

Relation between the Actual and Estimated Maximum Ground Level Concentration of Air Pollutant and Its Downwind Locations

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Abstract

In this work we used the Gaussian plume model to calculate the actual maximum ground level concentration (MGLC) of air pollutant and its downwind location by using different systems of dispersion parameters and for different stack heights. An approximate formula for the prediction of downwind position that produces the MGLC of a pollutant based on the Gaussian formula was derived for different diffusion parameters. The derived formula was used to calculate the approximate MGLC. The actual and estimated values are presented in tables. The comparison between the actual and estimated values was investigated through the calculation of the relative errors. The values of the relative errors between the actual and estimated MGLC lie in the range from: 0 to 70.2 and 0 to 1.6 for Pasquill Gifford system and Klug system respectively. The errors between the actual and estimated location of the MGLC lies in the range from: 0.2 to 227 and 0.7 to 9.4 for Pasquill Gifford system and Klug system respectively.

Keywords

Gaussian Plume Model, Actual and Estimated Values, Relative Errors

1. Introduction

The atmospheric diffusion models that are widely used for regulatory purposes have been reviewed by [1]. The most important parameters predicted by these models are the magnitude and location of the maximum ground level concentration. The Gaussian Plume Model (GPM) is a widely used dispersion model in more regulatory applications than the other models. It was derived by many authors such as [2] [3] [4] [5]. Maximum ground level concentration of air pollu-

tant was studied by [6].

The GPM has been expressed in terms of the horizontal and vertical diffusion parameters, $\sigma_y(x)$ and $\sigma_z(x)$. Different formulae for $\sigma_y(x)$ and $\sigma_z(x)$ have been collected from different References and presented by [7].

The concentration of a pollutant is a function of a number of variables, such as the emission rate, the location of the receptor from the source and the atmospheric conditions as: wind speed, wind direction and the vertical temperature changes in the local atmosphere.

The major purpose of the present study is to derive an approximate formula for estimating the downwind location of the maximum concentration that is used to find the MGLC of the released pollutant based on the Gaussian model.

The actual MGLC of air pollutant and its location was calculated by using the Gaussian formula for different dispersion parameters $\sigma_y(x)$ and $\sigma_z(x)$, and different stack heights. Different formulae of $\sigma_z(x)$ are used to derive an approximate formula for the downwind location of the maximum concentration that is used to predict the approximate MGLC.

The actual and estimated values are presented in tables. The comparison between the actual and estimated values was investigated through the calculation of the relative errors.

2. Model Description

The Gaussian plume model for estimating the concentration of pollutant released from a continuous point source at some point above the ground is given by [8] [9]:

$$C(x, y, z, H) = \frac{Q}{2\pi u \sigma_y \sigma_z} e^{-y^2/2\sigma_y^2} \left[e^{-(z-H)^2/2\sigma_z^2} + e^{-(z+H)^2/2\sigma_z^2} \right] \quad (1)$$

where,

C = Concentration of pollutant in air ($\text{g}\cdot\text{m}^{-3}$),

Q = Emission rate ($\text{g}\cdot\text{s}^{-1}$),

u = Wind speed at the effective release height ($\text{m}\cdot\text{s}^{-1}$),

x = Downwind distance from the source (m),

y = Lateral distance from the plume center line (m),

z = Vertical height above ground (m),

H = Effective release height above the ground (m),

σ_y and σ_z are the lateral and vertical dispersion parameters (m).

The second exponential term within the brackets is the term due to reflection at the ground surface. The ground level concentration for an elevated release below the centerline of the plume is obtained by setting y and $z = 0$ in Equation (1):

$$C(x, 0, 0, H) = \frac{Q}{\pi u \sigma_y \sigma_z} \exp\left[-\frac{H^2}{2\sigma_z^2}\right] \quad (2)$$

The variables σ_y and σ_z depend on the downwind distance from the source (x), and the atmospheric stability.

The actual MGLC below the plume centerline (C_{ma}) and its location (x_{ma}) can be calculated by using Equation (2). The rough estimation of the position of the MGLC (x_{me}) is calculated [10]:

$$\sigma_z = \frac{H}{\sqrt{2}} \quad (3)$$

Therefore, the rough estimation of the MGLC can be calculated from the following equation at $x = x_{me}$:

$$C_{me}(x, 0, 0, H) = \frac{2Q}{\pi euH^2} \left(\frac{\sigma_z}{\sigma_y} \right) \quad (4)$$

The formula of x_{me} can be derived by solving Equation (3) using different systems of the vertical diffusion parameters [11] as follows:

2.1. Pasquill-Gifford System

The values of σ_y and σ_z as functions of distance for use with his suggested stability categories were suggested by [12]. The modified values of σ_y and σ_z for use with the original Pasquill stability categories were suggested by [13]. The combination of Pasquill and Gifford parameters is called P-G scheme. In this scheme σ_y and σ_z are obtained from graphs as a function of downwind distance, x , for each stability class. These curves can be approximated by the following equations [14]

$$\sigma_y(x) = (a_1 \ln x + a_2)x \quad (5)$$

$$\sigma_z(x) = \frac{1}{2.15} \exp(b_1 + b_2 \ln x + b_3 \ln^2 x) \quad (6)$$

where the constants a_1 , a_2 , b_1 , b_2 , and b_3 depend on the atmospheric stability and their values are presented in **Table 1**.

From Equation (6) and Equation (3) we find the formula of the estimated position of the MGLC as:

$$x_{me} = \exp \left(\frac{-b_2 \pm \sqrt{b_2^2 - 4b_3 \left[b_1 - \ln \left(\frac{2.15H}{\sqrt{2}} \right) \right]}}{2b_3} \right) \quad (7)$$

Table 1. Coefficient of the Pasquill-Gifford system for all stability classes [14].

Coefficient	Stability Categories					
	A	B	C	D	E	F
a_1	-0.0234	-0.0147	-0.0117	-0.0059	-0.0059	-0.0029
a_2	0.3500	0.2480	0.1750	0.1080	0.0880	0.0540
b_1	0.8800	-0.9850	-1.1860	-1.3500	-2.8800	-3.8000
b_2	0.1520	0.8200	0.8500	0.7930	1.2550	1.4190
b_3	0.1475	0.0168	0.0045	0.0420	-0.0420	-0.0550

where, the positive square root is selected to estimate x_{me} [14].

2.2. Power Law Method (Klug System)

Specified a system of diffusion parameters that is applicable for short-term ground-level release over terrain with a low surface roughness [15] [16]. Klug does not exceed source distances of 2 or 3 km. In this range the diffusion parameters can be described by power law functions as:

$$\sigma_y(x) = p_y x^{q_y} \quad (8)$$

$$\sigma_z(x) = p_z x^{q_z} \quad (9)$$

where the coefficients p and q are specified in **Table 2**. The formula of the estimated location of the MGLC will be as:

$$x_{me} = \left(\frac{H}{\sqrt{2P_z}} \right)^{\frac{1}{q_z}} \quad (10)$$

2.3. Standard Scheme

In this scheme, the crosswind dispersion parameter $\sigma_y(x)$ and the vertical dispersion parameter $\sigma_z(x)$ for various stability classes can be analytically expressed based on Pasquill-Gifford (P-G) curves as follows [17]:

$$\sigma_y = \frac{rx}{(1+x/a)^p} \quad (11)$$

$$\sigma_z = \frac{sx}{(1+x/a)^q} \quad (12)$$

where r , s , a , p and q are constants depending on the atmospheric stability. Their values are given in **Table 3** (Green *et al.*, 1980). The derived formula of the estimated location of the MGLC is:

$$x_{me} = \frac{a}{q-1} \quad (13)$$

The actual MGLC (C_{ma}) and its location (x_{ma}) were calculated by using Equation (2) by differentiating it with respect to “ x ” and equate the result with zero, then find the value of maximum downwind distance and substituting in Equation (2) to find the actual MGLC. Equation (7), Equation (10) and Equation (13)

Table 2. Values of parameters of the power law (Klug System) for all stability classes [14].

Coefficient	Stability			Categories		
	A	B	C	D	E	F
p_y	0.4690	0.3060	0.2300	0.2190	0.2370	0.2730
q_y	0.9030	0.8850	0.8550	0.7640	0.6910	0.5940
p_z	0.0170	0.0720	0.0760	0.1400	0.2170	0.2620
q_z	1.3800	1.0210	0.8790	0.7270	0.6100	0.5000

Table 3. Values of the dispersion parameters corresponding to Pasquill stability.

Atmospheric			Stability	Categories	
Stability	r (m/km)	s (m/km)	a (km)	p	q
A	250	102	0.927	0.189	-0.918
B	202	96.2	0.37	0.162	-0.101
C	134	72.2	0.283	0.134	0.102
D	78.7	47.5	0.707	0.135	0.465
E	56.6	33.5	1.07	0.137	0.624
F	37	22	1.17	0.134	0.70

are used to estimate the location of the MGLC (x_{me}). The values of x_{me} are used to estimate the values of the MGLC by using Equation (4). The maximum concentrations of pollutant and their downwind locations were calculated using $Q = 3$ g/s and $u = 3$ m/s for different effective heights (5 m, 45 m, 100 m, and 250 m) and different atmospheric stabilities. The comparison between the actual and estimated values is investigated through the calculation of the relative error:

$$\text{Relative error} = \left| \frac{\text{actual value} - \text{estimated value}}{\text{actual value}} \right| \quad (14)$$

The results of this study are presented in **Tables 4-6**.

3. Results and Discussion

The actual and estimated maximum ground level concentrations of pollutant and their downwind locations were calculated using “emission rate” $Q = 3$ g/s and “wind speed” $u = 3$ m/s for different effective source heights (5 m, 45 m, 100 m, and 250 m) and for different atmospheric stabilities.

We see from Equation (7), Equation (10) and Equation (13) that the formula for x_{me} derived by using Pasquill-Gifford system and Power law method is dependent on the stability of air and the effective source height (H), while the derived formula using the Standard scheme was found to be dependent on the atmospheric stability only.

The results of this study are presented in tables. The comparisons between the actual and estimated values are investigated through the calculation of the relative error.

Tables 4-6 reveal that for each effective stack height (H) as the atmospheric stability tends to be stable both the actual and estimated position of the MGLC tends to be far from the stack except for stability class B at $H = 5$ m in **Table 5**. Also, the increase in the effective source height gives the same trend. These tables clarify that for each stability class the higher H gives the lower and the more distant concentration.

4. Conclusions

From **Table 4** for Pasquill Gifford system, one can see that higher H gives higher error for the estimation of maximum concentration (C_m) and its location (x_m).

Table 4. Relation between actual and estimated values for MGLC and its location using Pasquill-Gifford system for different stack heights.

Stability class	C_{ma} g/m ³	x_{ma} m	C_{me} g/m ³	x_{me} m	Relative C_m	Error% x_m
<i>H</i> = 5 m						
A	1.101E-2	9.9	1.099E-2	10.2	0.1	3.6
B	5.415E-3	31.3	5.415E-3	31.0	0.0	0.2
C	6.172E-3	40.4	6.171E-3	40.8	0.0	1.0
D	5.926E-3	64.6	5.910E-3	67.4	0.3	4.4
E	5.339E-3	101.1	5.338E-3	102.3	0.0	1.2
F	5.036E-3	164.5	5.032E-3	168.3	0.1	2.3
<i>H</i> = 45 m						
A	2.086E-4	77.2	2.094E-4	71.9	1.2	6.9
B	7.575E-5	304.1	7.565E-5	296.1	0.1	2.6
C	7.521E-5	478.8	7.521E-5	476.0	0.0	0.6
D	5.528E-5	959.9	5.520E-5	991.2	0.1	3.3
E	4.385E-5	1721.6	4.308E-5	1977.2	1.7	14.8
F	3.050E-5	3226	3.861E-5	4330.0	6.2	34.2
<i>H</i> = 100 m						
A	5.528E-5	132.6	5.684E-5	122.8	2.5	7.4
B	1.682E-5	667	1.678E-5	645.7	0.2	3.2
C	1.563E-5	1159.2	1.563E-5	1145.2	0.0	1.2
D	1.033E-5	2537.1	1.032E-5	2607.8	0.1	2.8
E	5.026E-6	6368.6	5.752E-6	8285.1	4.5	30.1
F	2.743E-6	14,699.4	2.013E-6	34,757.0	26.6	136.5
<i>H</i> = 250 m						
A	1.427E-5	231.7	1.382E-5	213.7	3.2	7.8
B	3.077E-6	1605.2	3.067E-6	1542.8	0.3	3.9
C	2.638E-6	3171	2.636E-6	3107.6	0.1	2.0
D	1.531E-6	7697.4	1.530E-6	7864.8	0.1	2.2
E	4.491E-7	41,722.4	3.828E-7	81,308.5	14.8	94.9
F	3.258E-8	122,425.9	5.544E-8	400,312.2	70.2	227.0

Table 5. Relation between actual and estimated values for MGLC and its location using power law method (Klug system) for different stack heights.

Stability class	C_{ma} g/m ³	x_{ma} m	C_{me} g/m ³	x_{me} m	Relative C_m	Error% x_m
<i>H</i> = 5 m						
A	2.184E-3	51.2	2.150E-3	47.8	1.6	6.6
B	3.713E-3	48.9	3.705E-3	45.3	0.2	3.3

Continued

C	3.440E-3	79.5	3.439E-3	78.9	0.0	0.7
D	5.085E-3	83.4	5.084E-3	84.9	0.0	1.8
E	5.937E-3	92	5.924E-3	97.0	0.2	5.5
F	5.539E-3	166.5	5.515E-3	182.1	0.4	9.4
<i>H</i> = 45 m						
A	5.763E-5	251.8	5.672E-5	235.1	1.6	6.6
B	6.143E-5	403.5	6.129E-5	389.9	0.2	3.4
C	4.509E-5	968.5	4.509E-5	962.2	0.0	0.8
D	5.614E-5	1714	5.612E-5	1743.8	0.0	1.7
E	5.475E-5	3375.5	5.463E-5	3558.0	0.2	5.4
F	4.524E-5	13,482.5	4.505E-5	14,750.0	0.4	9.4
<i>H</i> = 100 m						
A	1.538E-5	449.1	1.514E-5	419.3	1.6	6.6
B	1.383E-5	881.6	1.380E-5	852.3	0.2	3.3
C	9.332E-6	2402.8	9.331E-6	2384.1	0.0	0.8
D	1.092E-5	5140.6	1.091E-5	5230.1	0.0	1.7
E	9.971E-6	12,497.7	9.950E-6	13,137.9	0.2	5.4
F	7.884E-6	66,581	7.850E-6	75,839.6	0.4	9.4
<i>H</i> = 250 m						
A	3.377E-6	872	3.324E-6	814.4	1.5	6.6
B	2.501E-6	2162.9	2.495E-6	2091.1	0.2	3.3
C	1.531E-6	6814.6	1.531E-6	6761.6	0.0	0.8
D	1.667E-6	18,129.1	1.666E-6	18,445.2	0.0	1.7
E	1.413E-6	56,129.5	1.410E-6	59,166.3	0.2	5.4
F	1.062E-6	416,131	1.057E-6	455,247.4	0.4	9.4

Table 6. Actual values for MGLC, relation between actual and estimated values for its location using standard scheme for different stack heights.

Stability class	C_{ma} g/m ³	x_{ma} m	x_{me} m	Relative error% x_m
<i>H</i> = 5 m				
A	4.11E-3	33	-317.7	10.6
B	4.575E-3	36.5	-336.1	10.2
C	5.076E-3	50	-315.1	7.3
D	5.465E-3	77.5	-1321.5	18.05
E	5.286E-3	111	-2845.7	26.6
F	5.149E-3	173.5	-3900	23.5
<i>H</i> = 45 m				

Continued

A	7.350E-5	223.5	-317.7	2.4
B	6.481E-5	320.1	-336.1	2.05
C	6.438E-5	491	-315.1	1.64
D	5.251E-5	932.9	-1321.5	2.41
E	4.395E-5	1476.5	-2845.7	2.92
F	3.166E-5	2939.7	-3900	1.24
<i>H</i> = 100 m				
A	1.968E-5	393.3	-317.7	1.8
B	1.467E-5	689.2	-336.1	1.48
C	1.330E-5	1165	-315.1	1.27
D	8.116E-6	2877.2	-1321.5	1.45
E	5.349E-6	5469.5	-2845.7	1.52
F	2.535E-6	15,718.2	-3900	1.24
<i>H</i> = 250 m				
A	4.752E-6	688.5	-317.7	1.46
B	2.769E-6	1632.4	-336.1	1.2
C	2.188E-6	3187.7	-315.1	1.09
D	8.209E-6	13,099.8	-1321.5	1.10
E	3.236E-7	41,423.9	-2845.7	1.06
F	8.250E-8	243,818.1	-3900	1.02

Table 6 shows that the values of x_{me} estimated by Equation (13) are negative values so the approximation $\sigma_z = H/\sqrt{2}$ is not suitable for the standard scheme.

The values of the relative errors between the actual and estimated MGLC lie in the range from: 0 to 70.2 and 0 to 1.6 for Pasquill Gifford system and Klug system respectively. The errors between the actual and estimated location of the MGLC lies in the range from: 0.2 to 227 and 0.7 to 9.4 for Pasquill Gifford system and Klug system respectively.

From this discussion we conclude that the approximation $\sigma_z = H/\sqrt{2}$ is most suitable for the Klug system and Pasquill Gifford system.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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