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An Effect Exclusively Generated by General Relativity Could Explain Dark Matter

Stéphane Le Corre

Ecole Polytechnique Fédérale de Lausanne, Lausanne, Switzerland
Email: stephane.lecorre@epfl.ch

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Abstract

It has been demonstrated that dark matter can theoretically be completely explained by a natural effect of General Relativity (GR) without exotic matter or exotic correction as MOND, an effect that exists exclusively in GR and which is traditionally considered negligible. We give the values of this effect necessary to fully explain the dark matter component. In the framework of GR, this solution is mathematically as valid and legitimate as the hypothesis of an "ad hoc" addition of exotic material. The difference between these 2 solutions is revealed in physical terms. Physically the hypothesis of a new material generates a first difficulty since it requires the creation of a new physical entity whereas in our solution there is no creation of a new physical entity. A second difficulty is that this dark matter generates major problems of physical coherence: its quantity which is not a simple correction of our physics (despite the theory of gravitation was formed without this dominant gravitational component), its distribution (the galaxies must be filled with this matter and yet at our scale, physics does not need dark matter and has never been faulted), its insensitivity to electromagnetism (EM) therefore without interaction with photons (unlike all known physical entities, hence its name qualified as exotic or dark). Our solution of a larger-than-expected GR effect avoids all these difficulties. We show that its value, even if it is higher than expected, remains low. Its mathematical expression then implies that its effects are only detectable for structures with very large radii and very high velocities. This dark component then appears to be a negligible effect at our scale in terms of quantity despite its omnipresence; only the large structures of the Universe can reveal it. Finally, this effect is naturally not sensitive to the EM because it is the 2nd component of the gravitational field of the GR (in the same way as the Newtonian field). We also show that this effect can be obtained by clusters of galaxies. Indeed, we calculate the value of this effect produced by the brightest cluster galaxy (BCG). This value corresponds to the order of magnitude expected to explain dark matter. This result constitutes an important point of this study because it removes the main lock, the main physical difficulty of this solution, namely the source of such a field of low magnitude but still greater than expected.

Subject Areas

Cosmology, Astrophysics, Theoretical Physics

Keywords

Dark Matter, Clusters, BCG, Gravitation

1. Introduction

The application of general relativity (GR) to the observations shows the presence of two forms of unknown energies. The first of them appears from the scale of galaxies for which its influence dominates that of known matter [1]. Its dominance is even stronger on the scale of galaxy clusters [2]. Nevertheless, at lower scales, this component appears to be negligible (or even non-existent), so within the solar system this component does not intervene. At these "sub-galactic" scales, no notable effect is associated with it. The 2nd form of energy also appears at very large scales beyond the galaxies [3]. This 2nd form of energy is distinguished from the 1st in particular by its action. The 1st is associated with an attractive influence. It makes it possible to obtain larger rotation speeds of galaxies and greater curvatures of light (as observed) than what is predicted by GR from the only known masses. The 2nd is associated with a repulsive influence. It makes it possible to explain the measured acceleration of the expansion of the Universe, of the large structures of the Universe. At these scales, the gravitational force dominates the dynamics of the Universe. The mass of the Universe should then naturally cause the slowing down of the expansion of the Universe under the influence of gravitation (a priori only attractive force) contrary to what is observed. These forms of energy are commonly referred to as dark matter for the 1st and dark energy for the 2nd.

Several theoretical approaches have been proposed to explain these two dark components. In a general way, there are three possible kinds of physical explanations:

1) The GR equations are correct. In this case, we can consider 2 possible solutions:

a) The most immediate, since the only gravitational parameter on which we can "play" is the mass, is to suppose the existence of an unknown form of mass. For dark matter, this is the most widespread explanation to date, the one that gave its name to this form of energy (dark matter). For dark energy, it would be for example a repulsive gravitation (negative mass form) or a particle generating negative pressure. Currently, no exotic particles have ever been detected.

b) The second solution is to consider that a currently negligible effect of GR becomes unexpectedly preponderant at large scales. For example, gravitational waves have a negligible effect on the level of the solar system. This explains the difficulty there was in detecting them. And yet this effect is noticeable on objects generating these waves in large quantities, for example, a remarkable loss of mass after their fusion. There is another effect that comes exclusively from GR. We will talk about it in the following. The theoretical solution proposed in this article will precisely follow this possible kind of explanation for dark matter. Note that the cosmological constant as a mathematical explanation of dark energy falls into this category. But it still requires a physical explanation to justify the presence of this cosmological constant, and such a physical explanation could fall into one of these three categories.

2) The GR equations on which this detection is based are "false", more precisely incomplete or not precise enough causing this need to complete our observations by adding these forms of energy. These explanations then require the establishment of a new theoretical framework that would make it possible to account for the observations without adding new components. One such solution is the MOND theory. Remember, however, that the GR equations provide the best results to date for all gravitational structures at scales smaller than galaxies. And at these scales, the GR has never been found wanting. We can also add that it is remarkable that the addition of these two energy components does not come to contradict or invalidate the GR but to complete it in a coherent way. There are thus nowadays many different ways of measuring their quantity and all of them are consistent with GR. Finally, these components seem to follow the rules of the GR game well.

Among these three kinds of explanation, the least costly in terms of new concepts is path 1.b. Indeed, in this path, there is no new concept, no new material and no new theoretical framework, only GR with its own specific effects. Such an approach would therefore have great advantages. The theoretical solution that we are going to present in this article follows this path. It will only concern dark matter but not dark energy. The only novelty (and difficulty) that this solution will present is a higher-than-expected value of the Lense-Thirring effect. But we will demonstrate that some observed structures in galaxy clusters can generate this expected value. This is certainly the most important result of this study because this calculation closes the loop (detecting the source with the expected amplitude of this DM component, explaining the physical mechanism of its large-scale extension, its negligible effect at our scale and its exotic behavior, *i.e.*, insensitive to EM and only a gravitational effect).

2. Dark Matter Explained by General Relativity

2.1. From General Relativity to Linearized General Relativity

From GR, one deduces the linearized GR (LGR) in the approximation of a quasi-flat Minkowski space ($g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$; $\left|h^{\mu\nu}\right| \ll 1$). With the following Lorentz gauge, it gives the following field equations as in [4] (with $\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta$ and $\Delta = \nabla^2$):

$$\partial_\mu \bar{h}^{\mu\nu} = 0;\ \ \Box \bar{h}^{\mu\nu} = -2\frac{8\pi G}{c^4}T^{\mu\nu} \tag{1}$$

with

$$\bar{h}^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}h;\ h \equiv h_\sigma^\sigma;\ h_\nu^\mu = \eta^{\mu\sigma}h_{\sigma\nu};\ \bar{h} = -h \tag{2}$$

The general solution of these equations is:

$$\bar{h}^{\mu\nu}(ct,\boldsymbol{x}) = -\frac{4G}{c^4}\int \frac{T^{\mu\nu}\left(ct - |\boldsymbol{x}-\boldsymbol{y}|,\boldsymbol{y}\right)}{|\boldsymbol{x}-\boldsymbol{y}|}\mathrm{d}^3\boldsymbol{y} \tag{3}$$

In the approximation of a source with low speed, one has:

$$T^{00} = \rho c^2;\ T^{0i} = c\rho u^i;\ T^{ij} = \rho u^i u^j \tag{4}$$

And for a stationary solution, one has:

$$\bar{h}^{\mu\nu}(\boldsymbol{x}) = -\frac{4G}{c^4}\int \frac{T^{\mu\nu}(\boldsymbol{y})}{|\boldsymbol{x}-\boldsymbol{y}|}\mathrm{d}^3\boldsymbol{y} \tag{5}$$

At this step, by proximity with electromagnetism, one traditionally defines a scalar potential φ and a vector potential H^i. There are in the literature several definitions as in [5] for the vector potential H^i. In our study, we are going to define:

$$\bar{h}^{00} = \frac{4\varphi}{c^2};\ \bar{h}^{0i} = \frac{4H^i}{c};\ \bar{h}^{ij} = 0 \tag{6}$$

With gravitational scalar potential φ and gravitational vector potential H^i:

$$\begin{aligned} \varphi(\boldsymbol{x}) &\equiv -G\int \frac{\rho(\boldsymbol{y})}{|\boldsymbol{x}-\boldsymbol{y}|}\mathrm{d}^3\boldsymbol{y} \\ H^i(\boldsymbol{x}) &\equiv -\frac{G}{c^2}\int \frac{\rho(\boldsymbol{y})u^i(\boldsymbol{y})}{|\boldsymbol{x}-\boldsymbol{y}|}\mathrm{d}^3\boldsymbol{y} = -K^{-1}\int \frac{\rho(\boldsymbol{y})u^i(\boldsymbol{y})}{|\boldsymbol{x}-\boldsymbol{y}|}\mathrm{d}^3\boldsymbol{y} \end{aligned} \tag{7}$$

With K (determined in [6]) a new constant defined by:

$$GK = c^2 \tag{8}$$

This definition gives $K^{-1} \sim 7.4\times 10^{-28}\ \mathrm{kg}\cdot\mathrm{m}^{-1}$ very small compared to G.

The field Equations (1) can be then written (Poisson equations):

$$\Delta\varphi = 4\pi G\rho;\ \Delta H^i = \frac{4\pi G}{c^2}\rho u^i = 4\pi K^{-1}\rho u^i \tag{9}$$

With the following definitions of $\boldsymbol{g}$ (gravity field) and $\boldsymbol{k}$ (gravitic field), those relations can be obtained from the following equations (also called gravitomagnetism) with the differential operators "$\boldsymbol{rot} = \nabla\wedge$", "$\boldsymbol{grad} = \nabla$" and "$div = \nabla\cdot$":

$$\begin{aligned} &\boldsymbol{g} = -\boldsymbol{grad}\,\varphi;\ \boldsymbol{k} = \boldsymbol{rot}\,\boldsymbol{H} \\ &\boldsymbol{rot}\,\boldsymbol{g} = 0;\ div\,\boldsymbol{k} = 0 \\ &div\,\boldsymbol{g} = -4\pi G\rho;\ \boldsymbol{rot}\,\boldsymbol{k} = -4\pi K^{-1}\boldsymbol{j}_p \end{aligned} \tag{10}$$

With the Equations (2), one has:

$$h^{00} = h^{11} = h^{22} = h^{33} = \frac{2\varphi}{c^2};\ h^{0i} = \frac{4H^i}{c};\ h^{ij} = 0 \tag{11}$$

The equations of geodesics in the linear approximation give:

$$\frac{\mathrm{d}^2 x^i}{\mathrm{d}t^2} \sim -\frac{1}{2}c^2\delta^{ij}\partial_j h_{00} - c\delta^{ik}\left(\partial_k h_{0j} - \partial_j h_{0k}\right)v^j \tag{12}$$

It then leads to the movement equations:

$$\frac{\mathrm{d}^2 \boldsymbol{x}}{\mathrm{d}t^2} \sim -\boldsymbol{grad}\varphi + 4\boldsymbol{v} \wedge \left(\boldsymbol{rot}\,\boldsymbol{H}\right) = \boldsymbol{g} + 4\boldsymbol{v} \wedge \boldsymbol{k} \tag{13}$$

Remark: All previous relations can be retrieved starting with the parameterized post-Newtonian (PPN) formalism and with the traditional gravitomagnetic field $\boldsymbol{B}_g$. From [7] one has:

$$g_{0i} = -\frac{1}{2}\left(4\gamma + 4 + \alpha_1\right)V_i;\ V_i\left(\boldsymbol{x}\right) = \frac{G}{c^2}\int \frac{\rho\left(\boldsymbol{y}\right)u_i\left(\boldsymbol{y}\right)}{\left|\boldsymbol{x} - \boldsymbol{y}\right|}\mathrm{d}^3\boldsymbol{y} \tag{14}$$

The traditional gravitomagnetic field and its acceleration contribution are:

$$\boldsymbol{B}_g = \nabla \wedge \left(g_{0i}\boldsymbol{e}^i\right);\ \boldsymbol{a}_g = \boldsymbol{v} \wedge \boldsymbol{B}_g \tag{15}$$

And in the case of GR (that is our case):

$$\gamma = 1;\ \alpha_1 = 0 \tag{16}$$

It then gives:

$$g_{0i} = -4V_i;\ \boldsymbol{B}_g = \nabla \wedge \left(-4V_i\boldsymbol{e}^i\right) \tag{17}$$

And with our definition:

$$H_i = -\delta_{ij}H^j = \frac{G}{c^2}\int \frac{\rho\left(\boldsymbol{y}\right)\delta_{ij}u^j\left(\boldsymbol{y}\right)}{\left|\boldsymbol{x} - \boldsymbol{y}\right|}\mathrm{d}^3\boldsymbol{y} = V_i\left(\boldsymbol{x}\right) \tag{18}$$

One then has:

$$g_{0i} = -4H_i;\ \boldsymbol{B}_g = \nabla \wedge \left(-4H_i\boldsymbol{e}^i\right) = \nabla \wedge \left(4\delta_{ij}H^j\boldsymbol{e}^i\right) = 4\nabla \wedge \boldsymbol{H} \tag{19}$$

$$\boldsymbol{B}_g = 4\boldsymbol{rot}\,\boldsymbol{H}$$

With the following definition of gravitic field:

$$\boldsymbol{k} = \frac{\boldsymbol{B}_g}{4} \tag{20}$$

One then retrieves our previous relations:

$$\boldsymbol{k} = \boldsymbol{rot}\,\boldsymbol{H};\ \boldsymbol{a}_g = \boldsymbol{v} \wedge \boldsymbol{B}_g = 4\boldsymbol{v} \wedge \boldsymbol{k} \tag{21}$$

The interest of our notation ($\boldsymbol{k}$ instead of $\boldsymbol{B}_g$) is that the field equations are strictly equivalent to Maxwell idealization, in particular the speed of the gravitational wave obtained from these equations is the light celerity with $c^2 = GK$ just like in EM with $c^2 = 1/\mu_0\varepsilon_0$. Only the movement equations are different with the factor "4". But of course, all the results of our study can be obtained in the traditional notation of gravitomagnetism with the relation $\boldsymbol{k} = \frac{\boldsymbol{B}_g}{4}$.

2.2. From Linearized General Relativity to DM

In the classical approximation ($\|\boldsymbol{v}\| \ll c$), the linearized general relativity gives the following movement equations from (13) with m_i the inertial mass and m_p the gravitational mass:

$$m_i \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = m_p \left[\boldsymbol{g} + 4\boldsymbol{v} \wedge \boldsymbol{k} \right] \tag{22}$$

The traditional computation of rotation speeds of galaxies consists in obtaining the force equilibrium from the three following components: the disk, the bugle and the halo of dark matter. More precisely, one has [8]:

$$\frac{v^2(r)}{r} = \frac{\partial \varphi(r)}{\partial r} \text{ with } \varphi = \varphi_{disk} + \varphi_{bulge} + \varphi_{halo} \tag{23}$$

Then the total speed squared can be written as the sum of squares of each of the three speed components:

$$\begin{aligned} v^2(r) &= r\left(\frac{\partial \varphi_{disk}(r)}{\partial r}\right) + r\left(\frac{\partial \varphi_{bulge}(r)}{\partial r}\right) + r\left(\frac{\partial \varphi_{halo}(r)}{\partial r}\right) \\ &= v_{disk}^2(r) + v_{bulge}^2(r) + v_{halo}^2(r) \end{aligned} \tag{24}$$

Disk and bulge components are obtained from gravity field. They are not modified in our solution. So, our goal is now to obtain only the traditional dark matter halo component from the linearized general relativity. According to this idealization, the force due to the gravitic field $\boldsymbol{k}$ takes the following form $\|\boldsymbol{F}_k\| = m_p 4\|\boldsymbol{v} \wedge \boldsymbol{k}\|$ and it corresponds to previous term $m_p \frac{\partial \varphi_{halo}(r)}{\partial r} = \|\boldsymbol{F}_k\|$. As explained in [6], the natural evolution to the equilibrium state justifies that one assumes the approximation $\boldsymbol{v} \perp \boldsymbol{k}$. This assumption is important because it leads to several important predictions. In particular, the motion of dwarf satellite galaxies of a host should be roughly in a plane ($\perp \boldsymbol{k}$). It then gives the following equation:

$$\begin{aligned} \frac{v^2(r)}{r} &= \frac{\partial \varphi_{disk}(r)}{\partial r} + \frac{\partial \varphi_{bulge}(r)}{\partial r} + 4k(r)v(r) \\ &= \frac{v_{disk}^2(r)}{r} + \frac{v_{bulge}^2(r)}{r} + 4k(r)v(r) \end{aligned} \tag{25}$$

Our idealization means that:

$$v_{halo}^2(r) = v^2(r) - v_{disk}^2(r) - v_{bulge}^2(r) = 4rk(r)v(r) \tag{26}$$

The equation of dark matter (gravitic field in our explanation) is then:

$$v_{halo}(r) = 2\left(rk(r)v(r)\right)^{1/2} \tag{27}$$

This equation gives us the curve of rotation speeds of the galaxies as we wanted. Because we know the curves of speeds that one wishes to have for DM component, one can then deduce the curve of the gravitic field $\boldsymbol{k}$ inside the galaxy:

$$k(r) = \frac{v_{halo}^2(r)}{4rv(r)} \tag{28}$$

2.3. Dark Matter as the 2nd Component of the Gravitational Field $\boldsymbol{k}$

This solution of DM as the gravitic field has been studied in [6] for 16 galaxies (Table 1). It shows that this solution is mathematically possible but with two physical mandatory unexpected behavior for $k(r)$. First, the curve of gravitic field $k(r)$ becomes necessarily flat at the end of the galaxies. For such a field (similar mathematically to a magnetic field in EM) it is only possible if the galaxies are immersed in a uniform graviticfield $\boldsymbol{k}_0$. Second, the value of this field for these 16 galaxies is in the interval:

$$10^{-16.62}\,\mathrm{s}^{-1} < \left\|\boldsymbol{k}_0\right\| < 10^{-16.3}\,\mathrm{s}^{-1} \tag{29}$$

At this step, it is important to analyze the situation because we have a solution that mathematically works and furthermore this theoretical solution is already more comfortable, acceptable conceptually than the solutions of DM with exotic matter or exotic theoretical framework. Indeed, our entire explanation uses only a native effect of GR (no need of new concepts). It is even more obvious in logical term for several reasons. First, with an exotic matter, one needs to be immersed in this exotic matter with quantities that cannot be neglected at the scale of the galaxies. Nevertheless, at our scale no need to involve this component (it can be oddly negligible). In our solution, the very low value of $\left\|\boldsymbol{k}_0\right\| \sim 10^{-16.5}\,\mathrm{s}^{-1}$ explains why DM cannot be detected. As demonstrated in [9], this value is two orders of magnitude inferior to the Lense-Thirring effect of the Earth. The Lense-Thirring effect of the Earth has been observed (hardly because it is very weak) in Probe B experiment [10] and the effect of "our $\left\|\boldsymbol{k}_0\right\|$" is inferior to the sensitivity of this experiment, explaining that DM is negligible at our scale. And because of its mathematical expression (27), in our solution DM become noticeable only at large "r" (at least size of galaxies) or/and at large "v". Second, the behavior of a matter to explain DM is necessarily qualified of "exotic" because it generates only gravitational effects. It is insensitive to EM. In our solution, this is naturally explained because $\left\|\boldsymbol{k}_0\right\|$ is the 2nd component of the gravitational field. It is only a gravitational effect just like the Newtonian field (the 1st component of the gravitational field).

However, at this step, our solution presents some physical difficulties, but just like the other DM solutions and even less complicated to solve as we are going to see it. There are concretely two physical difficulties. How can we explain a uniform field $\boldsymbol{k}_0$ at the scale of galaxies? And how can the value of $\boldsymbol{k}_0$ be generated? For the first question, the solution can be found in EM. Because the gravitic field $\boldsymbol{k}_0$ is mathematically similar to the magnetic field in EM, it can have the same behavior. And precisely, the phenomenon of magnetization (magnet, ferromagnetic material…) can generate a uniform field at large scale. The spin of the atoms (at very small scale) generate a magnetization at the scale of the material (at our scale) because the field of each individual atoms can mutualize to generate an effect at large scale. A uniform magnetic field is then immersed the whole atoms of the material. By this way, our solution is then

Table 1. Distance r_0 to the center of the galaxy where the internal gravitic field $\frac{K_1}{r^2}$ generated by the galaxy becomes equivalent to the external gravitic field k_0 generated by the galaxies' cluster. k_0 dominates for $r > r_0$.

	K_1	k_0	$r_0\left[\frac{K_1}{r^2} \sim k_0\right]$	$r_0\,[\text{kpc}]$
NGC 5055	$10^{24.60}$	$10^{-16.62}$	$10^{20.61}$	13
NGC 4258	$10^{24.85}$	$10^{-16.54}$	$10^{20.695}$	16
NGC 5033	$102^{4.76}$	$10^{-16.54}$	$10^{20.65}$	15
NGC 2841	$10^{24.85}$	$10^{-16.33}$	$10^{20.59}$	13
NGC 3198	$10^{24.90}$	$10^{-16.55}$	$10^{20.725}$	18
NGC 7331	$10^{24.18}$	$10^{-16.30}$	$10^{20.24}$	6
NGC 2903	$10^{24.71}$	$10^{-16.30}$	$10^{20.505}$	11
NGC 3031	$10^{24.15}$	$10^{-16.57}$	$10^{20.36}$	8
NGC 2403	$10^{24.59}$	$10^{-16.39}$	$10^{20.49}$	10
NGC 247	$10^{24.30}$	$10^{-16.30}$	$10^{20.3}$	7
NGC 4236	$10^{24.00}$	$10^{-16.34}$	$10^{20.17}$	5
NGC 4736	$10^{24.54}$	$10^{-16.30}$	$10^{20.42}$	9
NGC 300	$10^{24.27}$	$10^{-16.31}$	$10^{20.29}$	6
NGC 2259	$10^{24.20}$	$10^{-16.30}$	$10^{20.25}$	6
NGC 3109	$10^{24.00}$	$10^{-16.58}$	$10^{20.29}$	6
NGC 224	$10^{24.00}$	$10^{-16.50}$	$10^{20.25}$	6

physically possible but it remains to find the astrophysical structure that could be this source (the equivalent of atomic spins in EM). Thanks to this mutualization, in [6] it has been showed that a likely candidate would be the cluster of galaxies. In the next section, we follow this way and answer concretely the second question by obtaining the expected order of magnitude of $\|\boldsymbol{k}_0\|$.

This previous explanation for $\boldsymbol{k}_0$ leads to two important predictions that could test this solution. If the source of $\boldsymbol{k}_0$ is effectively the cluster of galaxies, it implies that, for isolated galaxies inside a cluster, their equatorial plane should statistically be roughly parallel. If the mutualization is effectively in action, the previous equatorial plane (of isolated galaxies) between two neighboring clusters should also statistically be roughly parallel, but certainly less parallel (greater dispersion) that inside one cluster. The galaxies must be isolated to avoid local interactions that would erase the weak effect of $\boldsymbol{k}_0$. Isolated galaxies could perhaps be replaced by dwarf satellite galaxies.

3. Gravitic Field of Clusters: Source of Dark Matter

3.1. Expression of the Gravitic Field

The equations of the motion for the spin four-vector S_μ of a spherical symmetric body in rotation have been studied in several papers. In GR, it leads to a precession of S_μ. It can be deduced from the equations seen in Section 2.1. From [10], one can write the following equations (in 3-vector notation and in PPN formalism):

$$\dot{\boldsymbol{S}} = \left[\left(\gamma + \frac{\alpha}{2}\right)\frac{1}{c^2}\left(\boldsymbol{grad}\varphi \wedge \boldsymbol{v}\right) + \frac{1}{4}\left(\gamma + \alpha\right)\boldsymbol{rot}\,\boldsymbol{h}\right] \wedge \boldsymbol{S} \tag{30}$$

Which lead to define a geodetic vector field $\boldsymbol{\Omega}_G$ and a "gravito-magnetic" (frame-dragging) vector field $\boldsymbol{\Omega}_{LT}$ (the Lense-Thirring effect):

$$\boldsymbol{\Omega}_G = \left(\gamma + \frac{\alpha}{2}\right)\frac{1}{c^2}\left(\boldsymbol{grad}\varphi \wedge \boldsymbol{v}\right);\ \boldsymbol{\Omega}_{LT} = \frac{1}{4}\left(\gamma + \alpha\right)\boldsymbol{rot}\,\boldsymbol{h} \tag{31}$$

These expressions use the PPN formalism. For GR, $\alpha = 1$ and as seen before (16), $\gamma = 1$, it leads to:

$$\boldsymbol{\Omega}_G = \frac{3}{2c^2}\boldsymbol{grad}\varphi \wedge \boldsymbol{v};\ \boldsymbol{\Omega}_{LT} = \frac{1}{2}\boldsymbol{rot}\,\boldsymbol{h} \tag{32}$$

In our notation (20):

$$\boldsymbol{H} = \frac{\boldsymbol{h}}{4};\ \boldsymbol{k} = \boldsymbol{rot}\,\boldsymbol{H} \tag{33}$$

One then has

$$\boldsymbol{\Omega}_{LT} = 2\boldsymbol{k} \tag{34}$$

This Lense-Thirring effect can explicitly be written [4] with $\boldsymbol{J}$ the angular momentum:

$$\boldsymbol{\Omega}_{LT} = \frac{G}{c^2}\left(\frac{\boldsymbol{J}}{r^3} - \frac{3\boldsymbol{r}}{r^5}\left(\boldsymbol{r}\cdot\boldsymbol{J}\right)\right) \tag{35}$$

One deduces for our gravitic field:

$$\boldsymbol{k} = \frac{1}{2}\frac{G}{c^2}\left(\frac{\boldsymbol{J}}{r^3} - \frac{3\boldsymbol{r}}{r^5}\left(\boldsymbol{r}\cdot\boldsymbol{J}\right)\right) \tag{36}$$

If we are along the $\boldsymbol{J}$ axis, we then obtain:

$$\|\boldsymbol{k}\| = \left\|\frac{G}{c^2}\left(\frac{\boldsymbol{J}}{r^3}\right)\right\| \tag{37}$$

And with $\boldsymbol{J} = m\boldsymbol{r} \wedge \boldsymbol{v}$, one finally obtains:

$$\|\boldsymbol{k}\| = \frac{G}{c^2}\left(\frac{mv}{r^2}\right) \tag{38}$$

One can note that this relation is equivalent to the Biot-Savart law in EM (equation describing the magnetic field generated by a constant electric current). It approximates the field for a point source and far from the source.

3.2. Value of the Gravitic Field in a Cluster

As explained before, the uniformity of the gravitic field at the scale of the galaxies is only possible thanks to the mutualization of the neighboring clusters. If we assume that we have 6 neighboring clusters, 2 by spatial dimension as in

[6], with (29) one can deduce than the gravitic field of one cluster $\boldsymbol{k}_{0_{Cl}}(r)$ could be:

$$10^{-17.4} < \left\|\boldsymbol{k}_{0_{Cl}}\right\| < 10^{-17.1} \text{ avec } \boldsymbol{k}_0 = \sum_{\text{Neighb. Clusters}} \boldsymbol{k}_{0_{Cl}}(r) \tag{39}$$

It should be understood here that $\left\|\boldsymbol{k}_0\right\|$ is relatively uniform (relatively independent of r) thanks to the mutualization of the different gravitic fields of the neighboring clusters (as for the spins within a ferromagnetic material). But, individually each gravitic field of a cluster is not uniform but decreases with$r$ and even precisely according to r^2, due to the Poisson Equations (9) in agreement with (38).

At the center of the clusters, very high temperatures are reached, the particles are relativistic and can have speeds close to that of light ($v \sim c$). From (38), we then have:

$$\left\|\boldsymbol{k}_{0_{Cl}}\right\| = \frac{G}{c}\left(\frac{m}{r^2}\right) \tag{40}$$

We know that at the center of the clusters, one can find some of the most massive galaxies in the Universe, named the Brightest Cluster Galaxies (BCG). Let's take the example of M87, for which one expects [11] a mass at least of $M_{M87} \sim 10^{14} M_{\odot} \sim 2\times10^{44}\ \text{kg}$. At $r \sim 200\ \text{kpc} \sim 10^{21.79}\ \text{m}$, one obtains:

$$\left\|\boldsymbol{k}_{0_{Cl}}\right\| \sim \frac{6\times10^{-11}}{3\times10^{8}} 2\times10^{44} \times \frac{1}{\left(10^{21.79}\right)^2} = 10^{-17.97}\ \text{s}^{-1} \tag{41}$$

This value is in the correct order of magnitude but less than expected. However, as indicated in [11], the mass of the giant elliptical galaxy M87 may be greater than $10^{14} M_{\odot}$. And we have considered the mutualization of 6 neighboring clusters, but it could be also greater. Furthermore, the previous expressions of spin are obtained for a spherical body. A galaxy like the BCG has matter more concentrated than in a sphere, increasing then the value of the gravitic field. Our result could then be a minimum value. For example, a natural sphere packing in 3D space can lead to 12 neighboring spheres (Kepler conjecture), in this case, the interval of $k_{0_{Cl}}$ for one cluster would be $10^{-17.7} < \left\|\boldsymbol{k}_{0_{Cl}}\right\| < 10^{-17.4}$ instead of (39). At $r \sim 300\ \text{kpc}$, one could then obtain $\left\|\boldsymbol{k}_{0_{Cl}}\right\| \sim 10^{-17.7}\ \text{s}^{-1}$ with $M_{M87} \sim 8\times10^{44}\ \text{kg}$. The only BCG could then explain, on his own, the major part of the gravitic field of the cluster explaining DM.

4. Discussion

Obtaining the value of $k_{0_{Cl}}$ is very interesting because it shows that this solution works not only mathematically but also physically. The gravitic field of the clusters can be this hypothetical DM and in particular it could be generated largely by the BCG (the core of the clusters). It certainly remains to adjust more finely all the parameters and the equations, but with this solution we have never been so close to an explanation of the DM, moreover by repositioning DM within the framework of our known physical theories. DM is potentially another splendid success of the GR.

Compared to the Newtonian theory, the GR differs by at least five experimental tests, five major effects which are unexplainable in the Newtonian framework: the curvature of the light rays, the advance of the perihelion of Mercury, the red shift, the gravitational waves and the Lense-Thirring effect. This last effect is generally not mentioned because it is negligible, but it is an effect which has been measured and which does not exist in the Newtonian theory. We can note that the first four major effects (curvature, advance of the perihelion, red shift and gravitational wave) all have remarkable, measurable effects and that they are in a way essential pillars of the GR without which modern astrophysics would not exist. Only the Lense-Thirring effect, however exclusively resulting from the GR, has to date no major implication. Its effect seems surprisingly always negligible. With this solution, the "Lense-Thirring" effect and more precisely the 2nd component of gravitation would become at the center of gravitational physics at scales larger than that of galaxies.

The interest in using LGR (that is justified because at the ends of the galaxies. The gravitational field is very weak and the uniform k_0 is also very weak) is that it allows us to easily understand physically why this solution works. Indeed, the equations of GR in their linearized version highlight an aspect that is not visible at first glance by bringing the GR closer to the EM. The linearized form makes it possible to see that the GR completes Newton's gravitational theory in a form similar to Maxwell's theory by adding a 2nd component of gravitation, similar to the magnetic field in EM, the Newtonian gravitational force being similar to the electric field of the EM. In this way, it is natural to implement the same procedures and solutions. If it works in EM, it naturally works in GR. The question is then not to know if the solution is physically possible (because it is necessarily possible, contrary to the other DM solutions that are not necessarily physically possible) but to know if this solution has been chosen by nature. And for that, we need to determine some consequences that would reveal the specificities of this solution. Thanks to their similarity with EM, it is easy to determine some of them. Let's take the elements that build our solution:

1) Use of the 2nd gravitational field $\boldsymbol{k}_0$ similar to the 2nd electromagnetic field $\boldsymbol{B}$;

2) Ability to generate agravitic field $\boldsymbol{k}_0$ at large scale thanks to the galaxy clusters similar to the ability to generate a magnetic field $\boldsymbol{B}$ at the scale of a ferromagnetic material thanks to the atomic spins. This possibility is explained by the mathematical expression force due to these fields $\boldsymbol{F}_B = \boldsymbol{v} \wedge \boldsymbol{B}$ and $\boldsymbol{F}_k = \boldsymbol{v} \wedge \boldsymbol{k}$ (which is not a central force unlike the electric force or the gravity force);

3) Generation of a field $k_{0_{Cl}}$ greater than expected by the existence of an internal structure to the cluster (BCG) that concentrate very specific physical properties, just like magnetic field is generated by many astrophysical objects (planet, stars…) because of an internal structure despite that the object is glo-

bally neutral.

Thanks to what we observe in EM with a uniform magnetic field, we can predict:

For the 1st point, the plane motion of dwarf satellite galaxies (like particles in a magnetic field) [8] [12];

For the 2nd point, the statistical tendency to parallelism of the equatorial planes of isolated galaxies (or dwarf satellite galaxies) in a cluster and even between neighboring clusters [6];

For the 3rd point, the maximum quantities of dark matter at the position of the BCGs, at the center of the galaxy clusters;

For this last point, one can add that the gravitic field could be directly detected in the measurement of the Lense-Thirring effect of Earth with experiments more sensitive than Probe B [9]. And indirectly because the gravitic field of GR would be greater than expected for large astrophysical structures, the stars SO-2 at the galactic center could be influenced by a gravitic field of the central super massive black hole greater than expected generating a modification in the speed at their closest distance and a spatial distribution different than expected [13].

This solution has several positive results to its credit. It obviously makes it possible to account for the rotation speeds of galaxies (because it was built for this purpose) [6]. Several observations have shown planar motions of dwarf satellite galaxies around a few host galaxies [14] [15]. It also makes it possible to account for the dynamics of the WLM galaxy and above all to obtain the density of the gaseous intergalactic medium and interstellar gaseous medium thanks to the value of $\|\boldsymbol{k}_0\|$ [16]. We have seen that this solution explains the absence of DM at our scale because the magnitude of this field is effectively negligible at our scale. The measuring instruments are not yet sensitive enough. Inversely, this weakness and the mathematical expression of the associated force explains that DM become detectable only for very large size (beyond the galaxies' size) and for large speeds. It also explains that the gravitic field (the DM) fills the Universe at the scale of the galaxies. The mechanism of mutualization of the gravitic fields of the clusters is necessary to explain the flat curve of $\|\boldsymbol{k}_0\|$ obtained from the velocity curves of the galaxies. But this mutualization has the consequence of necessarily applying to the entire Universe since the clusters finally define a covering (in the geometric sense) of the Universe. The gravitic field resulting from the clusters then covers the universe on a large scale. In this explanation, it is still possible to have galaxies without dark matter. But in this case, these galaxies must be in isolated positions, far from the clusters or in areas of gravitic field inversion (where the field would cancel out). These galaxies should be the exception, not the rule. This solution also explain naturally why DM is unsensitive to EM and sensitive only to gravity, because $\boldsymbol{k}_0$ is precisely a gravitational field.

5. Conclusions

A possible explanation of DM (without exotic matter and in the frame of GR) is the gravitic field (2^{nd} component of the gravitational field of GR) of the clusters. The main result of our study is to propose the BCG as the main source of this field obtaining the good order of magnitude of the value to explain entirely the dark matter component. Thus, the cluster would be composed of an internal structure (the BCGs) that would form a generator core of the gravitic field of the cluster just like electrically neutral astrophysical bodies can be composed of an electrically charged core generating a magnetic field, a field greater than expected because without this internal structure there wouldn't be any. By considering the cluster in its globality, we would therefore tend to underestimate the value of the gravitic field.

The nature of this hypothesis is to be compared with the hypotheses of the other most popular dark matter solutions. For the MOND solution change of paradigm of the theory is required, in our case the gravitic field is a native component of the field of GR. For the most widespread hypothesis of dark matter, an "ad hoc" creation of particles (still not detected) is required, in our case the known ordinary matter is sufficient to explain DM. Furthermore, DM would have extraordinarily strange behavior: insensitive to EM, ubiquitous in the universe but absent at our scale. In our case, gravitic field is naturally insensitive to EM and both its value and its mathematical expression explain why it is undetectable at our scale but ubiquitous at a larger scale than galaxies.

This solution makes some predictions: the plane motion of dwarf satellite galaxies (like particles in a uniform magnetic field), the statistical tendency to parallelism of the equatorial planes of isolated galaxies (or dwarf satellite galaxies) in a cluster and even between neighboring clusters, the dark matter must be maximum at the center of the galaxy clusters (in the BCGs).

One can finish by specifying that this solution is a solution exclusively resulting from the GR because this component (gravitic field) is not known from the theory of Newton. DM would be a demonstration of the power of GR.

Conflicts of Interest

The author declares no conflicts of interest.

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