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General Relativity Can Explain Dark Matter without Exotic Matter

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Abstract

Dark matter is one of the most important mystery in astrophysics. Several explanations have been proposed. The most accepted one is the existence of an exotic matter, exotic because unsensitive to electromagnetism, unlike baryonic matter. One problem is that except on scales beyond the galaxies, no new matter is needed. This is an ad hoc addition only for large structures of the universe. But then another problem is that this added matter that should concern large astrophysical structures is not a slight correction of the quantities of matter, but it makes the only known baryonic matter a slight correction of this exotic matter. It means that gravitation theory would have been founded on a particular matter. From this point of view, baryonic matter would finally be the exotic matter and dark matter the normal one. But despite its dominance, no new matter has been discovered to date. In this article we propose a new explanation of dark matter compliant with General Relativity and without new matter. Furthermore, MOND-based theories appear as particular solutions of this explanation and allows demonstrating the Tully-Fisher relation. The clusters of galaxies would generate an extremely light Lense-Thirring effect on galaxies in the form of a relatively uniform and very weak field.

Subject Areas

Cosmology

Keywords

(Cosmology) Dark Matter, Cosmology, Theory, Galaxies, Formation, Galaxies, Fundamental Parameters

1. Introduction

The observations associated with the theory of General Relativity (GR) make it necessary to add two new terms to the energy balance of the universe. One generates an attractive action perceptible on the large structures of the universe from the scale of the galaxies, called dark matter (DM), and the other a repulsive action perceptible on even larger structures of the universe, called dark energy. These two unknown components are not currently understood and constitute the most important mysteries of our current knowledge in astrophysics because they represent the vast majority of the energy content of the universe. Our study focuses on the problem of DM. The most accepted explanation is to postulate the existence of an exotic matter, exotic because it has a behavior different from the only known baryonic matter. The name of this component, DM, comes from this explanation. At this day, no exotic matter has been observed. Another explanation is to postulate that it is necessary to modify the theoretical framework of gravitation starting with the Newtonian framework, which gave the name of MOND to this explanation. However, GR is extremely well verified and it is therefore extremely difficult to justify a modification of the theoretical framework of gravitation.

In our study, we propose a new explanation of the DM component. More precisely, our goal is to propose an explanation of the flat rotation curve of galaxies. This explanation is compliant with GR and does not require exotic matter. As demonstrated in several papers, the Linearized General Relativity (LGR), also called gravitoelectromagnetism or gravitomagnetism, can't explain this curve if we consider the own gravitation field of the galaxy (even by taking in account the non-linear terms). Nevertheless, LGR sheds new light on the interpretation of GR. This will allow to very naturally introduce our new hypothesis. The LGR shows that the gravitation can be interpreted in the same way than ElectroMagnetism (EM) when the gravitational field is weak. We will see that at the end of the galaxies, where the rotation speed curve flattens, the gravitational field is effectively weak enough to justify this approximation. Concretely, in addition to the Newtonian component, $\boldsymbol{g}$, LGR (and consequently GR) makes necessary the existence of a 2nd component, $\boldsymbol{k}$ (that we will call the gravitic field) giving the following movement equations $m_i \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = m_p \left[\boldsymbol{g} + 4\boldsymbol{v} \wedge \boldsymbol{k}\right]$, as reminded hereafter. This 2nd component is at the origin of the Lense-Thirring effect which was measured for the earth (Adler, 2015) [1] and whose results agree with GR. We can now introduce our hypothesis. Since the LGR modeling is similar to the EM and that $\boldsymbol{k}$ is similar to the magnetic field $\boldsymbol{B}$, we can use the entire EM toolbox and apply it to the LGR. In particular, the mathematical form of $\boldsymbol{B}$ in EM allows the existence of large-scale magnetic fields (unlike the central electric field $\boldsymbol{E}$) in the form of a uniform field. For example, we have the case in particles accelerators which make it possible to have high-speed trajectories maintained in an orbit which would not be possible without this uniform field. We will therefore make the hypothesis that:

HP: The galaxies are embedded in a relatively uniform external gravitic field

$\boldsymbol{k}_0$ (maintaining orbital speeds much higher than they could be without this field $\boldsymbol{k}_0$)

This uniform field is called external because it can only be produced by a structure larger than the galaxy itself, as we will see it. Nevertheless, in our study we will not focus on the origin of such a field but only on the fact that such a field makes it possible to explain DM without exotic matter and within the framework of GR. But it is very likely that this gravitic field $\boldsymbol{k}_0$ is generated by galaxy clusters (Le Corre, 2015) [14] and (Le Corre, 2023) [10]. If that is the case, we can therefore predict that a map of the universe indicating the presence of DM (such as EUCLID could produce) would reveal larger quantities of DM at the center of the galaxies' clusters than elsewhere.

2. DM Term in Linearized General Relativity

Our study will focus on the equations of GR in weak gravitational field. These equations are obtained from the linearization of GR (LGR). They are very close to the modeling of EM. Let's remind how we obtain the equations of LGR.

2.1. From General Relativity to Linearized General Relativity

From GR, one deduces the LGR in the approximation of a quasi-flat Minkowski space ($g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$; $\left|h^{\mu\nu}\right| \ll 1$). With the following Lorentz gauge, it gives the following field equations as in (Hobson *et al.*, 2006) [2] (with $\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta$ and $\Delta = \nabla^2$):

$$\partial_\mu \bar{h}^{\mu\nu} = 0;\ \Box \bar{h}^{\mu\nu} = -2\frac{8\pi G}{c^4} T^{\mu\nu} \tag{1}$$

With:

$$\bar{h}^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} h;\ h \equiv h_\sigma^\sigma;\ h_\nu^\mu = \eta^{\mu\sigma} h_{\sigma\nu};\ \bar{h} = -h \tag{2}$$

The general solution of these equations is:

$$\bar{h}^{\mu\nu}(ct, \boldsymbol{x}) = -\frac{4G}{c^4}\int \frac{T^{\mu\nu}(ct - |\boldsymbol{x}-\boldsymbol{y}|, \boldsymbol{y})}{|\boldsymbol{x}-\boldsymbol{y}|} \mathrm{d}^3\boldsymbol{y} \tag{3}$$

In the approximation of a source with low speed, one has:

$$T^{00} = \rho c^2;\ T^{0i} = c\rho u^i;\ T^{ij} = \rho u^i u^j \tag{4}$$

And for a stationary solution, one has:

$$\bar{h}^{\mu\nu}(\boldsymbol{x}) = -\frac{4G}{c^4}\int \frac{T^{\mu\nu}(\boldsymbol{y})}{|\boldsymbol{x}-\boldsymbol{y}|} \mathrm{d}^3\boldsymbol{y} \tag{5}$$

At this step, by proximity with electromagnetism, one traditionally defines a scalar potential φ and a vector potential H^i. There are in the literature several definitions as in (Mashhoon, 2008) [3] for the vector potential H^i. In our study, we are going to define:

$$\bar{h}^{00} = \frac{4\varphi}{c^2};\ \bar{h}^{0i} = \frac{4H^i}{c};\ \bar{h}^{ij} = 0 \tag{6}$$

With gravitational scalar potential φ and gravitational vector potential H^i:

$$\varphi(\boldsymbol{x}) \equiv -G\int \frac{\rho(\boldsymbol{y})}{|\boldsymbol{x}-\boldsymbol{y}|}\mathrm{d}^3\boldsymbol{y}$$

$$H^i(\boldsymbol{x}) \equiv -\frac{G}{c^2}\int \frac{\rho(\boldsymbol{y})u^i(\boldsymbol{y})}{|\boldsymbol{x}-\boldsymbol{y}|}\mathrm{d}^3\boldsymbol{y} = -K^{-1}\int \frac{\rho(\boldsymbol{y})u^i(\boldsymbol{y})}{|\boldsymbol{x}-\boldsymbol{y}|}\mathrm{d}^3\boldsymbol{y} \tag{7}$$

With K a new constant defined by (Le Corre, 2015) [14]:

$$GK = c^2 \tag{8}$$

This definition gives $K^{-1} \sim 7.4\times10^{-28}\ \mathrm{kg}\cdot\mathrm{m}^{-1}$ very small compare to G.

The field Equations (1) can be then written (Poisson equations):

$$\Delta\varphi = 4\pi G\rho;\ \Delta H^i = \frac{4\pi G}{c^2}\rho u^i = 4\pi K^{-1}\rho u^i \tag{9}$$

With the following definitions of $\boldsymbol{g}$ (gravity field) and $\boldsymbol{k}$ (gravitic field), those relations can be obtained from the following Equations (also called gravitomagnetism) with the differential operators "$\boldsymbol{rot} = \nabla\wedge$", "$\boldsymbol{grad} = \nabla$" and "$div = \nabla\cdot$":

$$\begin{aligned} &\boldsymbol{g} = -\boldsymbol{grad}\,\varphi;\ \boldsymbol{k} = \boldsymbol{rot}\,\boldsymbol{H} \\ &\boldsymbol{rot}\,\boldsymbol{g} = 0;\ div\,\boldsymbol{k} = 0; \\ &div\,\boldsymbol{g} = -4\pi G\rho;\ \boldsymbol{rot}\,\boldsymbol{k} = -4\pi K^{-1}\boldsymbol{j}_p \end{aligned} \tag{10}$$

With the Equations (2), one has:

$$h^{00} = h^{11} = h^{22} = h^{33} = \frac{2\varphi}{c^2};\ h^{0i} = \frac{4H^i}{c};\ h^{ij} = 0 \tag{11}$$

The equations of geodesics in the linear approximation give:

$$\frac{\mathrm{d}^2x^i}{\mathrm{d}t^2} \sim -\frac{1}{2}c^2\delta^{ij}\partial_j h_{00} - c\delta^{ik}\left(\partial_k h_{0j} - \partial_j h_{0k}\right)v^j \tag{12}$$

It then leads to the movement equations:

$$\frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2} \sim -\boldsymbol{grad}\,\varphi + 4\boldsymbol{v}\wedge(\boldsymbol{rot}\,\boldsymbol{H}) = \boldsymbol{g} + 4\boldsymbol{v}\wedge\boldsymbol{k} \tag{13}$$

Remark: All previous relations can be retrieved starting with the parameterized post-Newtonian (PPN) formalism and with the traditional gravitomagnetic field $\boldsymbol{B}_g$. From (Clifford, 2014) [4] one has:

$$g_{0i} = -\frac{1}{2}(4\gamma + 4 + \alpha_1)V_i;\ V_i(\boldsymbol{x}) = \frac{G}{c^2}\int \frac{\rho(\boldsymbol{y})u_i(\boldsymbol{y})}{|\boldsymbol{x}-\boldsymbol{y}|}\mathrm{d}^3\boldsymbol{y} \tag{14}$$

The traditional gravitomagnetic field and its acceleration contribution are:

$$\boldsymbol{B}_g = \nabla\wedge\left(g_{0i}\boldsymbol{e}^i\right);\ \boldsymbol{a}_g = \boldsymbol{v}\wedge\boldsymbol{B}_g \tag{15}$$

And in the case of GR (that is our case):

$$\gamma = 1;\ \alpha_1 = 0 \tag{16}$$

It then gives:

$$g_{0i} = -4V_i;\ \boldsymbol{B}_g = \nabla \wedge \left(-4V_i \boldsymbol{e}^i\right) \tag{17}$$

And with our definition:

$$H_i = -\delta_{ij} H^j = \frac{G}{c^2} \int \frac{\rho(\boldsymbol{y}) \delta_{ij} u^j(\boldsymbol{y})}{|\boldsymbol{x} - \boldsymbol{y}|} \mathrm{d}^3 \boldsymbol{y} = V_i(\boldsymbol{x}) \tag{18}$$

One then has:

$$\begin{gathered} g_{0i} = -4H_i; \\ \boldsymbol{B}_g = \nabla \wedge \left(-4H_i \boldsymbol{e}^i\right) = \nabla \wedge \left(4\delta_{ij} H^j \boldsymbol{e}^i\right) = 4\nabla \wedge \boldsymbol{H} \\ \boldsymbol{B}_g = 4\,\boldsymbol{rot}\,\boldsymbol{H} \end{gathered} \tag{19}$$

With the following definition of gravitic field:

$$\boldsymbol{k} = \frac{\boldsymbol{B}_g}{4} \tag{20}$$

One then retrieves our previous relations:

$$\boldsymbol{k} = \boldsymbol{rot}\,\boldsymbol{H};\ \boldsymbol{a}_g = \boldsymbol{v} \wedge \boldsymbol{B}_g = 4\boldsymbol{v} \wedge \boldsymbol{k} \tag{21}$$

The interest of our notation ($\boldsymbol{k}$ instead of $\boldsymbol{B}_g$) is that the field equations are strictly equivalent to Maxwell idealization, in particular the speed of the gravitational wave obtained from these equations is the light celerity with $c^2 = GK$ just like in EM with $c^2 = 1/\mu_0 \varepsilon_0$. Only the movement equations are different with the factor "4". But of course, all the results of our study can be obtained in the traditional notation of gravitomagnetism with the relation $\boldsymbol{k} = \frac{\boldsymbol{B}_g}{4}$.

2.2. From Linearized General Relativity to DM Term

In the classical approximation ($\|\boldsymbol{v}\| \ll c$), the LGR gives the following movement equations from (13) with m_i the inertial mass and m_p the gravitational mass:

$$m_i \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = m_p \left[\boldsymbol{g} + 4\boldsymbol{v} \wedge \boldsymbol{k}\right] \tag{22}$$

The simplified Figure 1 can help us to visualize how the two components of the linearized general relativity intervene in the equilibrium of forces. The gravitic field $\boldsymbol{k}$, perpendicular to galaxy rotation plane, with the velocity of the matter $\boldsymbol{v}$ generates a centripetal force increasing the Newtonian gravitation.

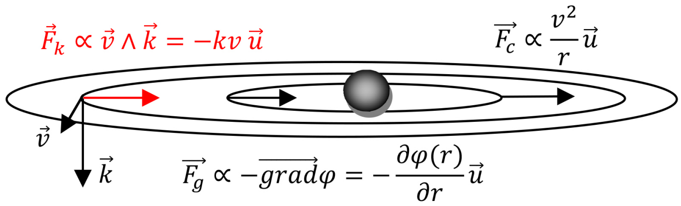


Figure 1. Simplified representation of the equilibrium of forces in a galaxy.

The traditional computation of rotation speeds of galaxies consists in obtaining the force equilibrium from the three following components: the disk, the bugle and the halo of dark matter. More precisely, one has (Kent, 1986) [5]:

$$\frac{v^2(r)}{r} = \frac{\partial \varphi(r)}{\partial r} \text{ with } \varphi = \varphi_{disk} + \varphi_{bulge} + \varphi_{halo} \tag{23}$$

Then the total speed squared can be written as the sum of squares of each of the three speed components:

$$\begin{aligned} v^2(r) &= r\left(\frac{\partial \varphi_{disk}(r)}{\partial r}\right) + r\left(\frac{\partial \varphi_{bulge}(r)}{\partial r}\right) + r\left(\frac{\partial \varphi_{halo}(r)}{\partial r}\right) \\ &= v_{disk}^2(r) + v_{bulge}^2(r) + v_{halo}^2(r) \end{aligned} \tag{24}$$

Disk and bulge components are obtained from the traditional Newtonian gravity field (1st component of LGR, *i.e.* the term "$\boldsymbol{g}$"). They are not modified in our solution. So, our goal is now to obtain only the dark matter halo component from the LGR. According to this idealization, the force due to the gravitic field $\boldsymbol{k}$ (2nd component of LGR) takes the following form $\|\boldsymbol{F}_k\| = m_p 4\|\boldsymbol{v} \wedge \boldsymbol{k}\|$ and it corresponds to previous term $m_p \frac{\partial \varphi_{halo}(r)}{\partial r} = \|\boldsymbol{F}_k\|$. As explained in (Le Corre, 2015), the natural evolution to the equilibrium state justifies that one requires the approximation $\boldsymbol{v} \perp \boldsymbol{k}$ because, with time, the particles with a speed not perpendicular to $\boldsymbol{k}$ must inexorably escape in an helicoidal trajectory outside the rotation's plane. It then gives the following equation:

$$\begin{aligned} \frac{v^2(r)}{r} &= \frac{\partial \varphi_{disk}(r)}{\partial r} + \frac{\partial \varphi_{bulge}(r)}{\partial r} + 4k(r)v(r) \\ &= \frac{v_{disk}^2(r)}{r} + \frac{v_{bulge}^2(r)}{r} + 4k(r)v(r) \end{aligned} \tag{25}$$

Our idealization means that:

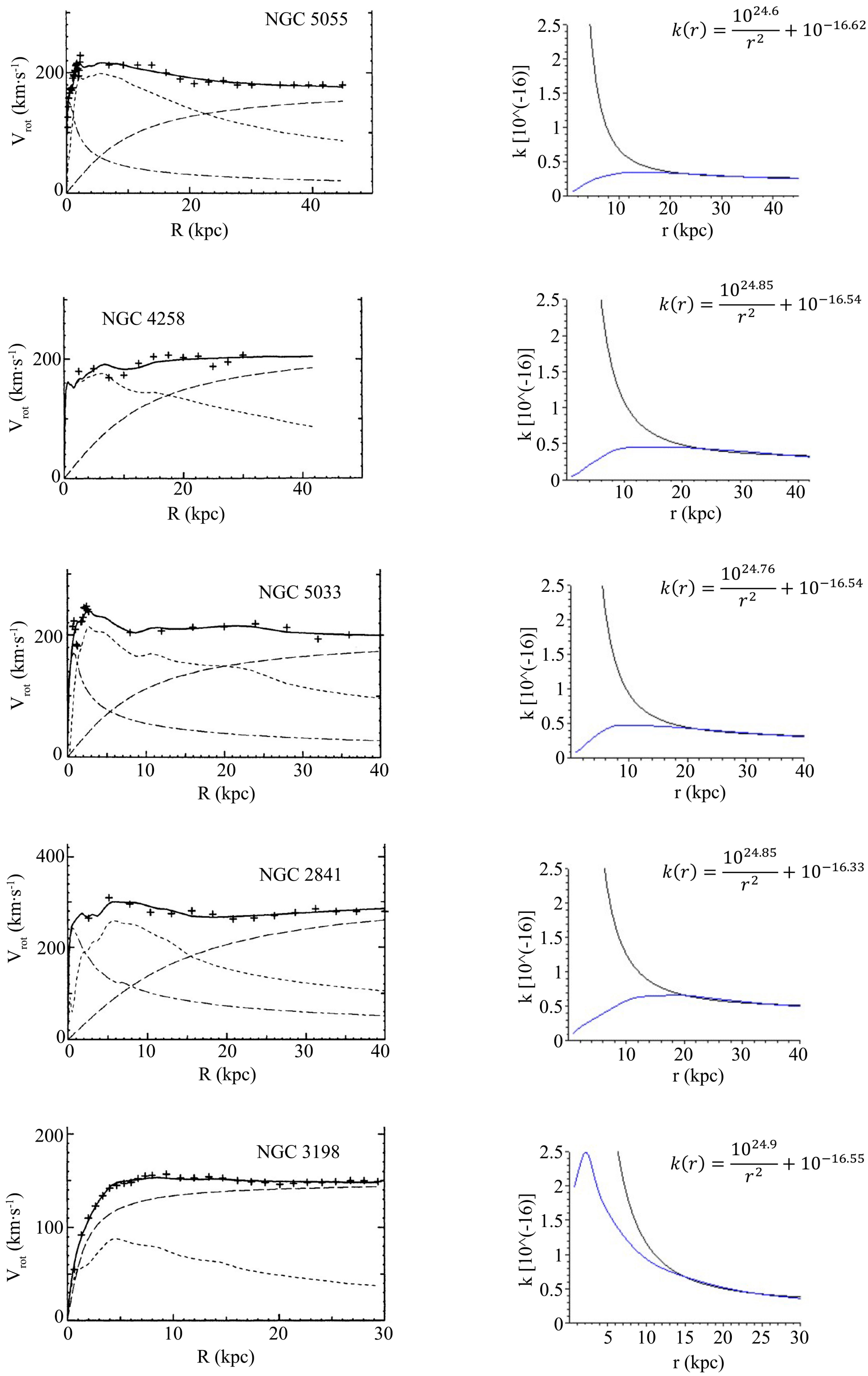
$$v_{halo}^2(r) = v^2(r) - v_{disk}^2(r) - v_{bulge}^2(r) = 4rk(r)v(r) \tag{26}$$

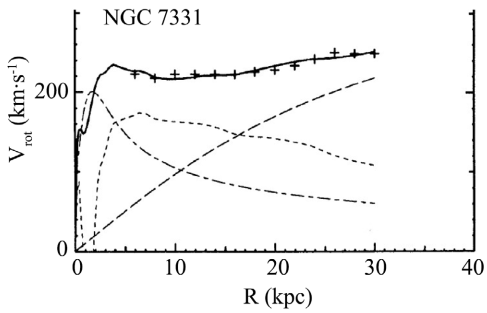
The equation of dark matter (gravitic field in our explanation) is then:

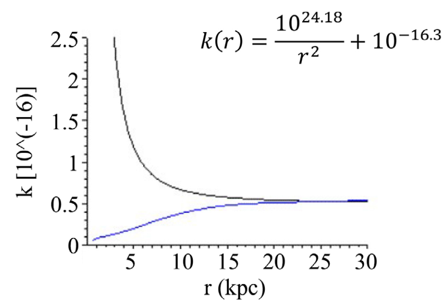
$$v_{halo}(r) = 2\left(rk(r)v(r)\right)^{1/2} \tag{27}$$

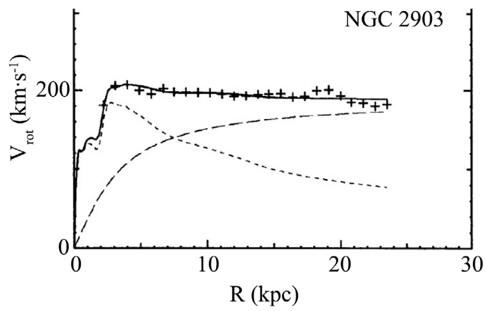
2.3. Dark Matter as the 2nd Component of an External Uniform Gravitational Field k_0: Principles

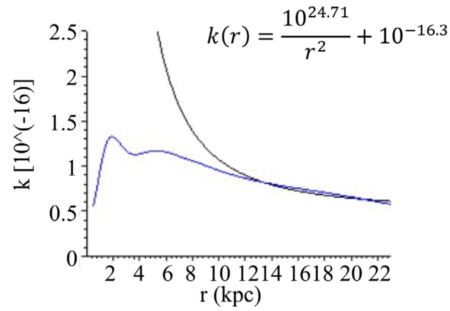
The observations allow obtaining the measurement of the different components of the rotational speed inside the galaxies. Each graph on the left in **Figure 2** gives $v_{disk}(r)$, $v_{bulge}(r)$, $v_{halo}(r)$ and $v(r)$ contributions along the radius of the galaxy. Because the LGR doesn't modify the components $v_{disk}(r)$ and $v_{bulge}(r)$, one can focus our study on the relation $v_{halo}^2(r) = 4k(r)v(r)r$. From these experimental measurements, one can then deduce our expected component of gravitic field inside the galaxies:

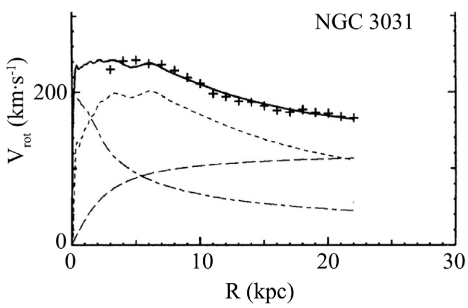

NGC 5055
V_rot (km·s^-1)
R (kpc)
$k(r) = \frac{10^{24.6}}{r^2} + 10^{-16.62}$
k [10^(-16)]
r (kpc)
NGC 4258
$k(r) = \frac{10^{24.85}}{r^2} + 10^{-16.54}$
NGC 5033
$k(r) = \frac{10^{24.76}}{r^2} + 10^{-16.54}$
NGC 2841
$k(r) = \frac{10^{24.85}}{r^2} + 10^{-16.33}$
NGC 3198
$k(r) = \frac{10^{24.9}}{r^2} + 10^{-16.55}$

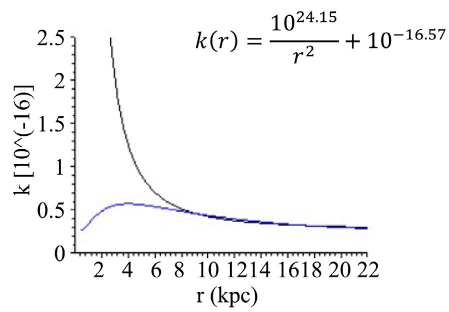

NGC 7331
V_rot (km·s⁻¹)
R (kpc)

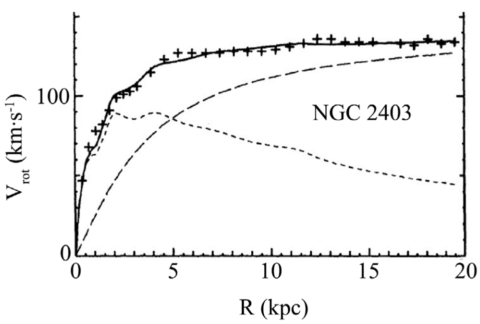

$k(r) = \frac{10^{24.18}}{r^2} + 10^{-16.3}$
k [10^(-16)]
r (kpc)

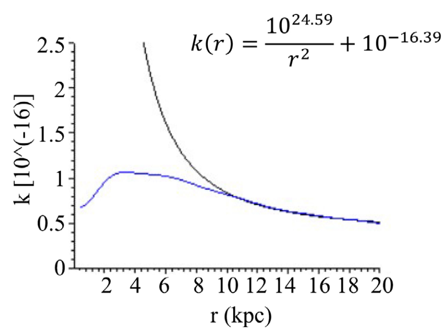

NGC 2903
V_rot (km·s⁻¹)
R (kpc)

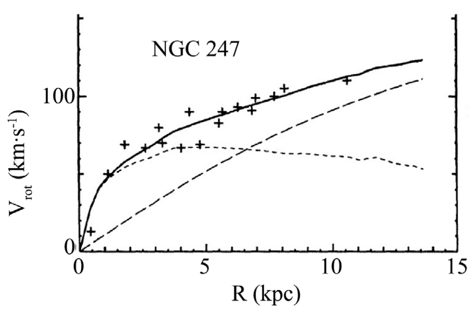

$k(r) = \frac{10^{24.71}}{r^2} + 10^{-16.3}$
k [10^(-16)]
r (kpc)

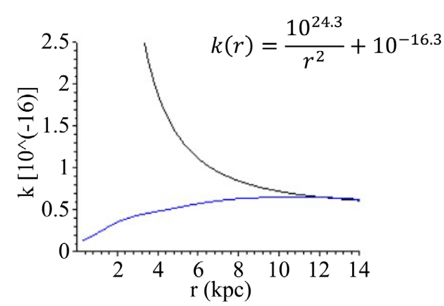

NGC 3031
V_rot (km·s⁻¹)
R (kpc)

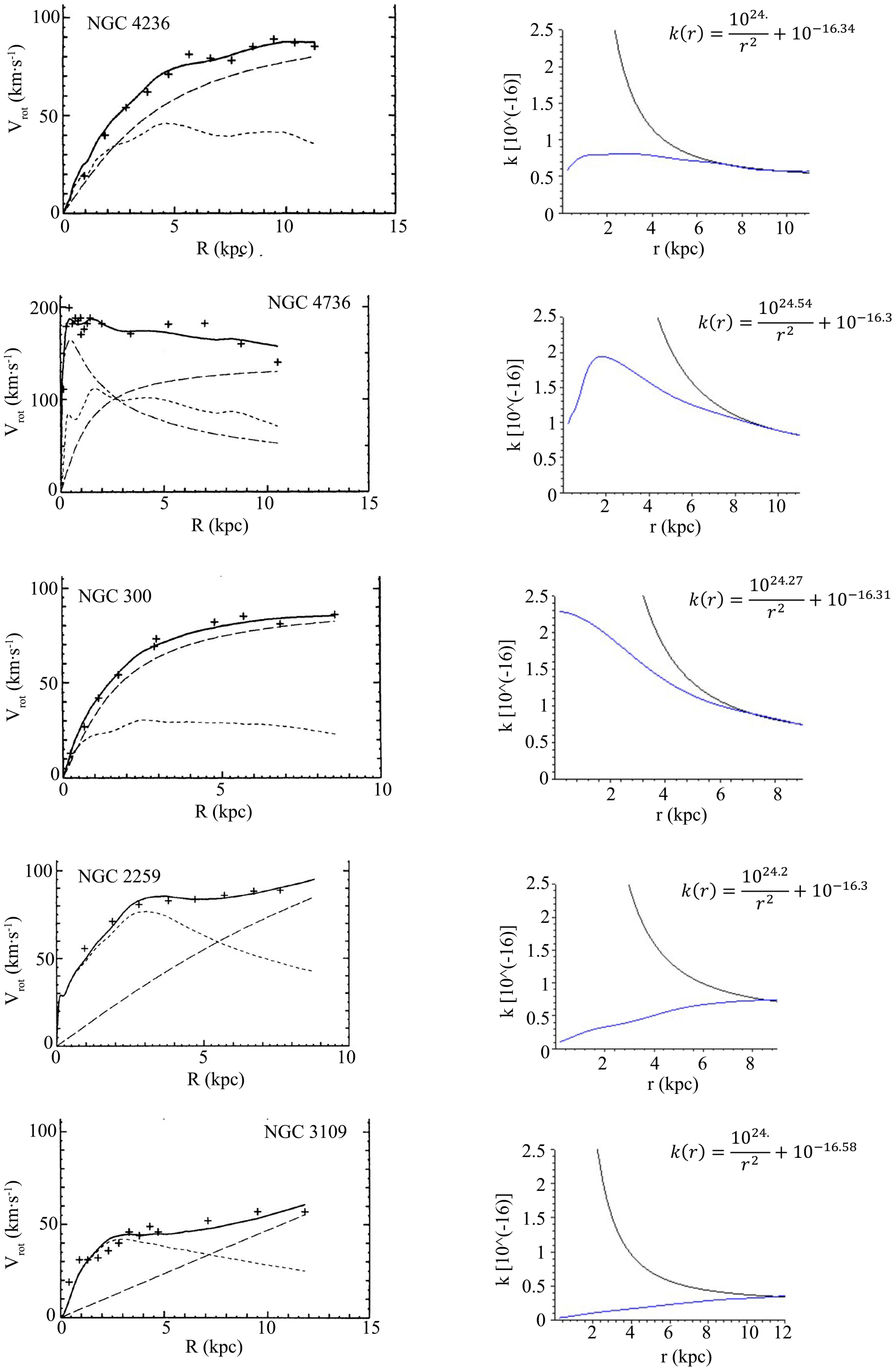

$k(r) = \frac{10^{24.15}}{r^2} + 10^{-16.57}$
k [10^(-16)]
r (kpc)

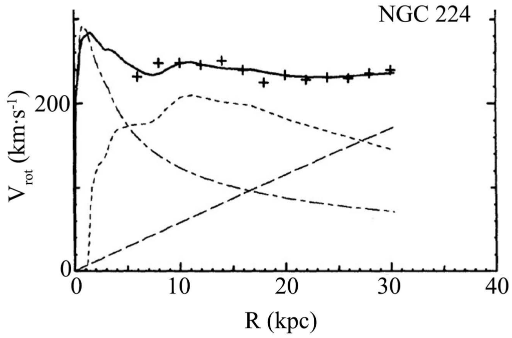

NGC 2403
V_rot (km·s⁻¹)
R (kpc)


$k(r) = \frac{10^{24.59}}{r^2} + 10^{-16.39}$
k [10^(-16)]
r (kpc)


NGC 247
V_rot (km·s⁻¹)
R (kpc)


$k(r) = \frac{10^{24.3}}{r^2} + 10^{-16.3}$
k [10^(-16)]
r (kpc)


NGC 4236
V_{rot} (km·s^{-1})
R (kpc)
$k(r) = \frac{10^{24.}}{r^2} + 10^{-16.34}$
k [10^(-16)]
r (kpc)
NGC 4736
V_{rot} (km·s^{-1})
R (kpc)
$k(r) = \frac{10^{24.54}}{r^2} + 10^{-16.3}$
k [10^(-16)]
r (kpc)
NGC 300
V_{rot} (km·s^{-1})
R (kpc)
$k(r) = \frac{10^{24.27}}{r^2} + 10^{-16.31}$
k [10^(-16)]
r (kpc)
NGC 2259
V_{rot} (km·s^{-1})
R (kpc)
$k(r) = \frac{10^{24.2}}{r^2} + 10^{-16.3}$
k [10^(-16)]
r (kpc)
NGC 3109
V_{rot} (km·s^{-1})
R (kpc)
$k(r) = \frac{10^{24.}}{r^2} + 10^{-16.58}$
k [10^(-16)]
r (kpc)



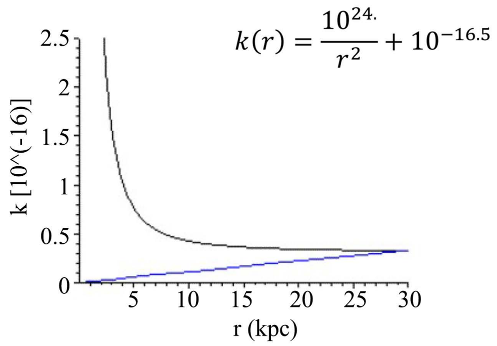


Figure 2. Left graphs are the basis experimental data from (KENT, 1987) [8] with observed rotation speeds (pluses), best-fitting full solution $v(r)$ (solid line) and contribution of components of bulge $v_{bulge}(r)$ (short and long dashed), disk $v_{disk}(r)$ (short-dashed) and dark matter $v_{halo}(r)$ (long dashed); Right graphs represent $k_{exp}(r)$ (in blue) and $k(r)=\frac{K_1}{r^2}+k_0$ (in black) explaining ends of rotation speed curves; Analytic expression of $k(r)$ is given at the top right of each graph.

$$k_{exp}(r)=\frac{v_{halo}^2(r)}{4rv(r)} \tag{28}$$

The calculated $k_{exp}(r)$ curves are the blue curves in the graphs on the right of **Figure 2**. Because the LGR is an approximation for weak gravitational field, these calculated $k_{exp}(r)$ curves should become a correct approximation for large radii. As we will see it, it is the case roughly for $r \gg 10\text{ kpc}$.

In this approximation, from Poisson Equations (9) and far from the center of the galaxy, the own gravitic field of the galaxy (the "internal" gravitic field $k_{gal}(r)$) should be of the following form $k_{gal}(r)=\frac{K_1}{r^2}$, equivalent to the Biot-Savart law in EM. As demonstrated in several articles (Lasenby *et al.*, 2023 [6]; Letelier, 2006 [7]) and as we will confirm it, this term $k_{gal}(r)$ can't explain DM. Our hypothesis HP to explain DM consists in adding another term, a uniform external gravitic field k_0 that embeds the galaxies. So explicitly, in our explanation the gravitic field along the galaxy is:

$$k(r)=k_{gal}(r)+k_0=\frac{K_1}{r^2}+k_0 \tag{29}$$

This 2nd calculated curve $k(r)$ is obtained by fitting the $k_{exp}(r)$ blue curve on its end part (where the approximation of LGR makes sense). The $k(r)$ curves are represented by the black curves in the graphs on the right of **Figure 2**.

2.4. Dark Matter as the 2nd Component of an External Uniform Gravitational Field k_0: Results

We are now going to verify our solution on different galaxies studied in (KENT, 1987) [8]. As explained in detail before, for that we determine $k_{exp}(r)$ curves (blue curves in the graphs on the right of **Figure 2**) from experimental data with the relation (28). And we determine $k(r)$ curves (black curves in the graphs on the right of **Figure 2**) from relation (29). The values of K_1 and k_0 (indicated at the top right for each graph in **Figure 2**) are obtained by fitting the $k_{exp}(r)$ curves at the end of the galaxies, places sufficiently distant to the center to justify the approximation $k(r)=\frac{K_1}{r^2}+k_0$. One can note that the fact that the experimental curves $k_{exp}(r)$ and the approximations $k(r)$ coincide along a large interval far from the center makes this approximation $k(r)$ relevant.

The first important result is the good accuracy of the approximation $k(r)=\frac{K_1}{r^2}+k_0$ at the end of $k_{exp}(r)$ along a distance of several kpc. It means that the existence of a relatively uniform external gravitic field k_0 is possible both mathematically but also physically. This fact was not at all guaranteed a priori.

This study allows retrieving the fact that the own gravitic field of the galaxy « $\frac{K_1}{r^2}$ » can't explain the DM and that for two reasons. The first because this own field should decrease according to r^2. But DM, *i.e.* $k(r)$, doesn't decrease as r^2. The second because the value of $\frac{K_1}{r^2}$ becomes negligible compare to the required embedding gravitic field k_0 explaining DM. For example, with $K_1 \sim 10^{24.5}$ at $r \sim 10\ \text{kpc}$ one has $\frac{K_1}{r^2} \sim 10^{-16.5}\ \text{s}^{-1} \sim k_0$. Consequently, in general, for $r \gg 10\ \text{kpc}$ one has $\frac{K_1}{r^2} \ll k_0$.

The value of the gravitation field, the Newtonian and $k(r)$ terms, at the end of the galaxy justifies the use of LGR. Indeed, for $r \gg 10\ \text{kpc}$, the acceleration due to the Newtonian term with the mass of a galaxy $M \sim 10^{41}\ \text{kg}$ is $\frac{GM}{r^2} \ll 6\times10^{-11}\ \text{m}\cdot\text{s}^{-2}$. With rotational speeds $v \sim 2\times10^{5}\ \text{m}\cdot\text{s}^{-1}$, the acceleration due to the own gravitic field at the end of the galaxy is $\left\|4\boldsymbol{v}\wedge\boldsymbol{k}_{gal}\right\| \sim 4v\frac{K_1}{r^2} \ll 8\times10^{5}\times10^{-16.5} \sim 10^{-10.5}\ \text{m}\cdot\text{s}^{-2}$. And the contribution of the external gravitic field is $\left\|4\boldsymbol{v}\wedge\boldsymbol{k}_0\right\| \ll 10^{-10.5}\ \text{m}\cdot\text{s}^{-2}$. At the end of the galaxies, the total gravitation field is weak enough to justify the use of LGR.

The value of $\left\|4\boldsymbol{v}\wedge\boldsymbol{k}_0\right\|$ also explains why this term is undetectable at our scale, and then why the DM is undetectable at our scale. Because, even with the light speed, this term contributes to the acceleration with a value of only $\left\|4\boldsymbol{v}\wedge\boldsymbol{k}_0\right\| \sim 4ck_0 \sim 10^{-7.5}\ \text{m}\cdot\text{s}^{-2}$.

Furthermore, this solution explains that this DM term becomes greater than the other term of gravitation field for large r (at the scale of the galaxy $r \gg 10\ \text{kpc}$) and for high speed ($v \gg 2\times10^{5}\ \text{m}\cdot\text{s}^{-1}$) because of the expression deduced from (27) $v_{halo}(r) \sim 2(rk_0v)^{1/2}$.

To summarize, the values required to explain the DM for this solution, with our sample of 16 galaxies, are in the following interval:

$$10^{-16.62}\ \text{s}^{-1} < \left\|\boldsymbol{k}_0\right\| < 10^{-16.3}\ \text{s}^{-1} \tag{30}$$

One can add a recent publication with more accurate results on the rotation speed curve of the Milky Way (Jiao *et al.*, 2023) [9]. In **Figure 3**, the measured speeds of Milky Way are presented just like in **Figure 2**. On the left graph, there are the blue curve for the total speed $v(r)$ and the black curve for the DM speed component $v_{halo}(r)$. On the right graph, there are the blue curve $k_{exp}(r)$ and the black curve $k(r)$. The accuracy of the approximation $k(r)$ is once again excellent on the last 10 kpc. One obtains $k(r)=\frac{10^{25.24}}{r^2}+10^{-16.9}$ and $\|\boldsymbol{k}_0\| \sim 10^{-16.9}\,\text{s}^{-1}$. We are in the same order of magnitude than the previous results. But the external field k_0 seems to be weaker. This result is interesting because a weaker value of k_0 would be easier to obtain, in particular if this gravitic field was generated by the brightest cluster galaxy as indicated in (Le Corre, 2023) [10].

2.5. The Uniform Gravitational Field k_0 in φ and g_{00}

We are going to search for an expression of the gravitational potential φ and of the component g_{00} of the metric tensor to be able to obtain orders of magnitude of DM for the CMB and for the weak gravitational lensing. Our previous study shows that an external gravitic field $\boldsymbol{k}_0$, uniform at the scale of a galaxy, explains the flat rotation's speed. And we have seen that for $r \gg 15\,\text{kpc}$, one can only consider this uniform $\boldsymbol{k}_0$ (the internal gravitic field becomes too small). It is the domain of validity of the linearized general relativity. In electromagnetism, when an atom is embedded in a constant and uniform magnetic field $\boldsymbol{B}$, one can take for the potential vector $\boldsymbol{A}=\frac{1}{2}\boldsymbol{B}\wedge\boldsymbol{r}$ (Basdevant, 1986) [11]. So, let's take for the potential vector:

$$\boldsymbol{H}=\frac{1}{2}\boldsymbol{k}\wedge\boldsymbol{r} \tag{31}$$

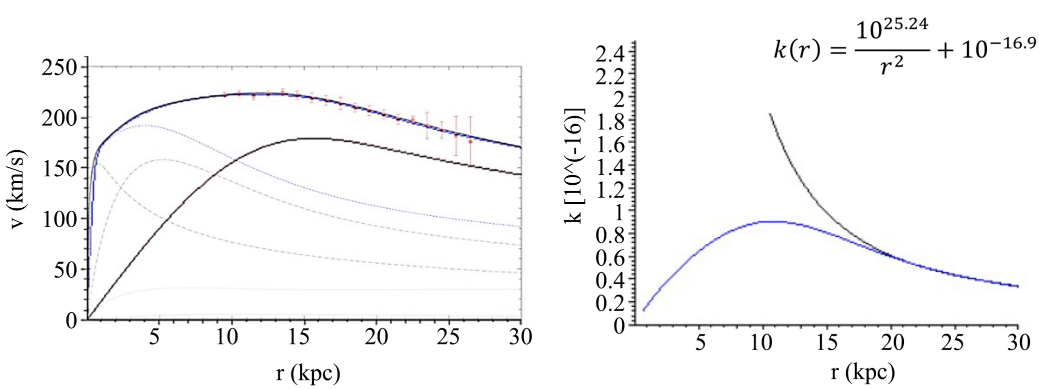


Figure 3. Left graphs are the basis experimental data from (Jiao *et al.*, 2023) [9] with observed rotation speeds (red points), best-fitting full solution $v(r)$ (blue line) and contribution of dark matter component $v_{halo}(r)$ (black line); Right graphs represent $k_{exp}(r)$ (in blue) and $k(r)=\frac{K_1}{r^2}+k_0$ (in black) explaining ends of rotation speed curves; Analytic expression of $k(r)$ is given at the top right.

One can note that this definition implies that $\boldsymbol{rot}\,\boldsymbol{H}=\boldsymbol{k}$ in agreement with Maxwell equations of linearized general relativity, as one can see it in the following calculation of our general configuration ($\boldsymbol{k}\perp\boldsymbol{r}$, $\boldsymbol{v}\perp\boldsymbol{k}$ and $\boldsymbol{v}\perp\boldsymbol{r}$): (Figure 4).

Explicitly, in the cylindrical coordinate system $(\boldsymbol{u}_r;\boldsymbol{u}_\varphi;\boldsymbol{u}_z)$ one has the external gravitic field and its potential vector:

$$\boldsymbol{k}\sim\boldsymbol{k}_0=\begin{pmatrix}0\\0\\k_0\end{pmatrix}$$

$$\boldsymbol{H}=\begin{pmatrix}H_r\\H_\varphi\\H_z\end{pmatrix}=\frac{1}{2}\boldsymbol{k}\wedge\boldsymbol{r}=\frac{1}{2}\begin{pmatrix}0\\0\\k_0\end{pmatrix}\wedge\begin{pmatrix}r\\0\\0\end{pmatrix}=\frac{1}{2}\begin{pmatrix}0\\k_0r\\0\end{pmatrix}\tag{32}$$

$$\boldsymbol{rot}\,\boldsymbol{H}=\begin{pmatrix}\frac{1}{r}\frac{\partial H_z}{\partial\varphi}-\frac{\partial H_\varphi}{\partial z}\\ \frac{\partial H_r}{\partial z}-\frac{\partial H_z}{\partial r}\\ \frac{1}{r}\left(\frac{\partial(rH_\varphi)}{\partial r}-\frac{\partial H_r}{\partial\varphi}\right)\end{pmatrix}=\frac{1}{2}\begin{pmatrix}-\frac{\partial(k_0r)}{\partial z}\\0\\ \frac{1}{r}\left(\frac{\partial(rk_0r)}{\partial r}\right)\end{pmatrix}=\begin{pmatrix}0\\0\\k_0\end{pmatrix}$$

That shows that one effectively has $\boldsymbol{rot}\,\boldsymbol{H}=\boldsymbol{k}$.

If we assume that, in the previous cylindrical coordinate system, we have a particle speed $\boldsymbol{v}=v\boldsymbol{u}_\varphi$ with v constant (one can note that it is approximately the case for the matter in the galaxy for $r\gg 15\text{ kpc}$), one has $\boldsymbol{grad}(\boldsymbol{H}\cdot\boldsymbol{v})=\frac{1}{2}k_0v\boldsymbol{u}_r$:

$$\boldsymbol{H}\cdot\boldsymbol{v}=\frac{1}{2}\begin{pmatrix}0\\k_0r\\0\end{pmatrix}\cdot\begin{pmatrix}0\\v\\0\end{pmatrix}=\frac{1}{2}k_0rv\tag{33}$$

and

$$\boldsymbol{grad}(\boldsymbol{H}\cdot\boldsymbol{v})=\begin{pmatrix}\frac{\partial}{\partial r}\\ \frac{1}{r}\frac{\partial}{\partial\varphi}\\ \frac{\partial}{\partial z}\end{pmatrix}\frac{1}{2}k_0rv=\begin{pmatrix}\frac{1}{2}k_0v\\0\\0\end{pmatrix}\tag{34}$$

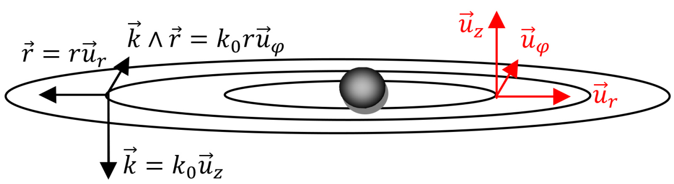


Figure 4. The potential vector $\boldsymbol{H}=\frac{1}{2}\boldsymbol{k}\wedge\boldsymbol{r}$.

Another explicit calculation gives $\boldsymbol{v} \wedge (\boldsymbol{rot}\,\boldsymbol{H}) = k_0 v \boldsymbol{u}_r$:

$$\boldsymbol{v} \wedge (\boldsymbol{rot}\,\boldsymbol{H}) = \boldsymbol{v} \wedge \boldsymbol{k} = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ k_0 \end{pmatrix} = \begin{pmatrix} vk_0 \\ 0 \\ 0 \end{pmatrix} \tag{35}$$

Finally, in this configuration, in the galaxy (for $r \gg 15\,\text{kpc}$) with $\boldsymbol{H} = \frac{1}{2}\boldsymbol{k} \wedge \boldsymbol{r}$ and $\boldsymbol{v} = v\boldsymbol{u}_\varphi$ (v and k constant), one has:

$$\boldsymbol{v} \wedge (\boldsymbol{rot}\,\boldsymbol{H}) = 2\,\boldsymbol{grad}(\boldsymbol{H} \cdot \boldsymbol{v}) \tag{36}$$

By this way, the movement equations become:

$$\frac{\mathrm{d}^2 \boldsymbol{x}}{\mathrm{d}t^2} \approx -\boldsymbol{grad}\,\varphi + 4\boldsymbol{v} \wedge (\boldsymbol{rot}\,\boldsymbol{H}) \approx -\boldsymbol{grad}\,\varphi + 8\,\boldsymbol{grad}(\boldsymbol{H} \cdot \boldsymbol{v})$$

$$\frac{\mathrm{d}^2 \boldsymbol{x}}{\mathrm{d}t^2} \approx -\boldsymbol{grad}(\varphi - 8\boldsymbol{H} \cdot \boldsymbol{v}) \tag{37}$$

In this configuration, the linearized general relativity modifies the Newtonian potential as:

$$\varphi \to \varphi - 8\boldsymbol{H} \cdot \boldsymbol{v} \tag{38}$$

And then it leads to an approximation of g_{00} containing g_{0i}:

$$g_{00} \sim 1 + 2\frac{\varphi}{c^2} - 16\frac{\boldsymbol{H} \cdot \boldsymbol{v}}{c^2} = 1 + \frac{2}{c^2}(\varphi - 8\boldsymbol{v} \cdot \boldsymbol{H}) \tag{39}$$

Remarks: This relation, valid in the configuration ($\boldsymbol{k} \perp \boldsymbol{r}$, $\boldsymbol{v} \perp \boldsymbol{k}$ and $\boldsymbol{v} \perp \boldsymbol{r}$) can be still a good approximation when the discrepancies to these angles are small.

2.6. Dark Matter and Cosmic Microwave Background (CMB)

So far, we have treated the problem of dark matter in the context of galaxies. But dark matter is also needed in the description of the CMB. We will always address this problem with linearized general relativity. Despite this approximation, we will see that once again the resulting magnitudes are surprisingly good.

Einstein's equations, with the impulse-energy tensor T_{kp} and the sign convention of (Hobson *et al.*, 2006) [2], are:

$$G_{kp} = R_{kp} - \frac{1}{2}g_{kp}R = -\frac{8\pi G}{c^4}T_{kp} \tag{40}$$

Let's write these equations in the equivalent form:

$$R_{kp} = -\frac{8\pi G}{c^4}\left(T_{kp} - \frac{1}{2}g_{kp}T\right) \tag{41}$$

In weak field and low speed ($T_{00} = \rho c^2 = T$), one can write

$$-\frac{1}{2}\Delta g_{00} = -\kappa\left(T_{00} - \frac{1}{2}g_{00}T\right) \tag{42}$$

With the traditional Newtonian approximation:

$$g_{00} = 1 + \frac{2}{c^2}\varphi \tag{43}$$

It gives:

$$\Delta\varphi = 4\pi G\rho\left(1 - \frac{2}{c^2}\varphi\right) \tag{44}$$

In this approximation ($\left|\frac{2}{c^2}\varphi\right| \ll 1$), it gives the Newtonian approximation (Hobson *et al.*, 2006) [2]:

$$\Delta\varphi = 4\pi G\rho \tag{45}$$

Now let's use the Einstein Equations (42) with our linearized general relativity approximation (39). It gives:

$$\frac{1}{c^2}\left(\Delta\varphi - 8\Delta(\boldsymbol{v}\cdot\boldsymbol{H})\right) = \frac{8\pi G}{c^4}\rho c^2\left(1 - \frac{1}{2}\left(1 + \frac{2}{c^2}(\varphi - 8\boldsymbol{v}\cdot\boldsymbol{H})\right)\right) \tag{46}$$

With the assumption of a uniform $\boldsymbol{v}$ (*i.e.* $\partial_i \boldsymbol{v} \sim 0$) and with Poisson Equation (9) ($\Delta\boldsymbol{H} = \frac{4\pi G}{c^2}\rho\boldsymbol{u}$ with $\boldsymbol{u}$ the speed of the source), (46) becomes:

$$\Delta\varphi = 4\pi G\rho\left(1 - \frac{2}{c^2}(\varphi - 8\boldsymbol{v}\cdot\boldsymbol{H})\right) + 32\pi G\rho\frac{\boldsymbol{v}\cdot\boldsymbol{u}}{c^2} \tag{47}$$

In our approximation ($\left|\frac{2}{c^2}(\varphi - 8\boldsymbol{v}\cdot\boldsymbol{H})\right| \ll 1$), it gives the linearized general relativity approximation:

$$\Delta\varphi = 4\pi G\rho + 32\pi G\rho\frac{\vec{v}.\vec{u}}{c^2} \tag{48}$$

We have then an equation in linearized general relativity approximation that can be interpreted as an idealization of the influence of visible matter ρ_b ("baryonic matter") for the first term "$4\pi G\rho$" and of the gravitic field ρ_{dm} (our dark matter explanation) for the second term "$32\pi G\rho\frac{\boldsymbol{v}\cdot\boldsymbol{u}}{c^2}$".

$$\Delta\varphi = 4\pi G\left(\rho + 8\rho\frac{\boldsymbol{v}\cdot\boldsymbol{u}}{c^2}\right) = 4\pi G\left(\rho_b + \rho_{dm}\right) \tag{49}$$

Even if it is an approximation, our idealization implies then to add a component similar to the traditional "ad hoc" dark matter term. One can then try to obtain an approximation of the Ω_{dm} term. Because of the disparities of the distribution of matter and its speed, one cannot use easily this relation to compute the equivalent dark matter quantities of gravitic field in our universe at current time. But at the time of CMB, distribution of matter can be considered homogeneous and speed of particles can be close to celerity of light $\|\boldsymbol{v}\| \sim \|\boldsymbol{u}\| \sim \alpha c$ with α a factor that must be close to 1. In this approximation, previous equation gives:

$$\Delta\varphi = 4\pi G\left(\rho + 8\rho\alpha^2\right) = 4\pi G\left(\rho_{b,CMB} + \rho_{dm,CMB}\right) \tag{50}$$

It gives the ratio of the gravitic field (equivalent to dark matter) compared to baryonic matter at the time of CMB:

$$\rho_{dm,CMB} \approx 8\alpha^2 \rho_{b,CMB} \tag{51}$$

In term of traditional Ω_b, it means that in the approximation of linearized general relativity, one has:

$$\frac{\Omega_{dm}}{\Omega_b} \approx 8\alpha^2 \tag{52}$$

The observations give (PLANCK Collaboration, 2014) [12]:

$$\frac{\Omega_{dm}}{\Omega_b} = \frac{\Omega_{dm}h^2}{\Omega_b h^2} \sim \frac{0.12}{0.022} \sim 5.45 \Rightarrow \alpha \sim 0.8 \tag{53}$$

With our approximation, this ratio can be obtained with the particles speed $\|\boldsymbol{v}\| \sim \|\boldsymbol{u}\| \sim 0.8c$, about 80% of the light celerity. The important result is not the accuracy of the value (because of our approximation) but it is the order of magnitude. And this order of magnitude is pretty good.

2.7. Dark Matter and Light Deviation (Weak Gravitational Lensing)

With this new relation (38), the traditional photon deviation expression due to gravitation $\delta_c = \frac{4\varphi}{c^2} = \frac{4GM}{c^2 r}$ can be modified by adding the term $\delta_{c\_DM} = 32\frac{\boldsymbol{v}\cdot\boldsymbol{H}}{c^2}$. It certainly gives an approximation not very accurate but it allows obtaining an order of magnitude of the curvature due to the DM compare to the baryonic matter.

For a photon, one can compute, with $v = c$:

$$\delta_{c\_DM} = 32\frac{\boldsymbol{v}\cdot\boldsymbol{H}}{c^2} \sim 32\frac{\|\boldsymbol{H}\|}{c} \tag{54}$$

As said before, one can take (31) for the potential vector. The photon deviation expression is then: (**Figure 5**)

$$\delta_{c\_DM} \sim 32\frac{\|\boldsymbol{H}\|}{c} \sim 16\frac{kr}{c} \tag{55}$$

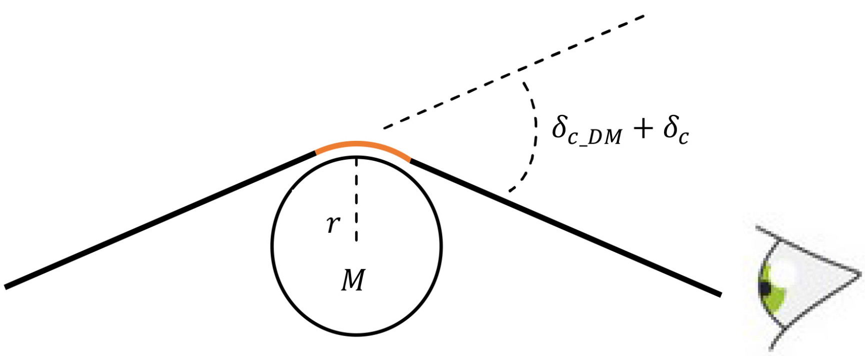


Figure 5. Light deviation due to a mass M of radius r.

To have an order of magnitude of the deviation, we can assume that the photons we receive have either been deflected at the end of the deviating structure or closer to the heart of the structure. Let's apply this relation to our previous studied galaxies. Roughly, at the end of these galaxies, at about $r \sim 20\,\text{kpc} \sim 60 \times 10^{19}\,\text{m}$, one has, for nearly all the galaxies, the following value of the gravitic field $k_0 \sim 10^{-16.5}$. It gives the order of magnitude of the correction due to the external gravitic field:

$$\delta_{c\_DM} \propto 16 \frac{10^{-16.5} \times 60 \times 10^{19}}{3 \times 10^{8}} \propto 6.3 \times 10^{-5} \quad (56)$$

And the curvature (due to gravity field) is about (for a typical mass of 10^{41} kg):

$$\delta_c \propto 4 \frac{7 \times 10^{-11} \times 10^{41}}{9 \times 10^{16} \times 60 \times 10^{19}} \propto 0.05 \times 10^{-5} \quad (57)$$

It means that δ_{c_DM} represents about 99.2% of the deviation ($\delta_{c\_DM} / \left(\delta_{c\_DM} + \delta_c\right)$). And closer to the heart of the galaxy, for example, at about $r \sim 2\,\text{kpc} \sim 6 \times 10^{19}\,\text{m}$, one has $\delta_{c\_DM} \propto 6.3 \times 10^{-6}$ and $\delta_c \propto 0.05 \times 10^{-4}$ meaning that δ_{c_DM} still represents about 56% of the deviation.

In our solution, the external gravitic field generates nearly all the curvature of the galaxies in agreement with experimental data (Neyman *et al.*, 1961) [13] which gives at least 90%. The previous calculated value $k_0 \sim 10^{-16.5}$ is enough to explain these deviations.

It is instructive to do this same calculation near the Sun to verify that the deviation on this local scale is extremely small compared to the Newtonian term. With a distance equal to the radius of the Sun $r \sim 7 \times 10^{8}\,\text{m}$ and the mass of the Sun $M \sim 2 \times 10^{30}\,\text{kg}$, one has $\delta_c \propto 4 \frac{7 \times 10^{-11} \times 2 \times 10^{30}}{9 \times 10^{16} \times 7 \times 10^{8}} \propto 0.89 \times 10^{-5}$ and $\delta_{c\_DM} \propto 16 \frac{10^{-16.5} \times 7 \times 10^{8}}{3 \times 10^{8}} \propto 37.3 \times 10^{-16.5} \ll \delta_c$. This explains that the DM at our scale is undetectable. And furthermore, it demonstrates that our solution is extremely consistent with the observations because in our explanation DM must be invisible on our scale but dominant on the scale of the large structures of the universe.

3. Discussion

We have seen that the own gravitic field of the galaxy $\boldsymbol{k}_{gal}$ is too weak to explain $\boldsymbol{k}(r)$ at the end of the galaxies. One can then expect that the gravitic field k_0 can only be generated by a structure larger than a galaxy hence the qualifier "external" for k_0. Furthermore, to obtain a field that maintains itself on large distance (giving a relatively uniform gravitic field), it must be generated not only from one object but also from its neighboring objects, just like the assemblies of spins in ferromagnetic material. In (Le Corre, 2015) [14] it is shown that the galaxies cluster with its nearest neighbors could be an excellent candidate to explain k_0. It is important to see that this capacity of mutualization of the gravitation field is only possible for this 2nd component of gravitation, the gravitic field, because of its mathematical expression like the magnetic field in EM. This capacity is impossible with the Newtonian field, like the electric field in EM. A prediction is that the map of the quantities of DM should reveal greater quantities at the centers of the galaxies' clusters than elsewhere, and perhaps around the brightest cluster galaxy (Le Corre, 2023) [10].

In this solution, with time, objects with velocity not perpendicular to $\boldsymbol{k}$ should escape from the galaxy because $\boldsymbol{F}_k = m_p 4(\boldsymbol{v} \wedge \boldsymbol{k})$ and then one should roughly tend to the equilibrium state $\boldsymbol{v} \perp \boldsymbol{k}$. This statistical tendency leads to several predictions. The motion of dwarf satellite galaxies of a host should be roughly in a plane ($\perp \boldsymbol{k}$) because they also undergo $\boldsymbol{k}_0$ which is relatively uniform at the scale of the galaxy (Le Corre, 2015) [15]. Such a behavior has been observed (Phillips *et al.* 2015) [16]. It also can explain the galactic disk warp of some galaxies (Le Corre, 2015 [14]; Le Corre, 2022 [17]) because at the center of the galaxy, the gravitic field is dominated by $\boldsymbol{k}_{gal}$ and at the ends of the galaxy, it is dominated by $\boldsymbol{k}_0$. If there is a notable difference in angle between $\boldsymbol{k}_0$ and $\boldsymbol{k}_{gal}$, the equilibrium states around the center and around the ends should imply a warping. It is important to note that most of the predictions about the influence of $\boldsymbol{k}_0$ should be taken in account in statistic terms because the value of $\|\boldsymbol{k}_0\|$ is very weak compare to the Newtonian component. If two galaxies are under their mutual gravitational influence, the effects of the weak $\|\boldsymbol{k}_0\|$ can be erased by the first component of gravitation, the Newtonian one.

In fact, we can go further than predictions. The expected values of the gravitic field (30), required to account for DM, allow retrieving the Tully-Fisher relation (Le Corre, 2023) [18] and obtaining MOND-based theories (as a specific solution of LGR with a uniform external gravitic field), and in particular by retrieving the MOND parameters $a_0 \sim 10^{-8}\,\text{cm}\cdot\text{s}^{-2}$ and $G' \sim 1.37G$ (Le Corre, 2023) [19]. It is very unlikely that these two demonstrations are due to hazard. For Tully-Fisher law and MOND-based theories, it is clearly both the idealization of LGR (with a uniform gravitic field $\boldsymbol{k}_0$) and the effective values of k_0 (explaining the quantities of DM) that allows obtaining these demonstrations. They are then an incredibly great positive point which validate this solution for explaining the DM component. Another remarkable result with the value of k_0 is that it allows obtaining the ratio of the density of the gaseous intergalactic medium and interstellar gaseous medium (Le Corre, 2022) [17].

It is interesting to compare this hypothesis HP to that of an exotic matter. We just showed that such modeling leads to extremely low values of the field $\boldsymbol{k}_0$ ($10^{-16.62}\,\text{s}^{-1} < \|\boldsymbol{k}_0\| < 10^{-16.3}\,\text{s}^{-1}$). This point is essential because it justifies the fact that we do not detect this field at our scale (explaining then the undetectability of DM). Its effects on the velocity ($v_{halo}(r) = 2(rk(r)v(r))^{1/2}$ as already seen) cannot be felt only at a large scale (because the term depends on "*r*") and for significant speeds (like that at the ends of galaxies). In the context of the hypothesis of an exotic matter, this point is very difficult on the one hand to explain because the quantity of exotic matter must be enormous (dominant compared to ordinary matter), which constitutes the first point which is extremely difficult to justify without calling into question the theory itself and on the other hand to maintain the coherence of the theory, because it implies an exotic behavior of this matter, in particular insensitive to EM. In our explanation, the insensitivity to EM is natural because it is simply the gravitational field (the 2nd component). If we also compare to MOND-based theories, our explanation does not require any modification of the theory and even the values of $\|\boldsymbol{k}_0\|$ justify the use of LGR. Furthermore, MOND-based theories finally appear as a particular solution of LGR with a uniform gravitic field $\boldsymbol{k}_0$.

The presence of DM has also been demonstrated in the Cosmic Microwave Background (CMB). In our explanation, the current presence of the $\boldsymbol{k}_0$ field could be explained by the energy conservation (similar to that of angular momentum). This distribution of $\boldsymbol{k}_0$ would be a decomposition of a more global (and perhaps even more homogeneous) pre-existing field. And so, going back in time, we would find this component of DM in the CMB.

The explanation of DM as a uniform gravitic field embedding the galaxies is therefore extremely consistent with reality and the least expensive in theoretical terms. Of course, this does not mean that it is the right solution but, according to the principle of Occam's razor, this explanation of the DM deserves a more honorable status.

4. Conclusions

In this study, we explain how an external uniform gravitic field $\boldsymbol{k}_0$ (2nd component of GR giving the Lense-Thirring effect) can solve the DM mystery. This explanation consists in the following steps. At the end of the galaxies, the gravitational field is low enough to justify the use of the LGR. In LGR, the movement equations are: $m_i \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = m_p\left[\boldsymbol{g} + 4\boldsymbol{v}\wedge\boldsymbol{k}\right]$ with the term $4\boldsymbol{v}\wedge\boldsymbol{k}$ which corresponds to the 2nd component of GR giving the Lense-Thirring effect in its linear form. This term $\boldsymbol{k}$ is equivalent to the magnetic field $\boldsymbol{B}$ in EM. Just like in EM with the mutualization of the atomic spins giving a magnetization for a magnet thanks to its mathematical expression, one can have a uniform gravitic field $\boldsymbol{k}_0$ (likely generated by the mutualization of the $\boldsymbol{k}$ of the clusters of galaxies). This term $\boldsymbol{k}_0$ would embed the galaxies. Mathematically such a solution is naturally possible (in the same way that a magnetic field exists in EM like in an accelerator of particles maintaining high speed particles on a steady orbit, higher than without this magnetic field). We also demonstrate that this solution is physically possible. Indeed, to explain the flat rotation speed of the galaxies, the physical values of this field must be around the interval $10^{-16.62}\,\mathrm{s}^{-1} < \|\boldsymbol{k}_0\| < 10^{-16.3}\,\mathrm{s}^{-1}$.

These values of $\|\boldsymbol{k}_0\|$ with the LGR idealization allows demonstrating the Tully-Fisher relation (Le Corre, 2023) [18]. In the same way, MOND-based theories can be obtained giving back the value of the MOND parameter $a_0 \sim 10^{-8}\,\mathrm{cm}\cdot\mathrm{s}^{-2}$ and deep MOND and AQUAL parameter $G' \sim 1.37G$ (Le

Corre, 2023) [19]. One can retrieve the density of the gaseous intergalactic medium and interstellar gaseous medium in the case of WLM galaxy and explain the warp of the galactic disk (Le Corre, 2022) [17]. This solution also allows obtaining movements of satellite dwarf galaxies in a plane perpendicular to the vector $\boldsymbol{k}_0$ (Le Corre, 2015) [15] as has already been observed (Phillips *et al.* 2015) [16].

One can note that this explanation of DM is self-consistent because the values of $\boldsymbol{k}_0$ to account for rotational speed of galaxies, are naturally very weak to justify the use of LGR. The mathematical expression of DM ($v_{halo}(r) = 2\left(rk(r)v(r)\right)^{1/2}$) in this solution explains the undetectability of DM at our scale and the detectability of DM for astrophysical structures greater than galaxies for large "*r*" and large "*v*". This solution explains too that DM term is unsensitive to EM because it is a component of the gravitation field. This explanation is completely compliant with GR and even more it is a solution exclusively possible inside the GR framework. All these facts are to be compared to the assumption of the existence of an exotic matter (with its exotic behavior and with its monstrous expected quantity) or to the MOND-based theories (with a questioning of the very well verified theoretical foundations). Furthermore, it has been demonstrated that the MOND-based theories can be seen as a particular solution of LGR and $\boldsymbol{k}_0$. This solution seems very more natural physically and theoretically than exotic matter or MOND.

Data Availability

There are no new data associated with this article.

Conflicts of Interest

The author declares no conflicts of interest.

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