n+1}, s^*\right)\right) &\le \tau + F\left(\alpha\left(s_n, s^*\right) d\left(s_{n+1}, Ts^*\right)\right) \\ &\le F\left(M\left(s_n, s^*\right) + LN\left(s_n, s^*\right)\right), \end{aligned} \tag{21}$$

where

$$\begin{aligned} &M\left(s_n, s^*\right) \\ &= \max\left\{d\left(s_n, s^*\right), d\left(s_n, Ts_n\right), d\left(Ts^*, s^*\right), \frac{d\left(s_n, Ts^*\right) + d\left(s^*, Ts_n\right)}{2}\right\} \\ &= \max\left\{d\left(s_n, s^*\right), d\left(s_n, s_{n+1}\right), d\left(Ts^*, s^*\right), \frac{d\left(s_n, Ts^*\right) + d\left(s^*, s_{n+1}\right)}{2}\right\}, \end{aligned} \tag{22}$$

$$N\left(s_n, s^*\right) = \min\left\{d\left(s^*, Ts_n\right), d\left(s_n, Ts_n\right)\right\} = \min\left\{d\left(s^*, s_{n+1}\right), d\left(s_n, s_{n+1}\right)\right\}. \tag{23}$$

Let $n \to \infty$ in (22) and (23), it shows

$$\lim_{n\to\infty} M\left(s_n, s^*\right) = d\left(Ts^*, s^*\right), \lim_{n\to\infty} N\left(s_n, s^*\right) = 0.$$

So

$$\lim_{n\to\infty} M\left(s_n, s^*\right) + LN\left(s_n, s^*\right) = d\left(Ts^*, s^*\right). \tag{24}$$

Take the right limits on the both sides of (21) and by (24), we get

$$\tau + F\left(d\left(Ts^*, s^*\right) + 0\right) \le F\left(d\left(Ts^*, s^*\right) + 0\right),$$

this is a contradiction. So $s^* = Ts^*$. Hence, T has a fixed point.

Remark 4. In the proof of Theorems 3, we only use (F1), (ζ2), it shows Theorems 3 is true as long as F and ζ satisfy (F1) and (ζ2), respectively.

Remark 5. Example 4 satisfies all the hypothesis of Theorem 3, so T has a fixed point. Indeed, $s = 0$ and $s = \frac{5}{2}$ are two fixed points of T.

Theorem 3 shows that T has a fixed point, but it can't guarantee the uniqueness of fixed point of T. Now in order to assure the uniqueness of fixed point of T, we consider the following condition:

4) For all s, $t \in Fix(T) \Rightarrow \alpha(s,t) \ge 1$, where $Fix(T)$ denotes the set of fixed points of T.

Theorem 4. Adding (4) to the conditions of Theorem 3, we can assure the uniqueness of fixed point of T.

Proof. We argue by contradiction, assume that there exist $s, t \in X$ such that $Ts = s$, $Tt = t$ with $s \ne t$. From (4), we have $\alpha(s,t) \ge 1$. Therefore, if follows from the definitions of T and ζ

$$\begin{aligned} \tau + F\left(d\left(Ts, Tt\right)\right) &= \tau + F\left(d\left(s,t\right)\right) \le \tau + F\left(\alpha\left(s,t\right) d\left(s,t\right)\right) \\ &\le F\left(M\left(s,t\right) + LN\left(s,t\right)\right), \end{aligned} \tag{25}$$

where

$$M(s,t) = \max\left\{d(s,t), d(s,Ts), d(t,Tt), \frac{d(s,Tt) + d(t,Ts)}{2}\right\} = d(s,t),$$

$$N(s,t) = \min\left\{d(s,Ts), d(t,Ts)\right\} = 0.$$

So (25) can be simplified to $\tau + F\left(d(s,t)\right) \le F\left(d(s,t)\right)$, it is a contraction. So $s = t$.

Remark 6. Example 5 satisfies all the hypothesis of Theorem 4, so T has a unique fixed point. In fact, $s = \frac{1}{2}$ is the unique fixed point of T.

Corollary 5. Let (X,d) be a complete metric space. Suppose T satisfies the following conditions: $d(Ts,Tt) > 0 \Rightarrow \zeta\left(\tau + \alpha(s,t) d(Ts,Tt), M(s,t)\right) \ge 0$, for all $s, t \in X$,

1) There exists $s_0 \in X$ such that $\alpha(s_0, Ts_0) \ge 1$;

2) T is triangular α-admissible;

3) T is continuous or T^2 is continuous and if $\left\{s_n = T^n s_0\right\}$ is a sequence in X

such that $\lim_{n\to\infty} s_n = s$, then $\alpha(Ts,s)\ge 1$ or if $\{s_n = T^n s_0\}$ is a sequence in X such that $\alpha(s_n, s_{n+1})\ge 1$ and $\lim_{n\to\infty} s_n = s$, then $\alpha(s_n, s)\ge 1$ for all $n\in N$, then T has a fixed point.

Proof. Take $L=0$ in Theorem 3.

Corollary 6. Let (X,d) be a complete metric space. If there exists $\phi:[0,\infty)\to[0,\infty)$ is a lower semi-continuous function with $\phi(t)=0$ if and only if $t=0$ such that for all $s,t\in X$,

$$d(Ts,Tt) < M(s,t) - \phi(M(s,t)), \text{ for all } s,t\in X, \tag{26}$$

then T has a unique fixed point.

Proof. From (26), so there exists $\tau>0$ such that $\tau + d(Ts,Tt) \le M(s,t) - \phi(M(s,t))$.

Let $\zeta(s,t) = t - s - \phi(t)$, $F(t)=t$, $L=0$, $\alpha(s,t)=1$, so T is an almost type α-F-Z-weak contraction. By Theorem 4, the proof is completed.

Corollary 7. [1] Let (X,d) be a complete metric space. If there exists $k\in(0,1]$ such that for all $s,t\in X$, $d(Ts,Tt)\le kd(s,t)$ then T has a unique fixed point.

Proof. Let $\phi(t)=(1-k)t, M(s,t)=d(s,t)$, so by Corollary 6, the proof is completed.

4. Consequences

4.1. Fixed Point Theorems in Partially Ordered Metric Spaces

Definition 10. $(X,d,\prec)$ is said to a complete partially ordered metric space, if (X,d) is a complete metric space and X is a nonempty set endowed with a partial order $\prec$.

Definition 11. $T:X\to X$ is non-decreasing endowed with a partial order $\prec$ if $s\prec t \Rightarrow Ts\prec Tt$.

Theorem 8. Let $(X,d,\prec)$ be a complete partially ordered metric space. If there exist $L\ge 0, \tau>0$ Such that T satisfies the following conditions:

1) There exists $s_0\in X$ such that $s_0 \prec Ts_0$;

2) T is non-decreasing;

3) for all $s\prec t$,
$d(Ts,Tt)>0 \Rightarrow \zeta\left(\tau + F\left(d(Ts,Tt)\right), F\left(M(s,t)+LN(s,t)\right)\right)\ge 0$;

4) T is continuous or if $\{s_n = T(s_0)\}$ is a sequence in X such that $s_n\prec s_{n+1}$ and $\lim_{n\to\infty} s_n = s$ for all $n\in N$, then $s_n\prec s$, then T has a fixed point.

Proof. Let $\alpha(s,t)=1, s\prec t$. Then T satisfies all the conditions of Theorem 3, so the proof is completed.

Now in order to assure the uniqueness of fixed point of T, we considesr the following condition:

4') For all $s,t\in Fix(T)$ such that $s\prec t$ or $t\prec s$.

Theorem 9. Adding (4') to the conditions of Theorem 8, we can assure the uniqueness of fixed point of T.

Proof. Let $\alpha(s,t)=1, s\prec t$. Then T satisfies all the conditions of Theorem 4, so the proof is completed.

4.2. Fixed Point Theorems of Cyclic Mappings

Definition 12. Let A and B be two nonempty subsets of a metric (X,d) and $T: A\cup B \to A\cup B$ be a mapping. If $T(A)\subset B$ and $T(B)\subset A$, then T is a cyclic mapping.

Theorem 10. Let (X,d) be a complete partially ordered metric space, A and B be two nonempty subsets of X such that $A\cap B\neq\varnothing$ and $T: X\times X\to X$. If there exist $L\geq 0$, $\tau>0$ such that T satisfies the following conditions:

1) There exists $s_0\in X$ such that $(s_0, Ts_0)\in (A\times B)\cup(B\times A)$;

2) T is a cyclic mapping;

3) For all $(s,t)\in (A\times B)\cup(B\times A)$,

$$d(Ts,Tt)>0 \Rightarrow \zeta\Big(\tau+F\big(d(Ts,Tt)\big), F\big(M(s,t)+LN(s,t)\big)\Big)\geq 0;$$

4) T is continuous or if there exists a sequences $\{s_n=T(s_0)\}$ such that $(s_n,s_{n+1})\in (A\times B)\cup(B\times A)$ and $\lim\limits_{n\to\infty} s_n = s$ for all $n\in N$, then $(s_n,s_{n+1})\in (A\times B)\cup(B\times A)$, then T has a fixed point, that is, there exists a $u\in A\cap B$ such that $Tu=u$.

Proof. Let $\alpha(s,t)=1$, $(s,t)\in (A\times B)\cup(B\times A)$. Then T satisfies all the conditions of Theorem 3, so T has a fixed point, the proof is completed.

Now in order to ensure the uniqueness of fixed point of T, we consider the following condition:

4") for all $s,t\in Fix(T)$ such that $(s,t)\in (A\times B)\cup(B\times A)$.

Theorem 11. Adding (4") to the conditions of Theorem 10, we can assure the uniqueness of fixed point of T.

5. Conclusion

In this paper, we investigate a new type of contraction named almost type α-F-Z-weak contraction, which is produced by the combination of F-contraction, Z-contraction, and almost contraction. In Section 3, sufficient conditions for the existence and uniqueness of the fixed point of such contraction in complete metric spaces are provided. There are some related fixed point results that can derive from our results. In Section 4, we propose the cases of partially ordered metric spaces and cycle mappings, some corresponding fixed point results are obtained.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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