Open Journal of Applied Sciences, 2024, 14, 976-982
https://www.scirp.org/journal/ojapps
ISSN Online: 2165-3925
ISSN Print: 2165-3917

Non-Darcy Flow in Molding Sands

Miguel A. Barron-Meza, Joan Reyes-Miranda, Daniel Flores-Sanchez

Departamento de Materiales, Universidad Autonoma Metropolitana Azcapotzalco, Ciudad de Mexico, Mexico
Email: bmma@correo.azc.uam.mx

How to cite this paper: Barron-Meza, M.A., Reyes-Miranda, J. and Flores-Sanchez, D. (2024) Non-Darcy Flow in Molding Sands. *Open Journal of Applied Sciences*, **14**, 976-982.
https://doi.org/10.4236/ojapps.2024.144064

Received: March 17, 2024
Accepted: April 15, 2024
Published: April 18, 2024

Abstract

Darcy's law is widely used to describe the flow in porous media in which there is a linear relationship between fluid velocity and pressure gradient. However, it has been found that for high numbers of Reynolds this law ceases to be valid. In this work, the Ergun equation is employed to consider the non-linearity of air velocity with the pressure gradient in casting sands. The contribution of non-linearity to the total flow in terms of a variable defined as a non-Darcy flow fraction is numerically quantified. In addition, the influence of the shape factor of the sand grains on the non-linear flow fraction is analyzed. It is found that for values of the Reynolds number less or equal than 1, the contribution of non-linearity for spherical particles is around 1.15%.

Keywords

Darcy's Law, Molding Sands, Non-Darcy Flow, Reynolds Number, Shape Factor

1. Introduction

Flow in porous media is essential in the metallurgical industry, particularly in the foundry industry, in which molten metals are poured into sand molds. The gases generated during the pouring of the molten metal into the sand molds must be able to escape to the outside so that the solidified pieces become free of pores and blowholes [1] [2]. The property of molding sands that measures their ability to allow gases to pass through them is called permeability. This property depends both on the physical characteristics of the sand (size and shape of the grains, grain size distribution, degree of compaction, porosity, tortuosity, humidity, additives, and so on), and on the physical properties of the gases [3] [4].

For laminar flow of gases in sands, velocity and pressure gradients have a linear relationship expressed by Darcy's Law [5]. However, for turbulent flows Darcy's Law is no longer valid. In the case of turbulent flow, several modifications to Darcy's Law have been proposed that introduce nonlinearity between gas velocity and pressure gradients. Nonlinear expressions between velocity and pressure gradients previously published in the literature are used for turbulent flows [6] [7]. Reported numerical results show that for Reynolds numbers greater than 1 the deviations from Darcy's Law begin to be significant and the linearity between the gas velocity and the pressure gradients is no longer valid. In this work, the effect of the Reynolds number on the validity of Darcy's law in molding sands is numerically studied using the Ergun equation. An expression that explicitly quantifies the dependence between the Reynolds number and the non-Darcy flow fraction of the air flow in molding sands is obtained.

2. Darcy Flow

Parameters such as humidity, additives, porosity, compactness, and tortuosity, determine the permeability of molding sands. Sometimes the Kozeny-Carman equation is proposed to determine the permeability of a porous media based on the parameters mentioned above [8] [9] [10] [11]. The Kozeny-Carman equation is a semi-empirical model that is obtained, among other considerations, from the exact solution for a laminar flow around a round tube [10]. In [12] the following expression is proposed to determine tortuosity τ as a function of porosity ϕ:

$$\tau^2 = A + n(1-\phi) \quad (1)$$

where $A = 1$ and $n = 2$ for molding sands. In [13] it is proposed that

$$\tau = 0.8(1-\phi)+1 \quad (2)$$

For low flow rates through porous media, *i.e.* laminar flows, Darcy's Law establishes a proportional relationship between flow and pressure gradient. Darcy's Law is a semi-empirical expression which was originally represented as follows [1] [14]:

$$Q = \frac{kA}{\mu}\left(\frac{\Delta P}{L}\right) \quad (3)$$

where Q is the volumetric flow of fluid through the porous media, k is the specific permeability, A is the cross-sectional area, μ fluid viscosity, ΔP is the pressure drop, and L is the thickness of the porous layer. In vector form, Darcy's law is expressed as follows:

$$\boldsymbol{u} = -\frac{k}{\mu}\nabla P \quad (4)$$

$$\nabla P = \left(\frac{\partial P}{\partial x}i + \frac{\partial P}{\partial y}j + \frac{\partial P}{\partial z}k\right) \quad (5)$$

where $\boldsymbol{u}$ is the velocity vector, and ∇P is the pressure gradient.

In [15] it is stated the Darcy's Law is valid only for flow rates for which the Reynolds number is less than 1, *i.e.*:

$$Re = \frac{\rho u d}{\mu} < 1 \quad (6)$$

where u is the mean microscopic velocity, and d is the mean pore diameter. Estimating pore diameter from the expression

$$d = 10\sqrt{\frac{k}{\phi}} \tag{7}$$

then, in accordance to [15], the criterion for the validity of Darcy's Law becomes

$$u < \frac{\mu}{10\rho}\sqrt{\frac{\phi}{k}} \tag{8}$$

3. Non-Darcy Flow

For turbulent gas flow in molding sands, Darcy's Law losses applicability since the relationship between velocity and pressure gradients is no longer linear. For porous materials such as molding sands, a macroscopic Reynolds number is defined as [1]

$$Re = \frac{d_p u \rho}{\mu(1-\phi)} \tag{9}$$

where $u = Q/A$ is the macroscopic velocity and d_p is the particle diameter. From Equation (9),

$$u = \frac{Re\mu(1-\phi)}{d_p \rho} \tag{10}$$

To handle turbulent flows, some corrections to Darcy's Law have been proposed. An example is the Forchheimer equation [16] [17] for spherical particles:

$$-\frac{\mathrm{d}P}{\mathrm{d}x} = Au + Bu^2 \tag{11}$$

where x is the flow direction and

$$A = \frac{150\mu(1-\phi)^2}{d_p^2 \phi^3} \tag{12}$$

$$B = \frac{1.75\rho(1-\phi)}{d_p \phi^3} \tag{13}$$

Ergun equation can be applied for non-uniform size of sand particles and large pressure drops [1]:

$$\frac{\Delta P}{L} = \left(\frac{150\mu\lambda^2(1-\phi)^2}{d_p^2\phi^3}\right)u + \left(\frac{1.75\rho\lambda(1-\phi)}{d_p\phi^3}\right)u^2 \tag{14}$$

where λ is the shape factor. It is assumed that $\lambda = 1$ for spherical particles. The above equation can be rewritten as follows:

$$\frac{\Delta P}{L} = A\lambda^2 u + B\lambda u^2 \tag{15}$$

Pressure gradient for turbulent flows in Equation (15) can be calculated by considering the sum of the linear contribution of the velocity given by Darcy's Law, DF, and the non-linear contribution of the velocity, NDF:

$$\frac{\Delta P}{L} = DF + NDF \tag{16}$$

where

$$DF = A\lambda^2 u \tag{17}$$

$$NDF = B\lambda u^2 \tag{18}$$

The linear and non-linear fraction contributions of velocity to the pressure gradient are defined here in this way:

$$x_{DF} = \frac{DF}{DF + NDF} \tag{19}$$

$$x_{NDF} = \frac{NDF}{DF + NDF} \tag{20}$$

Of course, it is verified that $x_{DF} + x_{NDF} = 1$.

Substitution of Equations (10), (17) and (18) into Equation (20) yields

$$x_{NDF} = \left(1 + \frac{C}{Re}\right)^{-1} \tag{21}$$

where

$$C = \frac{A\lambda d_p \rho}{B\mu(1-\phi)} \tag{22}$$

For laminar flows in which Reynolds number is low, x_{NDF} tends to zero, while for turbulent flows in which Reynolds number is high the value of x_{NDF} tends to 1, therefore

$$\lim_{Re\to 0} x_{NDF} = 0 \tag{23}$$

$$\lim_{Re\to \infty} x_{NDF} = 1 \tag{24}$$

Table 1 shows some values of Non-Darcy flow fraction X_{NDF} for several values of the Reynolds number obtained from Equations (21) and (22) for spherical particles and air using $\lambda = 1.0$, $\phi = 0.1$, $\rho = 1.223\ \text{kg}\cdot\text{m}^{-3}$, $\mu = 1.9 \times 10^{-5}\ \text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-1}$, $d_p = 1.0 \times 10^{-4}$ m. It is observed that as the Reynolds number increases, the value of the Non-Darcy contribution of velocity to the flow is increased too. According to Table 1, a Reynolds number equal to 1 gives a contribution of 0.0115, *i.e.* 1.15%, of the non-Darcy flow to the total air flow through the sand grains. This result seems to be compatible with the one reported in [15] which establishes that Darcy's law is valid for Reynolds number less than 1.

Table 1. Non-Darcy flow fraction as function of Reynolds number for spherical grains, using $\lambda = 1.0$, $\phi = 0.1$, $\rho = 1.223\ \text{kg}\cdot\text{m}^{-3}$, $\mu = 1.9 \times 10^{-5}\ \text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-1}$, $d_p = 1.0 \times 10^{-4}$ m.

Re	x_{NDF}
0.1	0.0012
0.5	0.0058
1.0	0.0115
2.0	0.0228
3.0	0.0339
4.0	0.0446
5.0	0.0551

Figure 1 depicts results of Table 1. Non-Darcy flow fraction of air in molding sands increases as the Reynolds number is increased. Figure 1 also shows that the non-Darcy flow fraction depends strongly on the shape factor of the sand grains. As the shape of the grains moves away from sphericity, that is, as the value of the shape factor increases, the contribution of the non-Darcy flow decreases. Figure 1 suggests that for sand grains with a shape factor greater than 5, the validity of Darcy's law is still maintained even for Reynolds number much greater than 1.

Figure 2 shows the nonlinear contribution of velocity, *i.e.* non-Darcy flow, to the air flow for a Reynolds number equal to 1. The nonlinear contribution becomes significantly smaller for shape factors with values greater than 6. The following expression was obtained through a statistical fit of the data shown in Figure 2:

$$x_{NDF} = 0.00223 + 0.02325e^{-\lambda/1.0906} \quad (25)$$

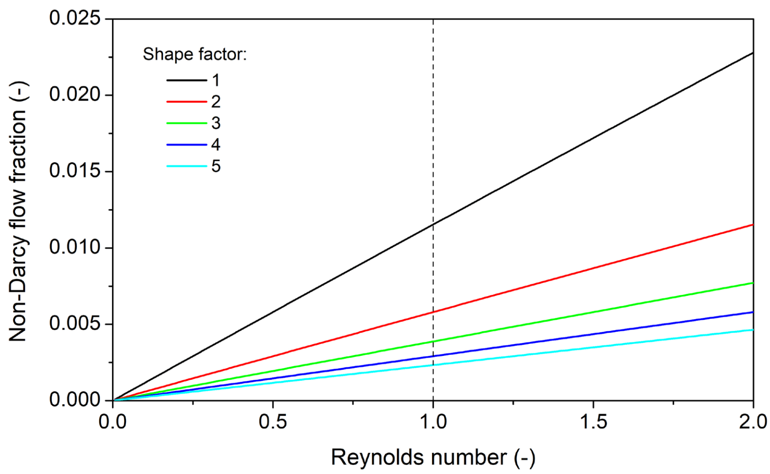

Figure 1. Non-Darcy flow fraction of air flow through sand particles as function of Reynolds number for several values of the shape factor.

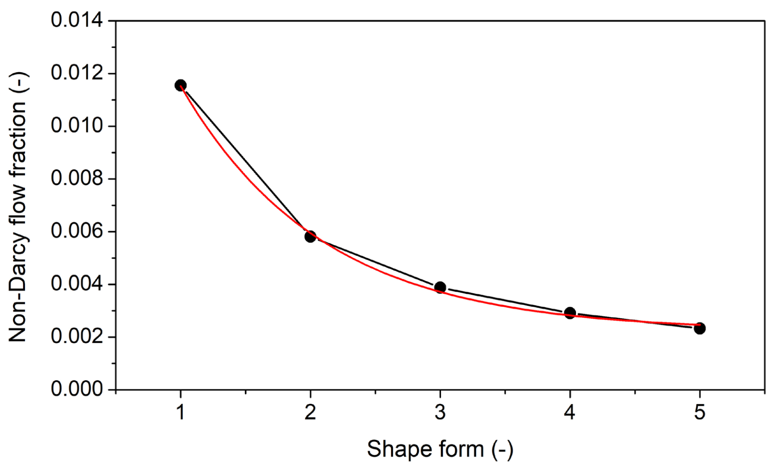

Figure 2. Non-darcy flow fraction of air as a function of the shape factor of the sand grains for a Reynolds number equal to 1.

The red line in **Figure 2** depicts an exponential decay of the non-linear flow with the shape factor. For example, for flake graphite with a factor form of 7.96 Equation (25) predicts a value of 0.0022 for non-linear flow fraction, which is extremely low. According to [1], the shape factor for screened sand grains varies from 1.15 to 2.54, which means that for values of the Reynolds number less than 1, Darcy's law adequately represents the flow of gases through molding sands.

4. Conclusions

The effect of the Reynolds number on the validity of Darcy's law in molding sands was numerically studied in this work using the Ergun equation. From numerical results, the following conclusions arise:

1) An expression that explicitly quantifies the dependence between the Reynolds number and the non-Darcy flow fraction of the air flow in molding sands was obtained and reported.

2) For values of the Reynolds number less than 1 the contribution of non-linearity between air velocity and pressure gradient in molding sands with spherical particles is only 1.15%.

3) This percentage of non-linearity decreases even more with the increase in the form factor of the sand grains, reaching 0.27% for a form factor equal to 7.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Poirier, D.R. and Geiger, G.H. (2016) Transport Phenomena in Materials Processing. TMS-Springer, Cham. https://doi.org/10.1007/978-3-319-48090-9

[2] Ajibola, O.O., Oloruntoba, D.T. and Adewuyi, B.O. (2015) Effects of Moulding Sand Permeability and Pouring Temperature on Properties of Cast 6061 Aluminium Alloy. *International Journal of Metals*, **2015**, Article ID: 632021. https://doi.org/10.1155/2015/632021

[3] Atanda, P.O., Olorunniwo, O.E., Alonge, K. and Oluwole, O.O. (2012) Comparison of Bentonite and Casava Starch on the Moulding Properties of Silica Sand. *International Journal of Materials and Chemistry*, **2**, 132-136.

[4] Ghanbarian, B., Hunt, A.G., Ewing, R.P. and Sahimi, M. (2013) Tortuosity in Porous Media: A Critical Review. *Soil Science Society of America Journal*, **77**, 1461-1477. https://doi.org/10.2136/sssaj2012.0435

[5] Adeyemi, T.S. (2021) Analytical Solution of Unsteady-State Forchheimer Flow Problem in an Infinite Reservoir: The Boltzmann Transform Approach. *Journal of Human, Earth, and Future*, **2**, 225-233. https://doi.org/10.28991/HEF-2021-02-03-04

[6] Anbar, S., Thompson, E.E. and Tyagi, M. (2018) The Impact of Compactation and Sand Migration on Permeability and Non-Darcy Coefficient from Pore-Scale Simulations. *Transport in Porous Media*, **11**, 1-21.

[7] Das, M.K., Mukherjee, P.P. and Muralidhar, K. (2018) Modeling Transport Phenomena in Porous Media with Applications. Springer, Cham.

https://doi.org/10.1007/978-3-319-69866-3

[8] Lu, J., Guo, Z., Chai, Z. and Shi, B. (2009) Numerical Study on the Tortuosity of Porous Media via Lattice Boltzmann Method. *Communications in Computational Physics*, **6**, 354-366. https://doi.org/10.4208/cicp.2009.v6.p354

[9] Nomura, S., Yamamoto, Y. and Sakaguchi, H. (2018) Modified Expression of Kozeny-Carman Equation Based on Semilog–Sigmoid Function. *Soils and Foundations*, **58**, 1350-1357. https://doi.org/10.1016/j.sandf.2018.07.011

[10] Srisutthiyakorn, N. and Mavko, G.M. (2017) What Is the Role of Tortuosity in the Kozeny-Carman Equation? *Interpretation*, **5**, 1F-T141. https://doi.org/10.1190/INT-2016-0080.1

[11] Masumura, R., Mikada, H. and Takekawa, J. (2019) The Relation between Permeability and Grain Size Distribution. *The* 23*rd International Symposium on Recent Advances in Exploration Geophysics* (*RAEG* 2019), 26 May 2019, Chiba, 1-4. https://doi.org/10.3997/2352-8265.20140242

[12] Iversen, N. and Jorgensen, B.B. (1993) Diffusion Coefficients of Sulfate and Methane in Marine Sediments: Influence of Porosity. *Geochimica et Cosmochimica Acta*, **57**, 571-578. https://doi.org/10.1016/0016-7037(93)90368-7

[13] Koponen, A., Kataja, M. and Timonen, J. (1996) Tortuous Flow in Porous Media. *Physical Review E*, **54**, 406-410. https://doi.org/10.1103/PhysRevE.54.406

[14] Hubbert, M.K. (1940) The Theory of Ground-Water Motion. *The Journal of Geology*, **48**, 785-944. https://doi.org/10.1086/624930

[15] Zimmerman, R.W. (2018) Fluid Flow in Porous Media. World Scientific, London.

[16] Zhang, T., Zhao, Y., Gan, Q., Yuan, L., Zhu, G. and Cai, Y. (2018) Experimental Investigation of the Forchheimer Coefficients for Non-Darcy Flow in Conglomerate-Confined Aquifer. *Geofluids*, **2018**, Article ID: 4209197. https://doi.org/10.1155/2018/4209197

[17] Wu, J., Hu, D., Li, W. and Cai, X. (2016) A Review of Non-Darcy Flow—Forchheimer Equation, Hydraulics Radius Model, Fractal Model and Experiment. *Fractals*, **24**, Article ID: 1630001. https://doi.org/10.1142/S0218348X16300014