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Exploring New Physics of Massive Particles and Charge Quantization

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Abstract

This research seeks to understand the fundamental properties of matter, such as mass, charge, and spin, and how they interact with each other. By studying the behaviour of these particles in different environments, scientists can gain insight into the nature of the universe. This research also has implications for our understanding of dark matter and dark energy, which are believed to make up the most of the universe's mass but remain largely mysterious. Additionally, this research may provide clues to the origin and evolution of the universe itself. The research began with a comprehensive review of existing theories that attempt to explain the origin and nature of charge quantization. Special attention is given to the standard model and quantum field theory, which provides a mathematical framework for describing both particles and their interactions. Also, this research aims to bridge the gap between our understanding of massive particles and charge quantization.

Subject Areas

Theoretical Physics

Keywords

Standard Model, Particle Physics, Higgs Field, Higgs Mechanism, Electroweak Interaction, Covariant Derivatives, Special Relativity, Klein Gordon Equation

1. Introduction

In the Standard Model, the concept of mass is one of the core elements of the Standard Model. Mass is the measure of an object's inertia or resistance to a change in its velocity. It is the property of a particle that enables it to interact with gravity. In the Standard Model, mass is identified as an inherent property of the fundamental particles that make up all matter in the universe. The Standard Model assigns mass to the particles, which in turn determines the strength of the associated forces. For example, the mass of the Higgs boson is responsible for the strength of the weak force. The Standard Model also assigns mass to the quark, the basic building block of matter. The mass of the quarks affects the behavior of protons and neutrons, the particles that make up the nucleus of an atom. The mass of the quarks determines the properties of all matter in the universe, including the properties that make up the wide variety of elements on the periodic table. The internal physical nature and structure of the most fundamental particles—leptons and quarks—is so far not known and not needed for the classification as such but of course, much wanted for the understanding of nature. [1]

In Special Relativity, the concept of mass plays an integral role in the theory of relativity. In special relativity, mass is an invariant quantity, meaning it remains constant no matter the speed or direction of the observer. In general relativity, the concept of mass is further developed, as it is seen as a form of energy and is used to explain the curvature of space time.

Mass is related to both energy and momentum, as stated by the famous Einstein equation, this equation shows that mass and energy are interchangeable, meaning mass can be converted into energy and vice versa. This phenomenon can be observed in nuclear fission, where energy is released through the splitting of atoms, [2] [3] which is a process in which the nucleus of an atom is split into two or more smaller nuclei, releasing a large amount of energy. This process occurs when a heavy atomic nucleus, such as uranium-235 or plutonium-239, absorbs a neutron and becomes unstable as shown below (**Figure 1**).

2. Physics of Standard Model

Electroweak Interaction Boson Masses

Physical world consists of matter particles like electrons and quarks beside field quanta (particles) like photons w^+, w^- and z^0 bosons. Matter particles are called fermions, while field quanta (particles) are called bosons. The standard model is a set of rules that describes the interactions between all the particles in the universe. The origin of mass generation is based on the breaking of one of these

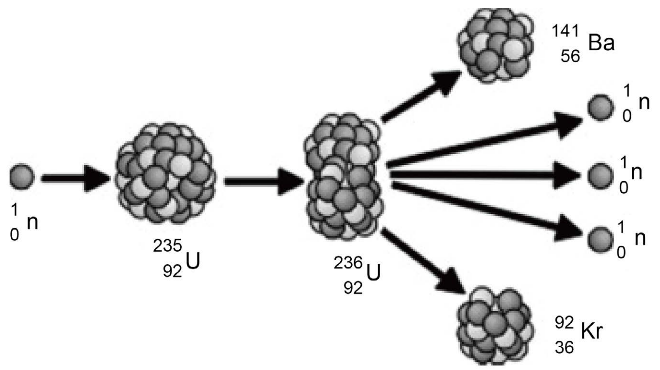

Figure 1. Diagram of nuclear fission.

rules, which leads to the elusive Higgs boson being accidentally created in a particle accelerator. The new theory explains how this happened and what it does to mass generation throughout the universe. [4]

The Lagrangian of the matter field is a mathematical description of the dynamics of the field, capturing how it changes with time and how it interacts with the rest of the system. It is an integral part of the classical description of physical systems like electromagnetism, gravity and quantum mechanics. The Lagrangian can be expressed in terms of the positions and momenta of the particles that make up the field and their interactions with each other. It is then used to calculate the dynamics of the field, including the equations of motion and the energy of the field. Furthermore, the Lagrangian can be used to calculate other quantities such as the entropy, pressure, temperature and chemical potential of the field. The Lagrangian is an indispensable tool for understanding the behavior of physical systems at the fundamental level. [5]

And the Lagrangian of the matter field is described by:

$$L = \bar{\Psi}\left(i\gamma^{\mu} - m\right)\Psi \tag{1}$$

m = the mass, Ψ = wave function, $\bar{\Psi}$ = wave fuction conjugation.

The conservation of quantities like energy and momentum, this can be achieved by space-time translation which leads to 4-momentum (energy + 3 momentum components). This leads to action and Lagrangian invariance. Noether theorem is the generalization of this symmetry by introducing continuous group of transformations of the fields. The corresponding Lagrangian determines a conserved tensor and an associated time independent observable. The Lagrangiancan be made invariant under local space time dependent gauge:

$$\begin{aligned} &\Psi \to \Psi' = e^{iq\Lambda(x)}\Psi(x) \\ &\bar{\Psi} \to \bar{\Psi}'(x) = e^{-iq\Lambda(x)}\Psi(x) \end{aligned} \tag{2}$$

Under this transformation, the first term of Lagrangian (1) transform as:

$$\begin{aligned} \bar{\Psi}' i\gamma^{\mu}\partial_{\mu}\Psi' &= e^{-iq\Lambda(x)}\bar{\Psi} i\gamma^{\mu}\partial_{\mu}\left[e^{iq\Lambda(x)}\Psi(x)\right] \\ &= e^{-iq\Lambda(x)}\bar{\Psi}\left[i\gamma^{\mu}\left(iq\left(\partial_{\mu}\Lambda\right)\Psi + \partial_{\mu}\Psi\right)\right]e^{iq\Lambda(x)} \\ &= \bar{\Psi}\left[i\gamma^{\mu}\partial_{\mu}\Psi + i^{2}q\gamma^{\mu}\Psi\partial_{\mu}\Lambda\right] \end{aligned} \tag{3}$$

where q is the charge and Λ is the conversion parameter.

Clearly this term is not invariant under the gauge transformation. Under a general gauge transformation, a given term in any Lagrangian is not invariant since the gauge field couples to the matter fields. However, if the gauge field is transformed through a unitary transformation, the term may be rendered invariant, potentially up to a total derivative. In other words, the term may be gauge invariant if the gauge field is transformed according to a unitary transformation. In this case, the term will not change under the gauge transformation, meaning it is invariant. [6]

The second term in (1) is given by:

$$\bar{\Psi}' m \Psi' = e^{-iq\Lambda} \bar{\Psi} m \Psi e^{iq\Lambda} = \bar{\Psi} m \Psi \tag{4}$$

The mass term is thus invariant. The invariance of the first term and the Lagrangian can be restored by replacing ordinary derivative by the covariant derivative which is a generalization of the ordinary derivative, which is used in the study of general relativity and other tensor calculus theories. It is represented by a curved arrow and used to differentiate tensors with respect to the associated metric tensor. The covariant derivative allows for differentiation in which the resulting tensor is guaranteed to transform in a specified manner under coordinate transformations. It is used to define the manner in which physical laws transform under coordinate transformations and is a powerful tool in the study of general relativity and other such theories, [7] then:

$$D_\mu \Psi = \partial_\mu \Psi + iqA_\mu(x) \tag{5}$$

where D is covariant derivative and A_μ is a field of boson quanta. A field of boson quanta is a system of particles composed of bosons, which are particles that obey Bose-Einstein statistics. These particles are typically characterized by the wave-particle duality, meaning they can exist as both waves and particles. Boson quanta fields have various applications, ranging from describing particles such as photons and gluons to field theories such as quantum electrodynamics and quantum chromodynamics. These fields are also useful for describing phenomena such as super fluidity and superconductivity. And which transform as:

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \partial_\mu \Lambda(x) \tag{6}$$

Under these constraints the Lagrangian is invariant, *i.e.*:

$$L' = \bar{\Psi}'\left(i\gamma^\mu D'_\mu \Psi' - m\Psi'\right)$$

$$L = \bar{\Psi}\left(i\gamma^\mu D_\mu \Psi - m\Psi\right) \tag{7}$$

This Lagrangian describes free matter particles and interaction of them with electromagnetic field (e.m) A_μ.

$$L_{e.m} = -\frac{1}{4} F_{\mu\upsilon} F^{\mu\upsilon} \tag{8}$$

where the $F^{\mu\upsilon}$ is electromagnetic field tensor.

The complete electroweak Lagrangian is:

$$\begin{aligned} L &= \bar{\Psi}\left(i\gamma^\mu D_\mu \Psi - m\Psi\right) - \frac{1}{4} F_{\mu\upsilon} F^{\mu\upsilon} \\ &= \bar{\Psi}\left(i\gamma^\mu \partial_\mu - m\right)\Psi - j^\mu A_\mu - \frac{1}{4} F_{\mu\upsilon} F^{\mu\upsilon} \end{aligned} \tag{9}$$

It's important to note that the Lagrangian invariance requires massless field quanta (bosons), since the mass term is not invariant:

$$m_\gamma^2 A_\mu A^\mu \neq m_\gamma^2 A'_\mu A'_\mu = m_\gamma^2 \left(A_\mu - \partial_\mu \Lambda\right)\left(A^\mu - \partial^\mu \Lambda\right) \tag{10}$$

And the mass term in quantum field theory is the part of the quantum Lagrangian that corresponds to the mass of a particle, it is often necessary to work with mass-independent Lagrangian, which are invariant under Lorentz transformations, in order to obtain consistent. [8]

3. Higgs Mechanism and Mass Generation

A cornerstone of the Standard Model is the mechanism that generates the particle masses while preserving the gauge invariance of the theory. Indeed, the direct introduction of masses for the fermions and for the gauge bosons that mediate the weak interaction violates the invariance with respect to the transformations of the electroweak symmetry group.

This mechanism explains why some particles have mass while others do not, and it also explains why some particles have different masses from one another. The discovery of the Higgs boson in 2012 confirmed the existence of this mechanism and provided further evidence for the Standard Model. [9]

It is the Higgs-Brout-Englert mechanism commonly called the Higgs mechanism, which allows the generation of particle masses while preserving the gauge symmetry of electroweak interactions.

The field quanta (particles) of electroweak interaction particles are not all massless. Some of these quanta bosons are massive. the gauge invariance of Lagrangian require massless quanta. Thus, one needs a certain mechanism (higgs mechanism) for mass generation. according to higgs mechanism (H.m) field quanta masses are generated at vacuum state, which can be found by considering the scalar (spin-less) Lagrangian, which describes klein-Gordon equation. Where:

$$\begin{aligned} L &= \left(\partial_{\mu}\Phi\right)^{*}\left(\partial^{\mu}\Phi\right) - V\left(\Phi\right) \\ &= \left(\partial_{\mu}\Phi\right)^{*}\left(\partial^{\mu}\Phi\right) - \frac{M^{2}}{2}\Phi^{*}\Phi - \frac{\lambda}{4}\left(\Phi^{*}\Phi\right)^{2} \end{aligned} \tag{11}$$

For real scalar field the potential V is given by:

$$V = \frac{M^{2}\Phi^{2}}{2} + \frac{\lambda\Phi^{4}}{4} \tag{12}$$

The potential is minimum, when:

$$\frac{\mathrm{d}V}{\mathrm{d}\Phi} = \frac{2M^{2}\Phi}{2} + \frac{\lambda\left(4\Phi^{3}\right)}{4} = 0$$

$$\Phi = \sqrt{\frac{-M^{2}}{\lambda}} = v = \text{vacuum} \tag{13}$$

Thus, the vacuum state potential is associated with a zero-point energy, and this zero-point energy has measurable effects. In the laboratory, it may be observed through the Casimir effect, and spontaneous emission. However, these energies may be regulated by introducing gradients in the vacuum states, Now assume existence of new higgs field H which fluctuate around vacuum, the newly discovered Higgs field, denoted H, has been proposed to exist in the Universe and to fluctuate around a vacuum expectation value. [10] This new field is said to be closely related to the Standard Model Higgs boson, as it shares a similar mass and spin. The Higgs field H is believed to be responsible for mass generation, as well as for the spontaneous symmetry breaking of the Standard Model. Furthermore, it is thought to be connected to the cosmological history of the Universe, as it could potentially influence the rate of vacuum decay. Thus:

$$\Phi = v + H(x) \tag{14}$$

where the $H(x)$ is higgs field.

Where v constant clearly the Lagrangian is not invariant under mirror reflection (parity).This means that when a system is reflected in a mirror, its Lagrangian will not remain the same. To understand this concept better, let us look at an example. Consider a particle moving in two dimensions. When this particle is reflected in a mirror, the equations of motion of the particle remain the same, but the Lagrangian of the particle changes. This is because the Lagrangian is dependent on the coordinates of the particle, which are changed by the reflection. Therefore, the Lagrangian is not invariant under mirror reflection (or parity). [11]

Then:

$$H \to -H$$

In this case, the term:

$$\frac{M^2}{2}\Phi^*\Phi = \frac{M^2}{2}(v+H)(v+H) = \frac{M^2}{2}\left(v^2 + 2vH + H^2\right) \neq -H$$

which is:

$$\Rightarrow \frac{M^2}{2}(v-H)(v-H) = \frac{M^2}{2}\left(v^2 - 2vH + H^2\right) \tag{15}$$

Thus, the symmetry is spontaneously broken, which is a concept that explains how certain physical and mathematical systems can spontaneously break a symmetric state of equilibrium, resulting in a state that is asymmetrical or "broken." This is commonly seen in things such as particle physics, where the laws of physics are symmetric but the physical states that occur are asymmetric. Symmetry breaking can be caused by external forces, such as energy, or by intrinsic forces, such as the nature of the system itself. The concept of symmetry breaking has been used to explain a wide range of physical phenomena, from crystal formation and the behavior of subatomic particles to the evolution of species and even the formation of stars, since parity is not conserved or reflection symmetry is hidden. [12]

Now consider the scalar filed to be:

$$\Phi = \frac{1}{\sqrt{2}}[v+H]e^{\frac{i\theta}{v}} \tag{16}$$

where:

$$H = H(x),\quad \theta = \theta(x) \tag{17}$$

The field can transform as:

$$\Phi(x)=\Phi=\mathrm{e}^{iq\Lambda(x)}\Phi(x)=\mathrm{e}^{iq\Lambda}\Phi \tag{18}$$

Then becomes:

$$\Phi=\frac{1}{\sqrt{2}}[v+H]\mathrm{e}^{\frac{i\theta}{v}+iq\Lambda} \tag{19}$$

By Comparing, the equation requires:

$$\bar{H}=H \tag{20}$$

$$\frac{\bar{\theta}}{v}=q\Lambda+\frac{\theta}{v}$$

$$\bar{\theta}=qv\Lambda+\theta \tag{21}$$

By choosing:

$$\Lambda=-\frac{1}{qv}\theta \tag{22}$$

The massless goldstone boson can be removed. The field Φ transforms as:

$$\Phi=[v+H]\mathrm{e}^{\frac{i\theta}{v}}\rightarrow\bar{\Phi}=[v+\bar{H}]\mathrm{e}^{\frac{i\bar{\theta}}{v}}$$

$$\Phi=[v+H] \tag{23}$$

where $\bar{\theta}$ vanishes, *i.e.*:

$$\bar{\theta}=0 \tag{24}$$

According to this choice the Lagrangian:

$$L=D_\mu^*\Phi^*D_\mu\Phi-\frac{M^2}{2}\Phi^*\Phi-\frac{\lambda}{4}(\Phi^*\Phi)^2-\frac{1}{4}F_{\mu\upsilon}F^{\mu\nu} \tag{25}$$

With the covariant derivative:

$$D_\mu=\partial_\mu+iqA_\mu D_\mu^*=\partial_\mu-iqA_\mu \tag{26}$$

And

$$\bar{A}_\mu=A_\mu+A_\mu\Lambda=A_\mu+\frac{1}{qv}\partial_\mu\theta \tag{27}$$

$$\bar{D}_\mu=\partial_\mu+iq\bar{A}_\mu=\partial_\mu+iqA_\mu+\frac{i}{v}\partial_\mu\theta \tag{28}$$

The above transformation is invariant. *i.e.*

$$\bar{L}=\bar{D}_\mu^*\bar{\Phi}^*\bar{D}_\mu\bar{\Phi}-\frac{M^2}{2}\bar{\Phi}^*\bar{\Phi}-\frac{\lambda}{4}(\bar{\Phi}^*\bar{\Phi})^2-\frac{1}{4}\bar{F}_{\mu\upsilon}\bar{F}^{\mu\nu}=L \tag{29}$$

In view of equations:

$$-M^2=\lambda v^2\Phi=\frac{1}{\sqrt{2}}(H+v)\mathrm{e}^{\frac{i\theta}{v}}$$

And then one gets:

$$L=\left[\frac{1}{2}\left(\partial_{\mu}H\right)^{2}-\lambda v^{2}H^{2}\right]+\frac{1}{2}q^{2}v^{2}A_{\mu}A^{\mu}+\frac{1}{2}q^{2}A_{\mu}A^{\mu}H^{2}+q^{2}vA_{\mu}A^{\mu}H$$
$$-\lambda vH^{3}-\frac{\lambda}{4}H^{4}-\frac{1}{4}F_{\mu\upsilon}F^{\mu\nu}+\frac{\lambda v^{4}}{4} \tag{30}$$

Thus, the field quanta bosons acquire mass, and Quanta bosons are the smallest known particles, and they have no mass. Quanta bosons have energy and momentum, but no mass or electric charge. They are influenced by the four fundamental forces—gravity, electromagnetism, strong nuclear force, and weak nuclear force—and can interact with other particles.

Where the mass term is:

$$\frac{1}{2}q^{2}v^{2}A_{\mu}A^{\mu}=\frac{1}{2}m_{A}^{2}A_{\mu}A^{\mu} \tag{31}$$

Their mass is given to be:

$$m_{A}=qv$$

The higgs quanta boson mass term is given by:

$$\lambda v^{2}H^{2}=\frac{1}{2}m_{H}^{2}H^{2}$$

Thus, the higgs particles mass are:

$$m_{H}=\sqrt{2\lambda v^{2}} \tag{32}$$

The Lagrangian invariance and form can be easily found by knowing that:

$$\bar{F}_{\mu\upsilon}=\partial_{\mu}\bar{A}_{\upsilon}-\partial_{\upsilon}\bar{A}_{\mu}=\partial_{\mu}\left(A_{\upsilon}+\frac{\partial_{\upsilon}\theta}{qv}\right)-\partial_{\upsilon}\left(A_{\mu}+\frac{\partial_{\mu}\theta}{qv}\right)$$
$$=\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}+\frac{\partial_{\mu\upsilon}\theta-\partial_{\nu\mu}\theta}{qv}=\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}=F_{\mu\upsilon}$$

where: $\partial_{\mu\upsilon}\theta=\partial_{\upsilon\mu}\theta$.

The above equation is Lagrangian invariant, meaning that it is invariant under the action of a Lagrangian transformation. This means that if we apply a Lagrangian transformation to both sides of the equation, the resulting equation will be identical to the original one. [13]

4. Discussion

The papers proposed that the Higgs field was responsible for the masses of particles, particularly the W and Z bosons that carry the weak nuclear force, and that the existence of the Higgs boson (a particle associated with the Higgs field) could be confirmed experimentally. The potential independent mass term in the Lagrangian is replaced by potential dependent mass to obtain anew Lagrangian, considering local transformation of ψ with ($\bar{\phi}=\phi-\phi_{0}$) the symmetry is restored when:

$$\partial_{\mu}\Lambda=m_{0}q^{-1}\gamma^{\mu-1}\left(\phi-\bar{\phi}\right)$$

To make the gauge field acquire mass the wave function is assumed to have constant background ϕ_0.

The gauge mass depends on this background ϕ_0 as well as the charge. The coefficients of ψ and ϕ_0 are related to the probability of existence of the system in dynamical or vacuum states. Another approach to acquire mass is suggested below by adding to the covariant derivative and additional charge term q_0. The mass term is shown to be dependent on this additional charge term according to the following equation:

$$\begin{aligned} D_\mu \Psi D_\mu \bar{\Psi} = {} & \partial_\mu \Psi \partial_\mu \bar{\Psi} + iqW_\mu \bar{\Psi} \partial_\mu \Psi + iq_0 \left(w - \phi_0 \right) \partial_\mu \Psi + iqW_\mu \Psi \partial_\mu \bar{\Psi} \\ & - q^2 W_\mu^2 \left| \Psi \right|^2 - qq_0 W_\mu \Psi \left(w - \phi_0 \right) + iq_0 \left(w - \phi_0 \right) \partial_\mu \bar{\Psi} \\ & - q_0 q W_\mu \bar{\Psi} \left(w - \phi_0 \right) - q_0^2 \left(W_\mu^2 - 2W_\mu \phi_0 + \phi_0^2 \right) \end{aligned}$$

A new development is made also to the quantum field theory by considering the Lagrangian to be dependent on the second order differential.

5. Conclusion

When introducing the new Lagrangian with the mass-dependent term, the transformation of the gauge field requires the inclusion of the potential and vacuum fields in the invariance parameter. The gauge field mediator can acquire mass by adding to the dynamical field constant vacuum background wave function related to the density of the mediator. One can also redefine the covariant derivative to add to the additional charge term. The research on massive particles and charge quantization provided strong evidence for the existence of a fundamental unit of electric charge and its discrete nature. In addition to that, the observed behavior of elementary particles, conservation laws, and experimental findings all support the concept of charge quantization as a fundamental principle in physics.

Conflicts of Interest

The authors declare no conflicts of interest.

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