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Analysis of a Steady MHD Mixed Convection Fluid Flow in a Microchannel within Permeable Walls with Suction and Injection Parameters

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Abstract

This research work focuses on the analysis of a steady fluid of a convection mixed flow within a permeable channel under the influence of suction and injection parameters. The fluid is presumed to be incompressible, and an existing model was used to formulate the governing equations with appropriate boundary conditions. The governing equations directing the flow system are modeled, non-dimensionalized, and transformed into ordinary differential equations where appropriate methods were used to solve the momentum and energy equations analytically. Solutions for momentum and energy equations of the incompressible fluid are presented graphically to display the impact of various thermos-physical parameters. The results revealed the significant effect of the magnetic strength reducing the fluid flow with increasing magnetic field strength values, which is due to the presence of electromagnetic force in the flow regime. Also, the pressure acts normally inward from the fluid towards the surface, the result reveals that an increase in the pressure gradient increases the fluid flow due to the suction and injection parameters with the presence of magnetic field strength, and an increase in Reynolds number decreases the flow regime. Meanwhile, an increase in the Prandtl number, Eckert number, and pressure gradient increases the fluid temperature within the permeable microchannel walls.

Subject Areas

Fluid Mechanics

Keywords

Reynolds Number, Prandtl Number, Eckert Number, Suction and Injection,

Mixed Convection

1. Introduction

The investigation of studies involving magnetic fluid flow through a channel has become a very demanding field of research due to its many applications in many industries, medical technology, and bio-engineering processes. Magnetic fluid is well known for its extensive use in industries and medicine, for instance, opening the blockage in the artery, magnetic wound or treatment of cancerous tumors, hyperthermia, and reduction of bleeding, ferrofluid sealing, high gradient magnetic separation, magnetic devices for cell separation, targeted transport of drugs, high-temperature plasmas, cooling of nuclear reactors, power generation systems. Several research studies on magnetic fluid flow dynamics in the presence of a magnetic field in a channel have been carried out to understand the interaction between a magnetic field and fluid flow characteristics [1]. Also, Saha and Chakrabartti [2] devoted their study to investigating the flow characteristics of a bio-magnetic fluid flowing through a channel under the action of the magnetic field by maintaining the magnet in the axial direction and changing its length considering the FHD phenomenon in relation to the formation of recirculation bubbles and the velocity contour. Hassanien and Mansour [3] discussed the magnetic flow through a porous medium between two infinite parallel plates. In addition to that, Tzirzilakis [4] investigated the fundamental problem of the bio-magnetic fluid flow in a channel with stenosis under the influence of a steady localized magnetic field.

Furthermore, Gbadeyan *et al.* [5] investigated the effect of radiation on temperature and velocity profiles of an electrodynamics froth flow process in vertical channels. Hassan [6] examined the influence of magnetic intensity and external heat generation on a flow system with variable viscosity channels with varying wall temperatures. Makinde [7] studied the impacts of heat source and magnetic strength on the irreversibility flow of a variable viscosity fluid through a channel with non-uniform wall temperature as well as on the entropy generation rate and other thermo-physical properties of the flow regime. Among the results obtained include the internal heat generation and Brinkman that allowed the fluid to interact and hence increase the fluid internal energy by moving faster while the magnetic strength parameter has a retarding effect due to the presence of Lorentz forces across the flow channel. Also, the increasing values of heat source, magnetic strength, and Brinkman number make the temperature rise due to the interaction of the particles in the fluid flow within the channel while the viscosity variation parameter reduces the fluid temperature.

Also, Cortell [8] studied the effects of suction, viscous dissipation, and thermal radiation on the flow and heat transfer of a power-law fluid past an infinite porous plate. Effect of suction and injection on unsteady free convection Couette flow and heat transfer of reactive viscous fluid in the vertical porous plate is to be found in the study by Jha *et al.* [9]. The investigation of the flow of an electrically conducting fluid in a porous channel in the presence of a transverse magnetic field is important because of its widespread engineering and industrial applications such as MHD marine propulsion, electronic packages, microelectronic devices, thermal insulation, petroleum reservoirs, MHD stirring of molten metal, the exothermic reaction in packaged reactors, and magnetic-levitation casting. However, Nandkeolyar *et al.* [10] investigated numerically and analytically the effect of suction/injection on unsteady hydromagnetic heat and mass transfer flow of a radiating and chemically reactive fluid via a flat porous plate with ramped wall temperature.

Makinde and Chinyoka [11] gave a numerical analysis of buoyancy effects on hydromagnetic unsteady flow through a porous channel while taking the suction and injection into account.

Hence, the present study aims to investigate the analysis of a steady MHD mixed convection fluid flow within a microchannel under the influence of suction and injection parameters. The fluid is assumed to be incompressible and electrically conducting flowing steadily within the channel. The existing model was used to formulate the equations governing the fluid flow in the form of differential equations with appropriate boundary conditions which are modeled, non-dimensionalized, and transformed into ordinary differential equations where appropriate methods were used to solve the momentum and energy equations analytically. The solutions for momentum and energy equations of the incompressible fluid are presented graphically to display the impact of various thermophysical parameters.

2. Mathematical Formulations

The study considers a steady flow of an incompressible MHD fluid within a channel of width a unit where the left side is kept at a temperature of T_0 which is fixed at $\bar{x}=0$ and the right side at T_1 fixed at $\bar{x}=a$ such that $T_1<T_0$. A uniform magnetic field of B_0 is applied in the direction parallel to the $\bar{y}$ axis. The $\bar{x}$-axis is aligned horizontally and $\bar{y}$–axis is perpendicular to the channel as shown in **Figure 1**. The fluid flow in the channel is induced as a result of the

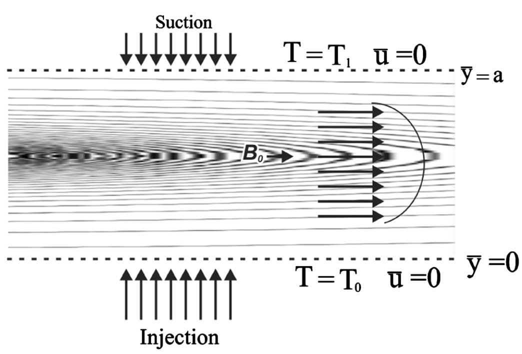

Figure 1. Geometry of the problem.

combined effect of magnetic parameters and pressure gradient.

Following Kefene, *et al.* [12], the governing equations for continuity, momentum, and energy are presented below:

$$\frac{\partial \bar{u}}{\partial \bar{x}} = 0 \tag{1}$$

$$V\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial \bar{x}} + \frac{\mu}{\rho}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma}{\rho}B_0^2\bar{u} \tag{2}$$

$$V\frac{\partial T}{\partial \bar{y}} = k\frac{\partial^2 T}{\partial \bar{y}^2} - \frac{\mu}{\rho c_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + \frac{\sigma}{\rho c_p}B_0^2\bar{u}^2 \tag{3}$$

with the initial and boundary conditions:

$$\bar{u}(\bar{y}) = 0, T(\bar{y},0) = 0 \quad \text{and} \quad \bar{u}(a) = T(a) = 0 \tag{4}$$

where $\bar{u}$ is the fluid velocity in $\bar{x}$ direction, T is the temperature of the fluid, V is the suction/ injection velocity, P is the fluid pressure at zero time, ρ is the density of the fluid, μ is the viscosity thermal diffusion coefficient, $\boldsymbol{g}$ is the acceleration due to gravity, α is the thermal diffusion coefficient, σ represents the electrical conductivity and a denotes the channel width.

Presenting the following non-dimensionless quantities:

$$\begin{aligned} &x = \frac{\bar{x}}{a},\ y = \frac{\bar{y}}{a},\ u = \frac{\bar{u}a}{v_0},\ \theta = \frac{T-T_0}{T-T_0},\ Pr = \frac{v_0}{\alpha},\ Re = \frac{va}{v_0},\ G = -\frac{\rho a \partial P}{\mu_0^2 \partial x}, \\ &P = \frac{\mu_0^2 \bar{P}}{\rho a^2},\ M^2 = \frac{\sigma B_0^2 a^2}{\mu_0} \text{ and } Ec = \frac{v^2}{c_p a^2 T - T_0}. \end{aligned} \tag{5}$$

Using the variables and parameters in Equation (5) on the momentum and energy Equations in (2) and (3), we obtain the equations below:

$$Re\frac{\mathrm{d}u}{\mathrm{d}y} = G + \frac{\mathrm{d}^2 u}{\mathrm{d}y^2} - M^2 u \tag{6}$$

$$PrRe\frac{\mathrm{d}\theta}{\mathrm{d}y} = \frac{\mathrm{d}^2\theta}{\mathrm{d}y^2} - PrEc\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2 + PrEcM^2u^2 \tag{7}$$

together with the boundary conditions

$$u(0) = u(1) = 0 \quad \text{and} \quad \theta(0) = \theta(1) = 0 \tag{8}$$

Solution Method

The linear second-order non-homogeneous differential Equation (6) which is the fluid velocity is solved by obtaining the complementary function, particular integral, and using the boundary conditions (8) to get the general exact solution below.

$$\begin{aligned} u(y) = -\Bigg(& e^{-\frac{Re}{2}}\Bigg(e^{-\frac{Re}{2}} - e^{\frac{Re}{2}+\sqrt{4M^2+R_e^2}} - e^{\frac{1}{2}\sqrt{4M^2+R_e^2}+\frac{1}{2}y\left(R_e-\sqrt{4M^2+R_e^2}\right)} \\ & + e^{\frac{R_e}{2}+\sqrt{4M^2+R_e^2}+\frac{1}{2}y\left(R_e-\sqrt{4M^2+R_e^2}\right)} - e^{\frac{R_e}{2}+\frac{1}{2}y\left(R_e+\sqrt{4M^2+R_e^2}\right)} \\ & + e^{\frac{1}{2}\sqrt{4M^2+R_e^2}+\frac{1}{2}y\left(R_e+\sqrt{4M^2+R_e^2}\right)} G \Bigg) - 1 + e^{\sqrt{4M^2+R_e^2}} M^2 \Bigg) \end{aligned} \tag{9}$$

The solution to energy Equation (7) is possible by using the results obtained in Equation (9) where appropriate. For convenience, the energy equation is re-written as:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}y^2} - PrRe\frac{\mathrm{d}\theta}{\mathrm{d}y} - PrEc\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2 + PrEcM^2u^2 = 0 \tag{10}$$

Now Equation (10) is also a second-order differential equation that could be solved analytically but the result is of a large volume that is represented graphically in the next section.

3. Results and Discussion

This section shows the graphical results of the momentum and energy equation where the impacts of a magnetic field, pressure gradient, Reynolds number, Prandtl number, Eckert number, and suction/injection parameter on a steady fluid flow within a channel are analyzed for more understanding of the flow regime.

Figures 2-4 respectively display the effects of the pressure gradient (G), suction/injection parameter (Re), and magnetic field strength (H) on the fluid velocity. As shown in Figure 2 that an increase in the pressure gradient also leads to an increase in the fluid velocity. This shows that the more pressure applied in the microchannel, the quicker the flow of the fluid, that is, the pressure gradient

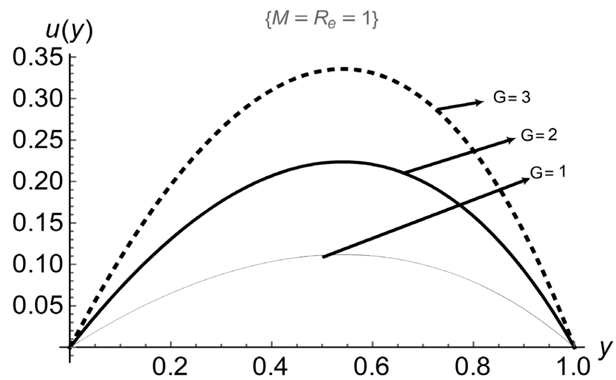

Figure 2. Velocity profile with variations in Pressure Gradient (G).

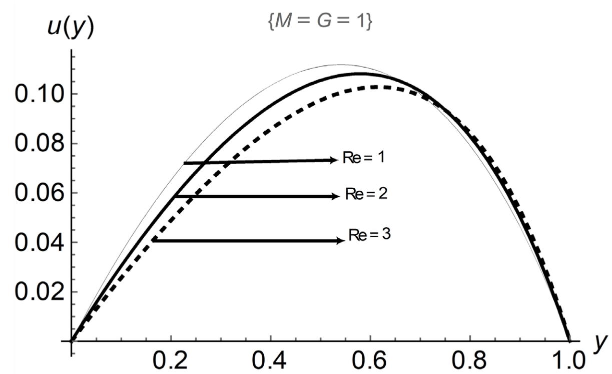

Figure 3. Velocity profile with variations in Reynolds Number (Re).

increases progressively the fluid velocity. Also, according to **Figure 3**, it can be observed that when there is an increase in the suction/injection parameter (*Re*), the fluid velocity decreases in the microchannel at the lower end of the channel towards the centerline before an increase in the fluid velocity occurs near the upper channel of the permeable wall which implies that the presence of suction is noticed at the lower plate of the channel towards the centerline and negligible at the upper plate. However, **Figure 4** displays the influence of magnetic field strength on the fluid velocity. It indicates that with an increase in magnetic strength, the fluid moves slower, that is, the fluid moves at a slower rate in a more powerful magnetic field. This implies that the presence of a magnetic field and the Lorentz force produces resistance to the movement of the flow which is physically true.

Figures 5-9 show the effect of Prandtl number (*Pr*), Eckert number (*Ec*), Pressure gradient (*G*), Magnetic Strength (*M*), and Reynolds number on the fluid temperature respectively. It reveals that an increase in the Prandtl number produces an increase in the diffusion of the temperature of the fluid, that is, the temperature rise has a significant effect on fluid properties together with the fluid properties of thermal conductivity.

Figure 6 presents the effect of Eckert number on the temperature of the fluid. The rise in the temperature of the fluid is due to an increase in Eckert's number. The rise in fluid temperature is a reason for internal heat generation due to viscous dissipation in the flow regime.

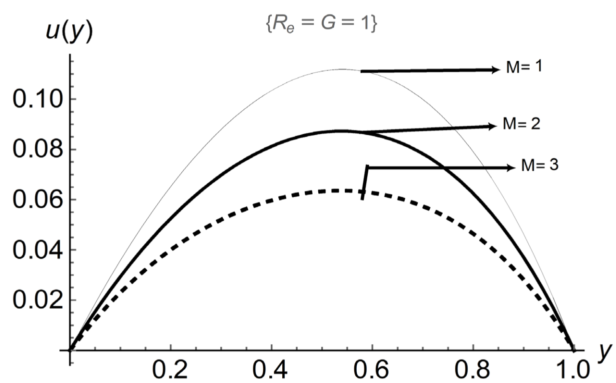

Figure 4. Velocity profile with variations in Magnetic Strength (*M*).

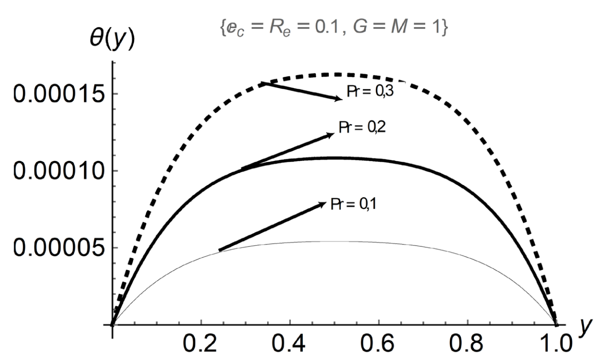

Figure 5. Temperature distribution with variations in Prandtl Number (*Pr*).

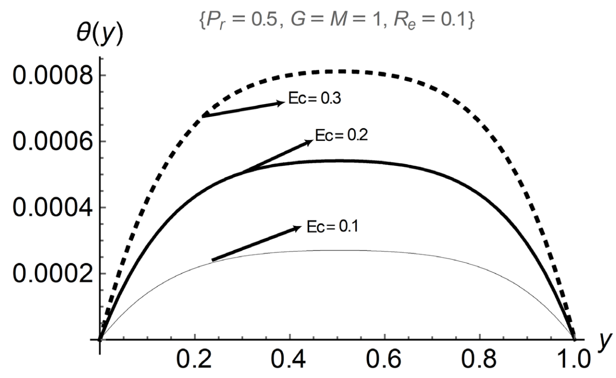

Figure 6. Temperature distribution with variations in Eckert Number (*Ec*).

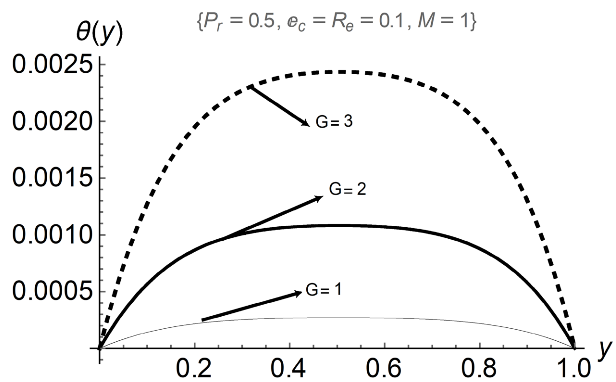

Figure 7. Temperature distribution with variations in Pressure Gradient (*G*).

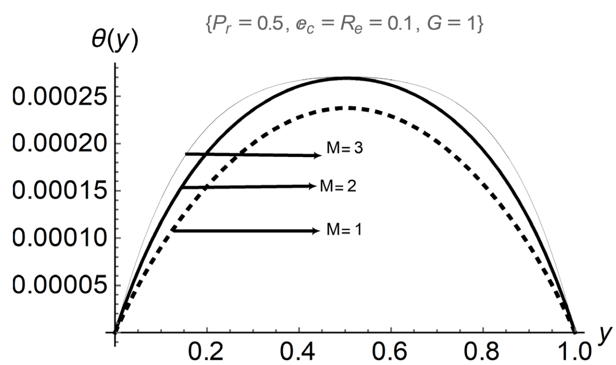

Figure 8. Temperature distribution with variations in Magnetic Strength (*M*).

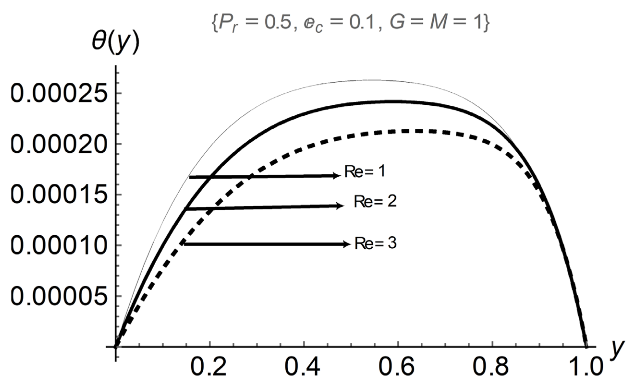

Figure 9. Temperature distribution with variations in Reynolds Number (*Re*).

The effect of pressure gradient on the temperature of the fluid is displayed in **Figure 7**. It shows that with an increase in the pressure gradient, the temperature of the fluid also increases which reveals that the more pressure applied in the microchannel, the more the increase in the temperature of the fluid. The presence of force is practically attributed to pressure over the flow surface, thereby increasing the fluid temperature.

Moreover, **Figure 8** presents the effect of magnetic strength on the temperature of the fluid. It reveals that with the increase in the magnetic strength, the temperature of the fluid reduces remarkably due to the Lorentz force's presence in the magnetic field. Thereby, allowing the maximum temperature to occur at the centerline of the flow regime with the maximum magnetic strength.

Finally, it was observed in **Figure 9**, that as the ratio of the inertia forces to viscous forces (Re) increases, the fluid temperature remarkably reduces. Also, the effect is observed at the lower plates of the channel towards the centerline and is negligible or has no effect at the upper plate.

4. Conclusions

It is presumed that the fluid is incompressible and electrically conducting flowing steadily within the channel. The existing model was used to formulate the fluid flow equations in the form of differential equations with appropriate boundary conditions. The momentum and energy equations are solved to consider the effect of various physical parameters such as magnetic strength (H), Eckert number (Ec), Prandtl number (Pr), and Reynolds number (Re) on the velocity and temperature fields. The boundary value problem, initial conditions, and the impact of flow parameters using the governing equations for conservation of mass or continuity, momentum, and energy which was intended to transform into an ordinary differential equation using suitable transformation and solved analytically. Mathematica software for an incompressible fluid was used to graphically present the results.

The result revealed the effect of the magnetic strength reducing the fluid flow with increasing magnetic field strength values, which is due to the presence of electromagnetic force in the flow regime. In addition, the pressure acts normally inward from the fluid towards the surface. This result shows that an increase in the pressure gradient increases the fluid flow due to the suction and injection parameters with the presence of magnetic field strength and an increase in Reynolds number decreases the flow. Meanwhile, an increase in the Prandtl number, Eckert number, and pressure gradient increases the fluid temperature within the permeable microchannel walls.

Conflicts of Interest

The authors declare no conflicts of interest.

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