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Periodic Solitary Wave Solutions of the (2 + 1)-Dimensional Variable-Coefficient Caudrey-Dodd-Gibbon-Kotera-Sawada Equation

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Abstract

In this paper, through symbolic computations, we obtain two exact solitary wave solitons of the (2 + l)-dimensional variable-coefficient Caudrey-Dodd-Gibbon-Kotera-Sawada equation. We study basic properties of l-periodic solitary wave solution and interactional properties of 2-periodic solitary wave solution by using asymptotic analysis.

Keywords

(2 + l)-Dimensional vc-CDGKS Equation, Solitary Wave Solution, Periodic Solitary Wave Solution, Asymptotic Analysis

1. Introduction

Nonlinear evolution equations appear in many fields of physics, such as fluids, quantum mechanics, condensed matter, superconductivity and nonlinear optics. Due to the fact that most systems in nature are complicated, many nonlinear evolution equations may possess variable coefficients. Recently, the investigation on exact solutions of the variable-coefficient nonlinear evolution equations has become the focus in the study of complex nonlinear phenomena in physics and engineering [1] [2] [3] [4].

In this paper, we study the (2 + 1)-dimensional variable-coefficient Caudrey-Dodd-Gibbon-Kotera-Sawada (vc-CDGKS) Equation (see [5] [6])

$$
\begin{aligned}
& u_t + a_1(t)u_{xxxxx} + a_2(t)u_x u_{xx} + a_3(t)uu_{xxx} + a_4(t)u^2 u_x + a_5(t)u_{xxy} \\
& + a_6(t)\partial_x^{-1}u_{yy} + a_7(t)u_x\partial_x^{-1}u_y + a_8(t)uu_y + a_9(t)u = 0,
\end{aligned}
\tag{1.1}
$$

where $a_i = a_i(t)$, $(i = 1, 2, \cdots, 9)$ are analytical functions with respect to the variable t. When $a_1(t) = -1$, $a_2(t) = a_3(t) = a_4(t) = a_5(t) = a_7(t) = a_8(t) = -5$, $a_6(t) = 5$, $a_9(t) = 0$ and $a_1(t) = -1$, $a_2(t) = -25/2$, $a_3(t) = a_4(t) = a_5(t) = a_7(t) = a_8(t) = -5$, $a_6(t) = 5$, $a_9(t) = 0$, two reduced (2 + 1)-dimensional equations were first proposed by Konopelchenko and Dubovsky through Lax pairs [7]. When $a_1(t) = 1/36$, $a_2(t) = a_3(t) = 5/12$, $a_4(t) = 5/4$, $a_5(t) = a_6(t) = -5/36$, $a_7(t) = a_8(t) = -5/12$, $a_9(t) = 0$, the vc-CDGKS Equation (1.1) becomes the (2 + 1)-dimensional constant-coefficient Caudrey-Dodd-Gibbon-Kotera-Sawada equation appeared in [8] [9] [10].

For the vc-CDGKS Equation (1.1), the bilinear form, bilinear Bäcklund transformation, Lax pair and the infinite conservation laws have been studied by Bell polynomials in [5] and N-soliton solutions have been constructed with the help of the Hirota bilinear method. In [6], non-traveling lump and mixed lump-kink solutions were investigated by Hirota bilinear form and symbolic computational software of Maple.

In this paper, we consider periodic solitary wave solutions of the (2 + 1)-dimensional vc-CDGKS Equation (1.1). The periodic solitary wave solution in this paper comes from Zaitsev [11] and this kind of solution is periodic in the direction of propagation and decays exponentially along the transverse direction. In [12] [13], some generalizations were given and the interactions between two *y*-periodic solitons were studied for the (2 + 1)-dimensional Kadomtsev-Petvinshvili equation. The periodic solitary wave solutions of the (2 + 1)-dimensional Sawada-Kotera equation, the (2 + 1)-dimensional KP I equation and the (3 + 1)-dimensional Jimbo-Miwa equation were studied in [14] [15] [16], respectively. In this paper, we present some generalizations and interactional properties between two periodic solitons for the (2 + 1)-dimensional vc-CDGKS Equation (1.1). The interactional properties will be analyzed based on the ideas in [17] [18], where the analysis was performed for constant-coefficient equations.

In the following section, we deduce the 1-periodic solitary wave solution which is periodic in the direction of one curve and decays exponentially along the proper transverse direction of the corresponding curve. We analyze the propagating curve and the center of the periodic solutions. We also deduce the 2-periodic solitary wave solution which is periodic in the direction of two curves and decays exponentially along two proper transverse directions of the corresponding curves. The interactional properties with $a_1(t) = t^{2n-2}$ and $a_1(t) = t^{2n-1}$ for $n = 1, 2, \cdots$ are analyzed separately.

2. Periodic Solitary Wave Solutions of the (2 + 1)-Dimensional vc-CDGKS Equation

In this paper, we study periodic solitary wave solutions of the (2 + 1)-dimensional vc-CDGKS Equation (1.1) with the following constraints in [5]

$$a_2(t) = a_3(t) = \frac{15a_1(t)}{c_0}e^{\int a_9(t)dt}, \quad a_5(t) = 5c_1 a_1(t), \quad a_6(t) = -5c_1^2 a_1(t),$$

$$a_4(t) = \frac{45a_1(t)}{c_0^2}e^{2\int a_9(t)dt}, \quad a_7(t) = a_8(t) = \frac{15c_1 a_1(t)}{c_0}e^{\int a_9(t)dt} \tag{2.1}$$

where c_0 and c_1 are nonzero real constants.

To obtain periodic solitary wave solutions, by using of the transformation in [5]

$$u = 2c_0 e^{-\int a_9(t)dt}(\ln G)_{xx}, \tag{2.2}$$

with

$$G = 1 + e^{\eta_1} + e^{\eta_2} + e^{\eta_1 + \eta_2 + A_{12}}, \tag{2.3}$$

or

$$\begin{aligned}
G = 1 &+ e^{\eta_1} + e^{\eta_2} + e^{\eta_3} + e^{\eta_4} + e^{\eta_1 + \eta_2 + A_{12}} + e^{\eta_1 + \eta_3 + A_{13}} + e^{\eta_1 + \eta_4 + A_{14}} + e^{\eta_2 + \eta_3 + A_{23}} \\
&+ e^{\eta_2 + \eta_4 + A_{24}} + e^{\eta_3 + \eta_4 + A_{34}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{12} + A_{13} + A_{23}} + e^{\eta_1 + \eta_2 + \eta_4 + A_{12} + A_{14} + A_{24}} \\
&+ e^{\eta_1 + \eta_3 + \eta_4 + A_{13} + A_{14} + A_{34}} + e^{\eta_2 + \eta_3 + \eta_4 + A_{23} + A_{24} + A_{34}} \\
&+ e^{\eta_1 + \eta_2 + \eta_3 + \eta_4 + A_{12} + A_{13} + A_{14} + A_{23} + A_{24} + A_{34}},
\end{aligned} \tag{2.4}$$

we can get two types of solitary wave solutions of the (2 + 1)-dimensional vc-CDGKS Equation (1.1), where

$$\eta_j = k_j x + k_j p_j y + \omega_j(t) + \eta_{j0}, \tag{2.5}$$

$$\omega_j(t) = -\left(k_j^5 + 5c_1 k_j^3 p_j - 5c_1^2 k_j p_j^2\right)\int a_1(t)dt, \tag{2.6}$$

$$e^{A_{ij}} = \frac{(k_i - k_j)^2 \left[c_1(p_i + p_j) + k_i^2 - k_i k_j + k_j^2\right] + c_1(k_i - k_j)(k_i p_i - k_j p_j) + c_1^2(p_i - p_j)^2}{(k_i + k_j)^2 \left[c_1(p_i + p_j) + k_i^2 + k_i k_j + k_j^2\right] + c_1(k_i + k_j)(k_i p_i + k_j p_j) + c_1^2(p_i - p_j)^2}, \tag{2.7}$$

and k_j, p_j and η_{j0} are arbitrary constants.

In the following discussion, we let $a_9(t) = 0$ for G in (2.3) and (2.4) and we can obtain 1-periodic and 2-periodic solitary wave solution of the (2 + 1)-dimensional vc-CDGKS Equation (1.1) by choosing special parameters of k_j and p_j $(j = 1, 2, 3, 4)$.

2.1. 1-Periodic Solitary Wave Solution

In order to get 1-periodic solitary wave solution of the (2 + 1)-dimensional vc-CDGKS Equation (1.1), we take parameters k_j, p_j and η_{j0} for $j = 1, 2$ in (2.3) as

$$k_1 = \alpha_1 + i\beta_1 = k_2^*, \quad p_1 = \gamma_1 + i\delta_1 = p_2^*, \quad \eta_{10} = \sigma_1 + i\varphi_1 = \eta_{20}^*, \tag{2.8}$$

where α_1, β_1, γ_1, δ_1, σ_1 and φ_1 are real constants. Substituting (2.8) into (2.3), we obtain

$$G = 1 + 2e^{\eta_{1R}}\cos(\eta_{1I}) + K_1 e^{2\eta_{1R}}, \tag{2.9}$$

where

$$\eta_{1R} = \alpha_1 x + (\alpha_1 \gamma_1 - \beta_1 \delta_1)y + \omega_{1R}\int a_1(t)dt + \sigma_1, \tag{2.10}$$

$$\eta_{1I} = \beta_1 x + \left(\beta_1 \gamma_1 + \alpha_1 \delta_1\right) y + \omega_{1I} \int a_1(t) \mathrm{d}t + \varphi_1, \tag{2.11}$$

$$K_1 = \frac{\alpha_1^2 \beta_1^2 - 3\beta_1^4 + 3c_1\beta_1^2\gamma_1 + c_1\alpha_1\beta_1\delta_1 + c_1^2\delta_1^2}{\alpha_1^2\beta_1^2 - 3\alpha_1^4 - 3c_1\alpha_1^2\gamma_1 + c_1\alpha_1\beta_1\delta_1 + c_1^2\delta_1^2}, \tag{2.12}$$

with

$$\begin{aligned}
\omega_{1R} &= -\alpha_1^5 + 5\alpha_1^3\left(2\beta_1^2 - c_1\gamma_1\right) + 15c_1\alpha_1^2\beta_1\delta_1 - 5c_1\beta_1\left(\beta_1^2 + 2c_1\gamma_1\right)\delta_1 \\
&\quad - 5\alpha_1\left(\beta_1^4 - 3c_1\beta_1^2\gamma_1 + c_1^2\left(\delta_1^2 - \gamma_1^2\right)\right), \\
\omega_{1I} &= -\beta_1^5 + 5\beta_1^3\left(2\alpha_1^2 + c_1\gamma_1\right) + 15c_1\alpha_1\beta_1^2\delta_1 - 5c_1\alpha_1\left(\alpha_1^2 - 2c_1\gamma_1\right)\delta_1 \\
&\quad - 5\beta_1\left(\alpha_1^4 + 3c_1\alpha_1^2\gamma_1 + c_1^2\left(\delta_1^2 - \gamma_1^2\right)\right).
\end{aligned} \tag{2.13}$$

Substituting (2.9) into (2.2), with $K_1 > 1$, we get a nonsingular exact solution of the (2 + 1)-dimensional vc-CDGKS Equation (1.1)

$$u = 2c_0 \frac{\alpha_1^2 K_1 - \beta_1^2 + \sqrt{K_1}\left[\left(\alpha_1^2 - \beta_1^2\right)A + 2\alpha_1\beta_1 B\right]}{\left[\sqrt{K_1}\cosh\left(\eta_{1R} + \ln\sqrt{K_1}\right) + \cos\left(\eta_{1I}\right)\right]^2}, \tag{2.14}$$

where $A = \cosh\left(\eta_{1R} + \ln\sqrt{K_1}\right)\cos\left(\eta_{1I}\right)$ and $B = \sinh\left(\eta_{1R} + \ln\sqrt{K_1}\right)\sin\left(\eta_{1I}\right)$.

The above solution describes a sequence of lumps, which is periodic in the direction of $\eta_{1R} + \ln\sqrt{K_1} = 0$.

The centers of the lumps are located at

$$\eta_{1R} + \ln\sqrt{K_1} = 0, \quad 2\beta_1^2 - \cos\left(\eta_{1I}\right)\sqrt{K_1}\left(\alpha_1^2 - \beta_1^2\right) - K_1\left(\alpha_1^2 + \beta_1^2\right) = 0. \tag{2.15}$$

The lumps decay exponentially along the proper transverse direction and this exact solution is called the 1-periodic solitary wave solution. The plots of the solution at $y = 0$ are given in **Figure 1** for $a_1(t) = 1$, $a_1(t) = t$ and $a_1 = \cos(t/4)$, respectively.

2.2. 2-Periodic Solitary Wave Solution

In order to obtain 2-periodic solitary wave solution of the (2 + 1)-dimensional vc-CDGKS Equation (1.1), we take parameters k_j, p_j and η_{j0} for $j = 1, 2, 3, 4$ in (2.4) as

$$\begin{aligned}
&k_1 = \alpha_1 + i\beta_1 = k_2^*, \ k_3 = \alpha_2 + i\beta_2 = k_4^*, \ p_1 = \gamma_1 + i\delta_1 = p_2^*, \\
&p_3 = \gamma_2 + i\delta_2 = p_4^*, \ \eta_{10} = \sigma_1 + i\varphi_1 = \eta_{20}^*, \ \eta_{30} = \sigma_2 + i\varphi_2 = \eta_{40}^*,
\end{aligned} \tag{2.16}$$

where α_j, β_j, γ_j, δ_j, σ_j, φ_j $(j = 1, 2)$ are real constants. In order to analyze the asymptotic properties of the solutions, we rewrite the function G in (2.4) as

$$\begin{aligned}
G &= 1 + 2\mathrm{e}^{\eta_{1R}}\cos\left(\eta_{1I}\right) + 2\mathrm{e}^{\eta_{2R}}\cos\left(\eta_{2I}\right) + K_1\mathrm{e}^{2\eta_{1R}} + K_2\mathrm{e}^{2\eta_{2R}} \\
&\quad + K_1 K_2\mathrm{e}^{2\left(\eta_{1R} + \eta_{2R} + d_1 + d_2\right)} + 2\mathrm{e}^{\eta_{1R} + \eta_{2R} + d_1}\cos\left(\eta_{1I} + \eta_{2I} + b_1\right) \\
&\quad + 2\mathrm{e}^{\eta_{1R} + \eta_{2R} + d_2}\cos\left(\eta_{1I} - \eta_{2I} + b_2\right) + 2K_1\mathrm{e}^{2\eta_{1R} + \eta_{2R} + d_1 + d_2}\cos\left(\eta_{2I} + b_1 - b_2\right) \\
&\quad + 2K_2\mathrm{e}^{\eta_{1R} + 2\eta_{2R} + d_1 + d_2}\cos\left(\eta_{1I} + b_1 + b_2\right),
\end{aligned} \tag{2.17}$$

where
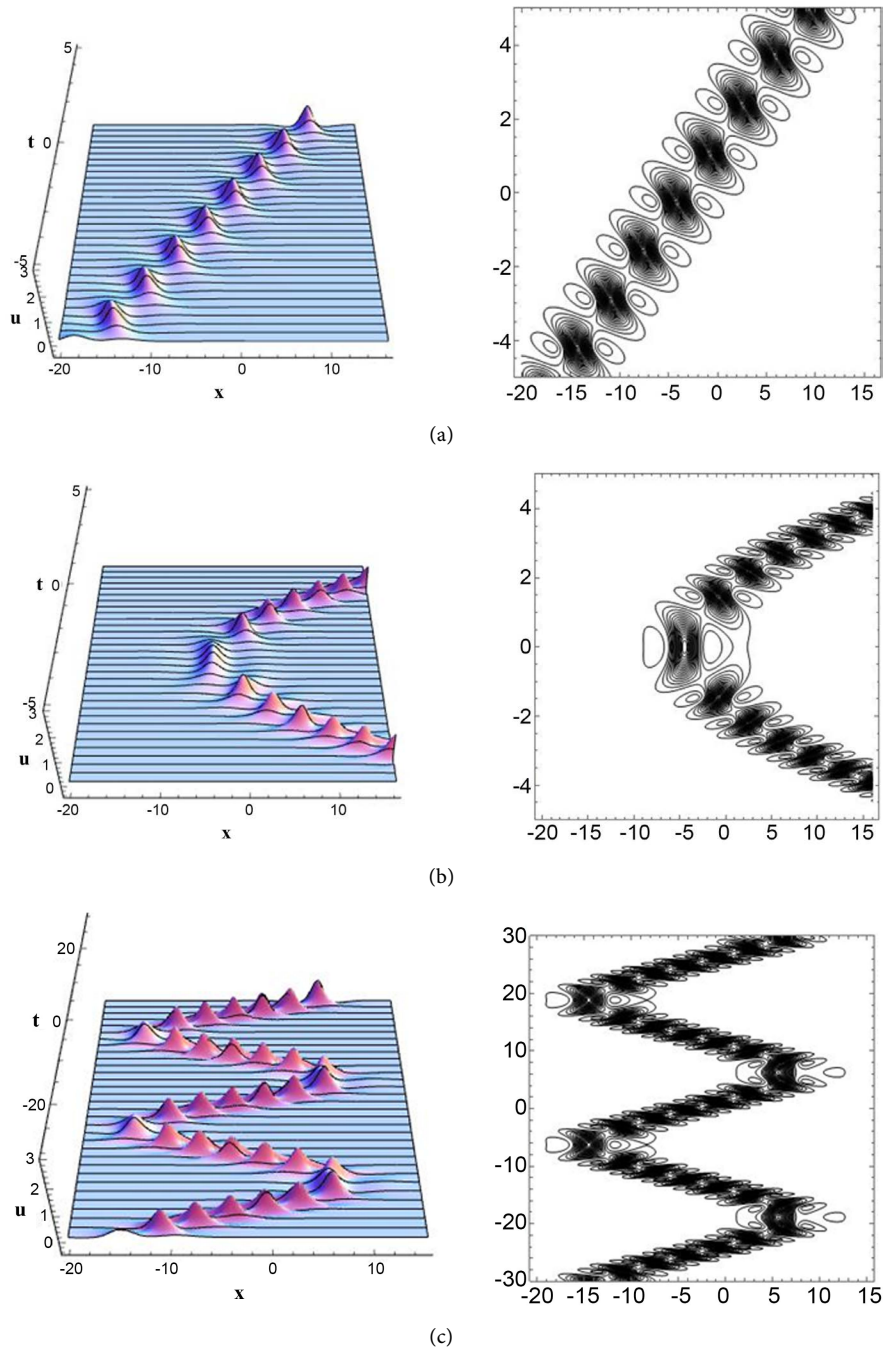
Figure 1. Plots of the 1-periodic solitary wave solutions (2.14) of the vc-CDGKS Equation (1.1) with $\alpha_1 = 1/2$, $\beta_1 = 3/5$, $\gamma_1 = 2/3$, $\delta_1 = 4/5$, $\sigma_1 = 1/6$, $\varphi_1 = 1$, $c_0 = c_1 = 1$. (a) $a_1 = 1$; (b) $a_1 = t$; (c) $a_1 = \cos(t/4)$.

$$\eta_{jR} = \alpha_j x + \left(\alpha_j \gamma_j - \beta_j \delta_j \right) y + \omega_{jR} \int a_1(t) \mathrm{d}t + \sigma_j, \tag{2.18}$$

$$\eta_{jI} = \beta_j x + \left(\beta_j \gamma_j + \alpha_j \delta_j \right) y + \omega_{jI} \int a_1(t) \mathrm{d}t + \varphi_j, \tag{2.19}$$

$$K_j = \frac{\alpha_j^2 \beta_j^2 - 3\beta_j^4 + 3c_1 \beta_j^2 \gamma_j + c_1 \alpha_j \beta_j \delta_j + c_1^2 \delta_j^2}{\alpha_j^2 \beta_j^2 - 3\alpha_j^4 - 3c_1 \alpha_1^2 \gamma_j + c_1 \alpha_j \beta_j \delta_j + c_1^2 \delta_j^2}, \quad j = 1, 2, \tag{2.20}$$

$$d_1 = \frac{1}{2}\ln\left[\left(\operatorname{Re}e^{A_{13}}\right)^2 + \left(\operatorname{Im}e^{A_{13}}\right)^2\right], \quad b_1 = \arctan\frac{\operatorname{Im}e^{A_{13}}}{\operatorname{Re}e^{A_{13}}}, \tag{2.21}$$

$$d_2 = \frac{1}{2}\ln\left[\left(\operatorname{Re}e^{A_{14}}\right)^2 + \left(\operatorname{Im}e^{A_{14}}\right)^2\right], \quad b_2 = \arctan\frac{\operatorname{Im}e^{A_{14}}}{\operatorname{Re}e^{A_{14}}}, \tag{2.22}$$

with

$$
\begin{aligned}
\omega_{jR} &= -\alpha_j^5 + 5\alpha_j^3\left(2\beta_j^2 - c_1\gamma_j\right) + 15c_1\alpha_j^2\beta_j\delta_j - 5c_1\beta_j\left(\beta_j^2 + 2c_1\gamma_j\right)\delta_j \\
&\quad - 5\alpha_j\left(\beta_j^4 - 3c_1\beta_j^2\gamma_j + c_1^2\left(\delta_j^2 - \gamma_j^2\right)\right), \\
\omega_{jI} &= -\beta_j^5 + 5\beta_j^3\left(2\alpha_j^2 + c_1\gamma_j\right) + 15c_1\alpha_j\beta_j^2\delta_j - 5c_1\alpha_j\left(\alpha_j^2 - 2c_1\gamma_j\right)\delta_j \\
&\quad - 5\beta_j\left(\alpha_j^4 + 3c_1\alpha_j^2\gamma_j + c_1^2\left(\delta_j^2 - \gamma_j^2\right)\right).
\end{aligned}
\tag{2.23}
$$

Substituting (2.17) into (2.2), we can get 2-periodic solitary wave solution of the (2 + 1)-dimensional vc-CDGKS Equation (1.1), which is composed of two sequences of lumps and the lumps decay along two directions.

In the following we study the interactional properties of this solution by using asymptotic analysis. Without loss of generality, we assume $\alpha_1 > 0$, $\alpha_2 > 0$, and $\alpha_1\omega_{2R} - \alpha_2\omega_{1R} > 0$.

Case (1): Let $a_1(t) = t^{2n-2}$, $n = 1,2,\cdots$, and we get

$$u \to \begin{cases} u_{1P}\left(\Lambda_1^-, \Gamma_1^-\right) + u_{2P}\left(\Lambda_2^-, \Gamma_2^-\right), & t \to -\infty \\ u_{1P}\left(\Lambda_1^+, \Gamma_1^+\right) + u_{2P}\left(\Lambda_2^+, \Gamma_2^+\right), & t \to +\infty \end{cases} \tag{2.24}$$

where

$$u_{jP}(\Lambda, \Gamma) = 2c_0 \frac{\alpha_j^2 K_j - \beta_j^2 + \sqrt{K_j}\left[\left(\alpha_j^2 - \beta_j^2\right)\cosh\Lambda\cos\Gamma + 2\alpha_j\beta_j\sinh\Lambda\sin\Gamma\right]}{\left[\sqrt{K_1}\cosh\Lambda + \cos\Gamma\right]^2}, \quad j = 1,2 \tag{2.25}$$

with

$$\Lambda_1^- = \eta_{1R} + \ln\sqrt{K_1}, \quad \Gamma_1^- = \eta_{1I}, \quad \Lambda_1^+ = \eta_{1R} + \ln\sqrt{K_1} + d_1 + d_2, \quad \Gamma_1^+ = \eta_{1I} + b_1 + b_2,$$

$$\Lambda_2^- = \eta_{2R} + \ln\sqrt{K_2} + d_1 + d_2, \quad \Gamma_2^- = \eta_{2I} + b_1 + b_2, \quad \Lambda_2^+ = \eta_{2R} + \ln\sqrt{K_2}, \quad \Gamma_2^+ = \eta_{2I}.$$

From the above analysis, we find that the shifts of the 2-periodic solitary waves before and after interactions are $d_1 + d_2$.

Case (2): Let $a_1(t) = t^{2n-1}$, $n = 1,2,\cdots$, and we get

$$u \to \begin{cases} \hat{u}_1 + \hat{u}_2, & t \to -\infty \\ \hat{u}_1 + \hat{u}_2, & t \to +\infty \end{cases} \tag{2.26}$$

where $\hat{u}_1$ and $\hat{u}_2$ are defined by $u_{1P}\left(\Lambda_1^+, \Gamma_1^+\right)$ and $u_{2P}\left(\Lambda_2^+, \Gamma_2^+\right)$, respectively. We find that there is no shift for the 2-periodic solitary waves before and after interactions.
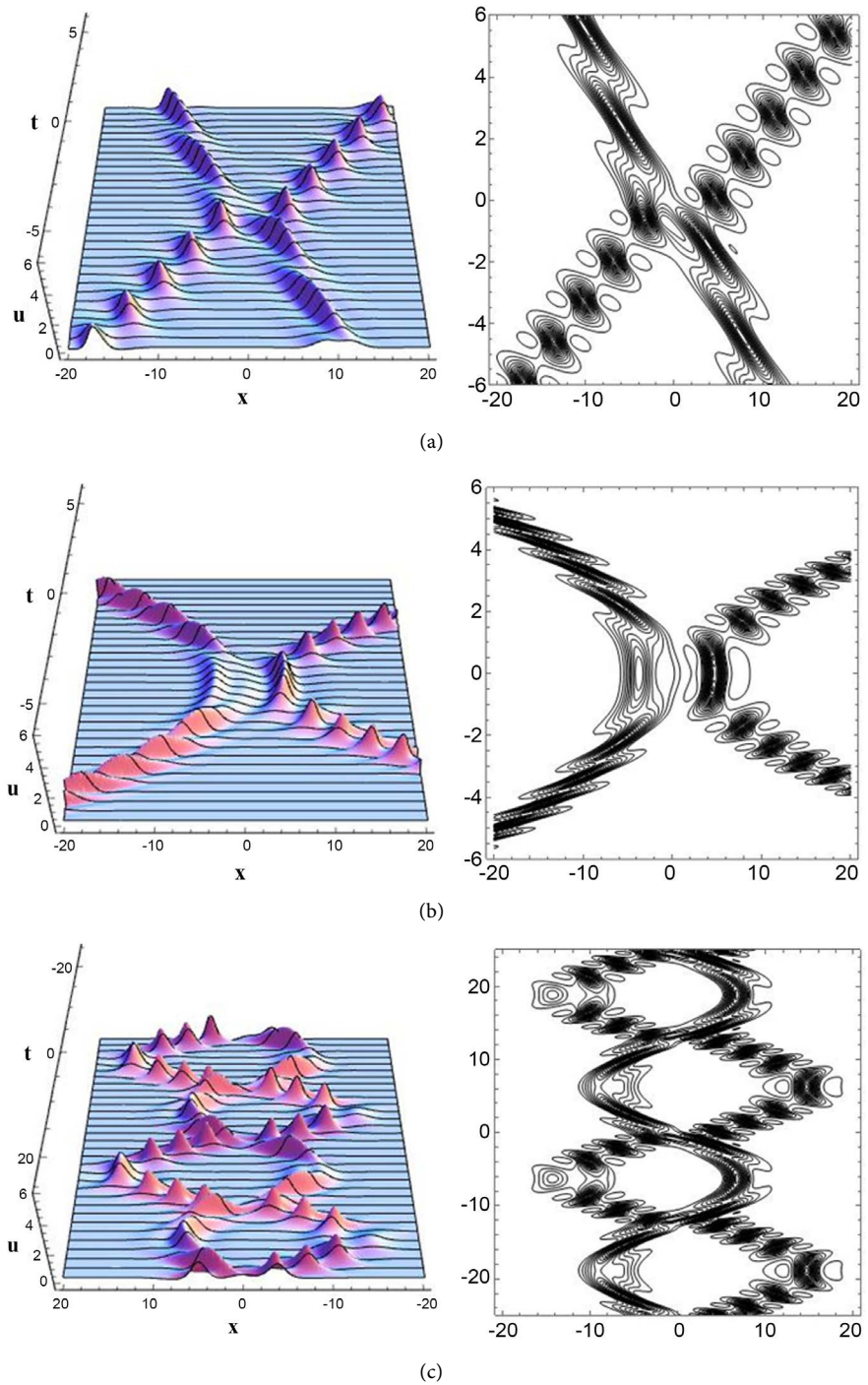
Taking the following set of parameters

$$\alpha_1 = \frac{1}{2}, \quad \beta_1 = \frac{3}{5}, \quad \alpha_2 = \frac{2}{3}, \quad \beta_2 = \frac{7}{8}, \quad \gamma_1 = \frac{2}{3}, \quad \delta_1 = \frac{4}{5}, \quad \gamma_2 = \frac{7}{100},$$

$$\delta_2 = \frac{1}{3}, \quad \sigma_1 = \frac{1}{6}, \quad \varphi_1 = 1, \quad \sigma_2 = \frac{1}{3}, \quad \varphi_2 = \frac{1}{2}, \quad c_0 = c_1 = 1 \tag{2.27}$$

The plots of the 2-periodic solution are given in **Figure 2** for $a_1(t)=1$, $a_1(t)=t$ and $a_1=\cos(t/4)$, respectively.

3. Conclusion

In this paper, we considered periodic solutions of the (2 + 1)-dimensional variable

(a)

(b)

(c)

Figure 2. Plots of the 2-periodic solutions of the vc-CDGKS Equation (1.1). (a) $a_1(t)=1$; (b) $a_1(t)=t$; (c) $a_1=\cos(t/4)$.

coefficient Caudrey-Dodd-Gibbon-Kotera-Sawada equation by using of two solitary wave solutions. By selecting different $a_1(t)$, we study basic properties of 1-periodic wave solution and interactional interactions of 2-periodic wave solution with asymptotic analysis method theoretically and graphically.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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