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MOND: An Approximation of a Particular Solution of Linearized General Relativity

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Abstract

It has been demonstrated that dark matter (DM) can theoretically be completely explained by a natural effect of General Relativity (GR) without exotic matter, the Lense-Thirring effect that exists exclusively in GR and that would be due to the clusters of galaxies. In this study, we show that this explanation of DM leads to a modelization that can be interpreted as MOND-based theories. More concretely, we retrieve from GR the value of MOND parameter $a_0 \sim 10^{-8}\,\mathrm{cm}\cdot\mathrm{s}^{-2}$ and deep MOND and AQUAL parameters $G' \sim 1.37G$. It means that MOND-based theories could be interpreted as an approximation of the linearized GR (*i.e.* GR in a weak gravitational field or small acceleration) in a particular physical case of a uniform gravitic field (2nd component of GR in its linearized form, similar to magnetic field of Electromagnetism). A publication has recently observed deviations from Newtonian acceleration with a 10σ significance for wide binary stars at weak gravitational acceleration. The author demonstrates that these deviations can be explained by MOND theory with the previous parameters' values. This situation leads to a difficulty. On one hand, the traditional DM hypothesis can't explain these deviations and on the other hand, empirical MOND theories are difficult to justify compared to the success of GR. With our result, no more difficulty, these deviations do not need to be explained by MOND theory but by linearized GR with the uniform gravitic field explaining the DM component (Lense-Thirring effect of the clusters of galaxies).

Subject Areas

Cosmology, Astrophysics, Theoretical Physics

Keywords

Dark Matter, MOND, Gravitation

1. Introduction

The component of Dark Matter (DM) is required in the frame of General Relativity (GR) to explain the rotational speeds of the galaxies, the curvature of the light due to the cluster of galaxies, the fluctuations of the CMB and so on. The most popular explanation is to assume the existence of an unknown exotic matter. This hypothesis seems nevertheless problematic because such a matter would follow very strange behavior, insensitive to Electromagnetism (EM) and only sensitive to gravitational effects. Furthermore, this unknown matter would dominate ordinary matter at a large scale while it has never been directly observed to date. An alternative solution (without exotic DM) has been proposed by Milgrom [1] [2], named Modified Newtonian Dynamics (MOND) consisting in modifying the Newtonian dynamics in the limit of small acceleration. Another solution without exotic matter has also been proposed [3], the galaxy clusters [4] would generate a gravitic field (the 2nd component of GR similar to the magnetic field of EM at the origin of the Lense-Thirring effect experimentally confirmed) that would embed large areas of the Universe (and then the galaxies) explaining this excess of gravitation.

In this study, we will show that MOND-based theories can be in fact obtained as an approximation of this second alternative explanation of DM. *i.e.* Linearized GR (LGR) with a uniform gravitic field, 2nd component of GR. First, we remind how LGR is obtained from GR, how LGR equations can explain DM and the expected values of the uniform gravitic field required to explain DM component. Second, we show how LGR, in the context of DM explanation, *i.e.* with a uniform gravitic field, can give a large family of MOND-based theories. In particular, one retrieves the value of MOND parameter $a_0 \sim 10^{-8}\,\mathrm{cm}\cdot\mathrm{s}^{-2}$ and the deep MOND and AQUAL parameter $G' \sim 1.37G$.

2. Dark Matter Explained by General Relativity

2.1. From General Relativity to Linearized General Relativity

From GR, one deduces the LGR in the approximation of a quasi-flat Minkowski space ($g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$; $\left|h^{\mu\nu}\right| \ll 1$). With the following Lorentz gauge, it gives the following field equations as in [5] (with $\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta$ and $\Delta = \nabla^2$):

$$\partial_\mu \bar{h}^{\mu\nu} = 0;\ \Box \bar{h}^{\mu\nu} = -2\frac{8\pi G}{c^4}T^{\mu\nu} \tag{1}$$

With:

$$\bar{h}^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}h;\ h \equiv h_\sigma^\sigma;\ h_\nu^\mu = \eta^{\mu\sigma}h_{\sigma\nu};\ \bar{h} = -h \tag{2}$$

The general solution of these equations is:

$$\bar{h}^{\mu\nu}(ct, \boldsymbol{x}) = -\frac{4G}{c^4}\int \frac{T^{\mu\nu}\left(ct - |\boldsymbol{x}-\boldsymbol{y}|, \boldsymbol{y}\right)}{|\boldsymbol{x}-\boldsymbol{y}|}\mathrm{d}^3\boldsymbol{y} \tag{3}$$

In the approximation of a source with low speed, one has:

$$T^{00} = \rho c^2;\ T^{0i} = c\rho u^i;\ T^{ij} = \rho u^i u^j \tag{4}$$

For a stationary solution, one has:

$$\bar{h}^{\mu\nu}(\boldsymbol{x}) = -\frac{4G}{c^4}\int \frac{T^{\mu\nu}(\boldsymbol{y})}{|\boldsymbol{x}-\boldsymbol{y}|}\mathrm{d}^3\boldsymbol{y} \tag{5}$$

At this step, by proximity with electromagnetism, one traditionally defines a scalar potential φ and a vector potential H^i. There are in the literature several definitions as in [6] for the vector potential H^i. In our study, we are going to define:

$$\bar{h}^{00} = \frac{4\varphi}{c^2};\ \bar{h}^{0i} = \frac{4H^i}{c};\ \bar{h}^{ij} = 0 \tag{6}$$

With gravitational scalar potential φ and gravitational vector potential H^i:

$$\varphi(\boldsymbol{x}) \equiv -G\int \frac{\rho(\boldsymbol{y})}{|\boldsymbol{x}-\boldsymbol{y}|}\mathrm{d}^3\boldsymbol{y}$$

$$H^i(\boldsymbol{x}) \equiv -\frac{G}{c^2}\int \frac{\rho(\boldsymbol{y})u^i(\boldsymbol{y})}{|\boldsymbol{x}-\boldsymbol{y}|}\mathrm{d}^3\boldsymbol{y} = -K^{-1}\int \frac{\rho(\boldsymbol{y})u^i(\boldsymbol{y})}{|\boldsymbol{x}-\boldsymbol{y}|}\mathrm{d}^3\boldsymbol{y} \tag{7}$$

With K (determined in [3]) a new constant defined by

$$GK = c^2 \tag{8}$$

This definition gives $K^{-1} \sim 7.4\times 10^{-28}\ \mathrm{kg}\cdot\mathrm{m}^{-1}$ very small compared to G.

The field Equations (1) can be then written (Poisson equations):

$$\Delta\varphi = 4\pi G\rho;\ \Delta H^i = \frac{4\pi G}{c^2}\rho u^i = 4\pi K^{-1}\rho u^i \tag{9}$$

With the following definitions of $\boldsymbol{g}$ (gravity field) and $\boldsymbol{k}$ (gravitic field), those relations can be obtained from the following equations (also called gravitomagnetism) with the differential operators "$\boldsymbol{rot} = \nabla\wedge$", "$\boldsymbol{grad} = \nabla$" and "$div = \nabla\cdot$":

$$\begin{gathered}\boldsymbol{g} = -\boldsymbol{grad}\,\varphi;\ \boldsymbol{k} = \boldsymbol{rot}\,\boldsymbol{H}\\ \boldsymbol{rot}\,\boldsymbol{g} = 0;\ div\,\boldsymbol{k} = 0;\\ \boldsymbol{div}\,\boldsymbol{g} = -4\pi G\rho;\ \boldsymbol{rot}\,\boldsymbol{k} = -4\pi K^{-1}\boldsymbol{j}_p\end{gathered} \tag{10}$$

With the Equations (2), one has:

$$h^{00} = h^{11} = h^{22} = h^{33} = \frac{2\varphi}{c^2};\ h^{0i} = \frac{4H^i}{c};\ h^{ij} = 0 \tag{11}$$

The equations of geodesics in the linear approximation give:

$$\frac{\mathrm{d}^2x^i}{\mathrm{d}t^2} \sim -\frac{1}{2}c^2\delta^{ij}\partial_j h_{00} - c\delta^{ik}\left(\partial_k h_{0j} - \partial_j h_{0k}\right)v^j \tag{12}$$

It then leads to the movement equations:

$$\frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2} \sim -\boldsymbol{grad}\,\varphi + 4\boldsymbol{v}\wedge(\boldsymbol{rot}\,\boldsymbol{H}) = \boldsymbol{g} + 4\boldsymbol{v}\wedge\boldsymbol{k} \tag{13}$$

Remark: All previous relations can be retrieved starting with the parameterized post-Newtonian (PPN) formalism and with the traditional gravitomagnetic field $\boldsymbol{B}_g$. From [7] one has:

$$g_{0i} = -\frac{1}{2}(4\gamma + 4 + \alpha_1)V_i;\ V_i(\boldsymbol{x}) = \frac{G}{c^2}\int \frac{\rho(\boldsymbol{y})u_i(\boldsymbol{y})}{|\boldsymbol{x}-\boldsymbol{y}|}\mathrm{d}^3\boldsymbol{y} \tag{14}$$

The traditional gravitomagnetic field and its acceleration contribution are:

$$\boldsymbol{B}_g = \nabla \wedge \left(g_{0i}\boldsymbol{e}^i\right);\ \boldsymbol{a}_g = \boldsymbol{v} \wedge \boldsymbol{B}_g \tag{15}$$

And in the case of GR(that is our case):

$$\gamma = 1;\ \alpha_1 = 0 \tag{16}$$

It then gives:

$$g_{0i} = -4V_i;\ \boldsymbol{B}_g = \nabla \wedge \left(-4V_i\boldsymbol{e}^i\right) \tag{17}$$

And with our definition:

$$H_i = -\delta_{ij}H^j = \frac{G}{c^2}\int \frac{\rho(\boldsymbol{y})\delta_{ij}u^j(\boldsymbol{y})}{|\boldsymbol{x}-\boldsymbol{y}|}\mathrm{d}^3\boldsymbol{y} = V_i(\boldsymbol{x}) \tag{18}$$

One then has:

$$g_{0i} = -4H_i;\ \boldsymbol{B}_g = \nabla \wedge \left(-4H_i\boldsymbol{e}^i\right) = \nabla \wedge \left(4\delta_{ij}H^j\boldsymbol{e}^i\right) = 4\nabla \wedge \boldsymbol{H} \tag{19}$$

$$\boldsymbol{B}_g = 4\,\boldsymbol{rot}\,\boldsymbol{H}$$

With the following definition of gravitic field:

$$\boldsymbol{k} = \frac{\boldsymbol{B}_g}{4} \tag{20}$$

One then retrieves our previous relations:

$$\boldsymbol{k} = \boldsymbol{rot}\,\boldsymbol{H};\ \boldsymbol{a}_g = \boldsymbol{v} \wedge \boldsymbol{B}_g = 4\boldsymbol{v} \wedge \boldsymbol{k} \tag{21}$$

The interest of our notation ($\boldsymbol{k}$ instead of $\boldsymbol{B}_g$) is that the field equations are strictly equivalent to Maxwell's idealization, in particular, the speed of the gravitational wave obtained from these equations is the light celerity $c^2 = GK$ just like in EM $c^2 = 1/\mu_0\varepsilon_0$. Only the movement equations are different with the factor "4". But of course, all the results of our study can be obtained in the traditional notation of gravitomagnetism with the relation $\boldsymbol{k} = \frac{\boldsymbol{B}_g}{4}$.

2.2. From Linearized General Relativity to DM

In the classical approximation ($\|\boldsymbol{v}\| \ll c$), the linearized general relativity gives the following movement equations from (13) with m_i the inertial mass and m_p the gravitational mass:

$$m_i\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = m_p\left[\boldsymbol{g} + 4\boldsymbol{v} \wedge \boldsymbol{k}\right] \tag{22}$$

The traditional computation of rotation speeds of galaxies consists of obtaining the force equilibrium from the three following components: the disk, the bulge and the halo of dark matter. More precisely, one has [8]:

$$\frac{v^2(r)}{r}=\left(\frac{\partial\varphi(r)}{\partial r}\right)\text{ with }\varphi=\varphi_{disk}+\varphi_{bulge}+\varphi_{halo} \tag{23}$$

Then the total speed squared can be written as the sum of squares of each of the three speed components:

$$\begin{aligned}v^2(r)&=r\left(\frac{\partial\varphi_{disk}(r)}{\partial r}\right)+r\left(\frac{\partial\varphi_{bulge}(r)}{\partial r}\right)+r\left(\frac{\partial\varphi_{halo}(r)}{\partial r}\right)\\&=v_{disk}^2(r)+v_{bulge}^2(r)+v_{halo}^2(r)\end{aligned} \tag{24}$$

Disk and bulge components are obtained from gravity field. They are not modified in this solution. So, the goal is now to obtain only the traditional DM halo component from the LGR. According to this idealization, the force due to the gravitic field $\boldsymbol{k}$ takes the following form $\|\boldsymbol{F}_k\|=m_p4\|\boldsymbol{v}\wedge\boldsymbol{k}\|$ and it corresponds to previous term $m_p\frac{\partial\varphi_{halo}(r)}{\partial r}=\|\boldsymbol{F}_k\|$. As explained in [3], the the natural evolution to the equilibrium state justifies that one assumes the approximation $\boldsymbol{v}\perp\boldsymbol{k}$. This assumption is important because it leads to several important predictions. In particular, the motion of dwarf satellite galaxies of a host should be roughly in a plane ($\perp\boldsymbol{k}$). It also can explain the galactic disk warp of some galaxies. It then gives the following equation:

$$\begin{aligned}\frac{v^2(r)}{r}&=\left(\frac{\partial\varphi_{disk}(r)}{\partial r}\right)+\left(\frac{\partial\varphi_{bulge}(r)}{\partial r}\right)+4k(r)v(r)\\&=\frac{v_{disk}^2(r)}{r}+\frac{v_{bulge}^2(r)}{r}+4k(r)v(r)\end{aligned} \tag{25}$$

Our idealization means that:

$$v_{halo}^2(r)=v^2(r)-v_{disk}^2(r)-v_{bulge}^2(r)=4rk(r)v(r) \tag{26}$$

The equation of dark matter (gravitic field in our explanation) is then:

$$v_{halo}(r)=2\left(rk(r)v(r)\right)^{1/2} \tag{27}$$

This equation gives us the curve of rotation speeds of the galaxies for the DM component. Because we know the curves of speeds that one wishes to have for DM component, one can then deduce the curve of the gravitic field $\boldsymbol{k}$ inside the galaxy:

$$k(r)=\frac{v_{halo}^2(r)}{4rv(r)} \tag{28}$$

2.3. Dark Matter as the 2nd Component of the Gravitational Field $\boldsymbol{k}$

This solution of DM as the gravitic field has been studied in [3] for 16 galaxies (**Table 1**). It shows that this solution is mathematically possible but with two physical mandatory unexpected behaviors $k(r)$. First, the curve of the gravitic field $k(r)$ becomes necessarily flat at the end of the galaxies. For such a field

Table 1. Distance r_0 to the center of the galaxy where the internal gravitic field $\frac{K_1}{r^2}$ generated by the galaxy becomes equivalent to the external gravitic field k_0 generated by the galaxies' cluster. k_0 dominates for $r > r_0$.

	K_1	k_0	$r_0\left[\frac{K_1}{r^2} \sim k_0\right]$	$r_0\,[\text{kpc}]$
NGC 5055	$10^{24.60}$	$10^{-16.62}$	$10^{20.61}$	13
NGC 4258	$10^{24.85}$	$10^{-16.54}$	$10^{20.695}$	16
NGC 5033	$10^{24.76}$	$10^{-16.54}$	$10^{20.65}$	15
NGC 2841	$10^{24.85}$	$10^{-16.33}$	$10^{20.59}$	13
NGC 3198	$10^{24.90}$	$10^{-16.55}$	$10^{20.725}$	18
NGC 7331	$10^{24.18}$	$10^{-16.30}$	$10^{20.24}$	6
NGC 2903	$10^{24.71}$	$10^{-16.30}$	$10^{20.505}$	11
NGC 3031	$10^{24.15}$	$10^{-16.57}$	$10^{20.36}$	8
NGC 2403	$10^{24.59}$	$10^{-16.39}$	$10^{20.49}$	10
NGC 247	$10^{24.30}$	$10^{-16.30}$	$10^{20.3}$	7
NGC 4236	$10^{24.00}$	$10^{-16.34}$	$10^{20.17}$	5
NGC 4736	$10^{24.54}$	$10^{-16.30}$	$10^{20.42}$	9
NGC 300	$10^{24.27}$	$10^{-16.31}$	$10^{20.29}$	6
NGC 2259	$10^{24.20}$	$10^{-16.30}$	$10^{20.25}$	6
NGC 3109	$10^{24.00}$	$10^{-16.58}$	$10^{20.29}$	6
NGC 224	$10^{24.00}$	$10^{-16.50}$	$10^{20.25}$	6

(similar mathematically to a magnetic field in EM) it is only possible if the galaxies are immersed in a uniform gravitic field $\boldsymbol{k}_0$. Second, the value of this field for these 16 galaxies is in the interval:

$$10^{-16.62}\,\text{s}^{-1} < \left\|\boldsymbol{k}_0\right\| < 10^{-16.3}\,\text{s}^{-1} \tag{29}$$

3. MOND Obtained by Linearized General Relativity

3.1. From Uniform Gravitic Field $\boldsymbol{k}_0$ of LGR to MOND-Based Theories

We are going to show that MOND-based theories finally correspond to an approximation of this dark matter solution. Let's remember that this solution of dark matter in the form of a $\boldsymbol{k}$ field corresponds to a particular solution of the linearization of general relativity for which it is assumed that neighboring clusters of galaxies generate a uniform $\boldsymbol{k}_0$ field on a large scale like the magnetic spins of atoms generate a uniform magnetic field across a ferromagnetic material (magnetization).

Let's note v_N the Newtonian rotational speed (the bulge and disk components), (25) can be written:

$$\frac{v^2(r)}{r} = \frac{v_N^2(r)}{r} + \frac{v_{halo}^2(r)}{r} \tag{30}$$

And more explicitly:

$$\frac{v^2(r)}{r} = \frac{GM}{r^2} + 4k(r)v(r) \tag{31}$$

Which gives:

$$\frac{v^2(r)}{r} = \frac{GM}{r^2}\left(1 + \frac{4k(r)v(r)r^2}{GM}\right) \tag{32}$$

This relation can be interpreted as a modification of the Newtonian dynamics. LGR finally gives a family of MOND-based theories depending on the values of the uniform gravitic field $k(r) = \|\boldsymbol{k}_0\|$.

3.2. From Uniform Gravitic Field k_0 of LGR to Value of MOND Parameter a_0

MOND at large radii [2] obtains:

$$v^4(r) \sim GMa_0 \tag{33}$$

With $\frac{a^2}{a_0} \sim \frac{GM}{r^2}$ and $a = \frac{v^2(r)}{r}$

In LGR, (32) gives:

$$v^4(r) = GM\left[\frac{GM}{r^2}\left(1 + \frac{4k(r)v(r)r^2}{GM}\right)^2\right] \tag{34}$$

At large radii, the MOND parameter a_0 can then be written in LGR:

$$a_0 \sim \frac{GM}{r^2}\left(1 + \frac{4k(r)v(r)r^2}{GM}\right)^2 \tag{35}$$

For the end of our Galaxy, one has:

$$M_{Gal} = 10^{42}\ \text{kg};\ R_{Gal} = 40\ \text{kpc} = 10^{21}\ \text{m};\ v_{Gal} = 240\ \text{km}\cdot\text{s}^{-1} = 2.4\times10^5\ \text{m}\cdot\text{s}^{-1} \tag{36}$$

From [3], the uniform gravitic field $k(r) = \|\boldsymbol{k}_0\|$, embedding the galaxies for a sample of 16 galaxies, is in the interval (29). Let's compute a_0 for the 2 extremities of this interval.

For $\|\boldsymbol{k}_0\| = 10^{-16.3}\ \text{s}^{-1}$:

$$a_0 \sim \frac{GM_{Gal}}{R_{Gal}^2}\left(1 + \frac{4k_0 v_{Gal} R_{Gal}^2}{GM_{Gal}}\right)^2$$

$$= \frac{6\times10^{-11}\times10^{42}}{10^{42}}\left(1 + \frac{4\times10^{-16.3}\times2.4\times10^5\times10^{42}}{6\times10^{-11}\times10^{42}}\right)^2$$

$$a_0 \sim 6\times10^{-11}\left(1 + 1.6\times10^{-0.3}\right)^2$$

$$a_0 \sim 2\times10^{-10}\ \text{m}\cdot\text{s}^{-2} \sim 2\times10^{-8}\ \text{cm}\cdot\text{s}^{-2} \tag{37}$$

For $\|\boldsymbol{k}_0\| = 10^{-16.62}\,\mathrm{s}^{-1}$:

$$a_0 \sim 6\times10^{-11}\left(1+1.6\times10^{-0.62}\right)^2$$
$$a_0 \sim 10^{-10}\,\mathrm{m}\cdot\mathrm{s}^{-2} \sim 10^{-8}\,\mathrm{cm}\cdot\mathrm{s}^{-2} \tag{38}$$

For the sample of 16 galaxies, one then has:

$$10^{-16.62}\,\mathrm{s}^{-1} < \|\boldsymbol{k}_0\| < 10^{-16.3}\,\mathrm{s}^{-1} \Leftrightarrow 10^{-8}\,\mathrm{cm}\cdot\mathrm{s}^{-2} < a_0 < 2\times10^{-8}\,\mathrm{cm}\cdot\mathrm{s}^{-2} \tag{39}$$

One obtains the expected values of $a_0 \sim 10^{-8}\,\mathrm{cm}\cdot\mathrm{s}^{-2}$ mentioned in [2]. The explanation of DM with a uniform gravitic field k_0 (without exotic matter and compliant with GR) allows them to obtain the results of the MOND theory.

3.3. From Uniform Gravitic Field k_0 of LGR to Value of Deep MOND and AQUAL Parameter *G′*

In [9], the observations of wide binary stars are well modelized with the deep MOND and AQUAL parameters:

$$1.33 < \frac{g_{obs}}{g_{pred}} < 1.43 \tag{40}$$

With

$$g_{pred} = \frac{GM}{r^2};\ g_{obs} = \frac{v^2(r)}{r} \sim \frac{G'M}{r^2};\ \frac{g_{obs}}{g_{pred}} \sim \frac{G'}{G} \tag{41}$$

This result agrees with the equivalent correction of the gravitational constant $G' = 1.37G$ [9] predicted by MOND and AQUAL at the position of the Sun. Let's verify that this MOND correction corresponds to the predicted values k_0.

From (32), one can write:

$$\frac{g_{obs}}{g_{pred}} \sim \frac{G'}{G} = 1 + \frac{4k(r)v(r)r^2}{GM} \tag{42}$$

At the position of the Sun, one should have:

$$\frac{4k_0 v_{Sun} R_0^2}{GM_{Gal/Sol}} \sim 0.37 \tag{43}$$

With [9]:

$$R_0 = 8.2\ \mathrm{kpc} = 2.5\times10^{20}\ \mathrm{m};\ v_{Sun} = 232.8\ \mathrm{km}\cdot\mathrm{s}^{-1} = 2.3\times10^{5}\ \mathrm{m}\cdot\mathrm{s}^{-1} \tag{44}$$

From (41), one can write:

$$\frac{v_{Sun}^2}{R_0} \sim \frac{G'M_{Gal/Sol}}{R_0^2} \Rightarrow M_{Gal/Sol} \sim \frac{R_0 v_{Sun}^2}{G'} \tag{45}$$

(43) can be written (with $G' = 1.37G$):

$$\frac{4k_0 v_{Sun} R_0^2}{R_0 v_{Sun}^2} \sim \frac{0.37}{1.37} \Rightarrow k_0 \sim \frac{0.37}{1.37}\frac{v_{Sun}}{4R_0} = 10^{-16.2} \tag{46}$$

This value is a very good approximation of the expected value k_0. Inversely, the explanation of DM $\boldsymbol{k}_0$ allows retrieving the results of deep MOND and

AQUAL.

4. Discussion

The astrophysical observations imply the existence of a new component called dark matter, the interpretation of which remains to be defined. Without exotic matter, on the one hand, this component of dark matter is explainable within the framework of the LGR in the particular case of the existence of a uniform gravitic field (resulting from the clusters of galaxies) which would embed the galaxies. This explanation avoids the addition of exotic material and is compliant with GR. On the other hand, this component can also be explained by a modification of the Newtonian theory of gravitation (MOND). But this approach is more difficult to justify because it affects the theoretical foundations of gravitation which are so far very well established. Our study sheds new light on MOND-based theories. They would thus correspond to a double approximation, first to the reduction of the GR to the weak field (LGR), and second to a particular case which is the existence of a uniform gravitational field embedding the galaxies which is only possible for a specific configuration of galaxy clusters [3].

The results observed in [9] reveal deviations from Newtonian acceleration with 10σ significance. This leads to a remarkable point. As mentioned in [9], these wide binary star deviations cannot be explained by the DM hypothesis. We are then left with 2 possible paths. Either we need the 2 hypotheses, DM on a large scale and MOND on the scale of small accelerations, or MOND would be sufficient to also explain the DM. These 2 ways are actually problematic. In the 1st case, we now find ourselves with 2 hypotheses (instead of only DM which is already quite problematic on its own). In the 2nd case, we end up with a theory that would work on particular cases whereas GR is infinitely better verified than MOND. But with our study, the situation radically changes. The fact that the MOND theory is a very particular case of GR (in the context of its linearization and in the specific case of the presence of a uniform gravitic field) makes it possible to encompass these 2 paths in 1 alone because the k_0 field is naturally present with the same intensity at all scales. Embedding the galaxies, k_0 with the values (29) explains on the one hand the component of DM on a large scale and on the other hand the deviations from the Newtonian acceleration on more local scales in agreement with the MOND approximation (39). One can also add a recent publication [10] which demonstrates that MOND could be an alternative to the planet nine hypothesis. In our explanation of MOND-based theories, it would then mean that the gravitic field k_0 would be an alternative to the planet nine hypothesis, which is possible because k_0 applies to all objects [11] even at our scale.

The fact that observations corroborate the MOND models would not mean that they reveal a problem in the theoretical framework of GR but rather that the MOND models do not define a generic theoretical framework for the gravitational interaction but only a theoretical solution of a particular physical case of GR (that of the presence of a uniform field similar in EM to the magnetic field found in some materials such as ferromagnetic materials).

One can also add a recent publication that demonstrates the Tully-Fisher Law [12] in the same framework, LGR with the expected value (29) of k0.

5. Conclusions

In this study, we show that the explanation of the DM by a uniform gravitic field k_0 embedding the galaxies (likely generated by the clusters of galaxies), which makes it possible to account for the DM component without adding exotic matter and compliant with GR, also makes it possible to define MOND-based models. Furthermore, the field values [3], $10^{-16.62}\,\text{s}^{-1} < \|\boldsymbol{k}_0\| < 10^{-16.3}\,\text{s}^{-1}$, required to explain the DM, allows us to find the expected values of the MOND parameter [2], $a_0 \sim 10^{-8}\,\text{cm}\cdot\text{s}^{-2}$, and of the parameter [9] $G' \sim 1.37G$ from deep MOND and AQUAL which account for the Newtonian acceleration deviations observed in [9] in the low acceleration regime. Consequently, the uniform gravitic field k_0 in addition to accounting for DM, can account for the observed Newtonian acceleration deviations.

While in MOND, these deviations are integrated as a correction of the theoretical framework (independent of the DM component) our study shows rather that it would be a correction due to the presence of a uniform field (the one explaining the DM) and that the MOND modeling would be more an approximation of a particular solution of the gravitational interaction rather than a generic theoretical framework of the gravitational interaction. More precisely, the MOND-based modeling would approximate the linearization of the GR in the particular case of the existence of a uniform gravitic field similar to the magnetic field of materials such as it exists in ferromagnetic materials.

It is reminded that this gravitic field (whose values can account for the DM and a large class of MOND-based theories) is physically justified because it is predicted by the GR and gives rise to the effect known as the Lense-Thirring effect and which has already been observed. In other words, the GR and its gravitic field k_0 make the DM hypothesis and the MOND-based theories useless.

Conflicts of Interest

The author declares no conflicts of interest.

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