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# An Evolutionary Game Theoretical Approach to the Teaching-Learning Techniques in the Post-Pandemic Era

**Humberto Muñiz[1], Elvio Accinelli[1], Erika Hernández[2*]**

[1]Universidad Autónoma de San Luis Potosí, San Luis Potosí (UASLP), México
[2]Universidad Politécnica de San Luis Potosí (UPSLP), San Luis Potosí, México
Email: *Erika.Hernandez@upslp.edu.mx

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## Abstract

In this paper, using evolutionary game theory tools, we analyze the evolution of the behaviors of professors, students and schools towards the use of the information and communication technologies (ICTs) in the teaching-learning process in the post-pandemic era, for this purpose we will use the replicator dynamics and we will show that solutions to this dynamical system could indicate the evolution that the teaching-learning processes will follow over time in the post-pandemic world.

## Subject Areas

Economics, Game Theory, Statistics

## Keywords

ICTs, Teaching-Learning Process, COVID-19, Replicator Dynamics, Game Theory

## 1. Introduction

In recent years, the use of digital technology has been increasing in all sectors of the society, especially in education. Increasingly, teaching and learning based on the use of Information and Communication Technologies (ICTs) are recognized as valid method for education and transmission of knowledge, by complementing or reinforcing traditional approaches, see for example [1]. This process was reinforced during the pandemic period in which virtual classes or distance education (first stage) became widespread in most countries [1] [2] [3]. Although, in many cases not with the expected results [4] [5] due to the lack of economic resources and infrastructure on the part of teachers, students and schools [6] [7].

Like many teachers, who had to acquire electronic equipment to continue teaching, students had to make investments in computers and acquisition of software for online classes, but those who did not have the resources to do so or did not adapt to this new methodology left their education aside [7]. Schools by their part had to train their teachers in the use of digital platforms and acquire licenses so that their staff could continue with distance education, which represented an investment not foreseen before pandemic [2] [8].

The first stage of education in times of pandemic made clear the great economic and social disparity of many students, professors and schools [7] [9] [10] [11]. This tried to be corrected with hybrid education, which offered to students the possibility of attending (some days) face-to-face classes and thus closing the educational gap that arose during the online period [12]. The hybrid methodology caused the educational authorities to make large investments to equip classrooms with adequate technology [13], which represented an investment in technology and training little contemplated by the schools in the days before COVID-19.

The accumulation of experiences lived during the pandemic is something that needs to be taken into account in today's educational systems, since with the return to face-to-face activity, the controversy over the use of ICTs as a main part of the teaching-learning process is revived, but now with an accumulated experience that is not alien to any of the protagonists, beyond nuances with greater or lesser intensity, all those involved in the educational task see the need to implement the use of ICTs in classes. Certainly the position towards the use of ICTs as part of the learning process depends on different factors such as economic, social or technological preferences, since these are the ones that mainly facilitate the access to technology [8].

Unlike works such as [1] [3] [14] where the challenges of the use of ICTs in the education in the pre- and post-pandemic times are addressed, and the role that teachers and students play in their implementation. In this paper we try to explain the factors that influence the evolution of the use of ICTs in the teaching-learning process in the post-pandemic world. The main novelty of this work is the implementation of evolutionary game theory to describe the trajectories that these techniques will follow over time and the role that ICTs will play. For this we will propose an evolutionary game theoretical model. Three populations, students, professors and educational authorities, will gather the participants in a game in which each participant must choose between two behaviors: using and promoting teaching-learning methods based on the use of ICTs or choosing traditional techniques in which the use of this is limited to certain activities, such as the delivery of homework. Although there is a large number of works that use evolutionary games to understand the learning and imitation processes, or the reproduction of the most successful behaviors (see for example [15]), as far as the authors of the work know, there is not literature regarding the analysis of the choice of teaching-learning methods based on evolutionary models that consider the consequences of the COVID-19 pandemic, which is the main objective of this work. To explain the evolution of teaching-learning methods over time and why one or the other tends to consolidate will use the replicator dynamics (see [16] and [17] for different applications).

The article is organized as follows: In section 0 we present our model about the behaviour towards the use of ICTs by professors, students and authorities in the post-pandemic era. In section 0, we present the payoff matrix of the game and in section 3 we characterize its possible Nash equilibria. To analyze the evolution of the teaching-learning techniques, we introduce the replicator dynamic in section 3 and we carry out the Liapunov-stability analysis of the stationary state of this system. Section 2 is dedicated to explaining the emergence of heteroclinical cycles. Finally in section 3 we give the final considerations and analyze future lines of work.

## 2. The Model

We introduce below our teaching-learning ($U\&T$) model in which professors, students and authorities must choose between to follow traditional education, similar to the one used in the pre-pandemic times, or educational methodologies based on the use of the ICTs, in this sense using the skills acquire during the COVID-19 pandemic. In game theoretical terms, we will present a three players game each of who has two different (pure) strategies (or behaviors), basically to be in favor or against the use of ICTs in the teaching-learning process.

### Professor, Students, Educational Authorities and Their Behaviors

In accordance with the literature of game theory, professors, students and educational authorities will follow the strategy whose associated expected value is greater. Hence we describe below the participants of the game and their behaviors.

*Professor* who, as a result of the global health crisis caused by the COVID-19 that provoke a massive use of the ICTs in the traditional teaching process [12], are now trained in its use [3] [14], can mainly take two opposite positions: to use teaching methods that favor the use of electrical devices and digital platforms, or returning to teaching processes in which the use of this type of application is scarce, perhaps only for delivery of tasks. We will say that professors using the first teaching strategy are in favor of the use of ICTs, we will denote this behaviour by $P_U$, while we will call to professors who follow the second strategy traditional professors $P_T$.

The use of ICTs in the teaching process seems to be a growing trend in educational systems [18] [19], so taking advantage of training acquired during the global health crisis, professors, upon returning to the classroom of classes in person, could implement the use of technology as part of their teaching work. However, implementing this type of teaching techniques will depend to a large extent on the levels of demand by the educational authorities, the tastes of professors towards the use of technology or the level of training achieved during this time.

*Students*, who are increasingly familiar with the use of technology can take two different positions towards its implementation as part of their learning process: 1) have a predilection for the use of technology to carry out online activities such as reviewing extra class material, academic advice, carrying out activities such as homework assignments, taking exams, or 2) prefer the limited use of these technologies, that is, prefer that the use of ICTs within the process of a subject be as little as possible. We will call students whose preferences point to the use of ICTs digital student $S_U$, while those who prefer the minimum use of these will call traditional students $S_T$.

These positions, beyond showing preferences for the use of ICTs in the classroom, can correspond to the different social and economic realities, because as we mentioned above, today's young people live immersed in a technological world [20], so we can think that young people are empirically trained in the use of technologies, however using them as a complementary means to classrooms can become unfeasible for a large majority, due to lack of financial resources (see [2] [7]). Beyond the economic factors that can help to the technological development of students, the role played by educational authorities is essential to achieve a better adaptation of students in the implementation of these. Schools better equipped with technological infrastructure and personnel trained in its use are of the utmost importance to help students, including those whose economic problems, to get a better develop in a technological learning environment.

As a result of the COVID-19 pandemic, the different *school authorities* worldwide made great efforts to continue education, as a result of these efforts large investments in technological infrastructure were made to replace the use of the blackboard and chalk during the pandemic, so the use of this infrastructure in the post-pandemic environment could be desirable. However, its continuous use will bring about natural wear and tear, which is why an investment must continue to be made in repair and maintenance, which, with the return to face-to-face classes, could be considered unnecessary causing educational authorities to implement education policies that lead back to traditional educational models. By $A_U$ we will denote the behavior of the educational authorities in favor of the use of ICTs in classrooms and by $A_T$ the contrary behavior.

As usual in evolutionary game theory, a mixed strategy for each player correspond with distribution or percentage of the total population who follows certain behavior (pure strategy). Since $P_U$ denotes the professors' choice of using ICTs as a part of their teaching process, then by $x_{T_{P_U}}$ we denote the proportion of professors who following strategy $P_U$, while by $1-x_{T_{P_U}}$ we denote the portion of professors who continues teaching using traditional methodologies. In the same way $x_{S_U}$ denotes the portion of students who decide to continue using the study techniques acquired during the pandemic. Consequently $x_{S_T}=1-x_{S_U}$ is the portion of students using the studying techniques they have before the COVID-19 pandemic. By $x_{A_U}$ we denote the probability that school authorities will act by encouraging the use ICTs and by $1-x_{A_U}$ the probability that this does not occurs.

## 3. Payoffs Matrices

In this section we present the payoff matrices of professors, students and scholar authorities.

### 3.1. Education Authorities

Schools located in urban areas tend to be better equipped, in terms of technological infrastructure than those located in rural units. In most cases, this is due to a greater investment by the government in urban areas than that made in rural [9] [21]. This fact suggest a positive relation between investment and infrastructure development of schools. Following this lines, in this paper we assume that schools in favor of using of ICTs have greater technological infrastructure than the use that prefer traditional teaching-learning process.

Even when we are a not considering the government to be part of our model, to give more realism to our model, we will consider its exogenous participation in the game. The income of public sector schools not only depends on the registration fees collected from students, but also on the budget allocated to education by government, which we will assume to be greater for schools in favor of use of ICTs and lower for schools with contrary position. By $G_I$ and $G_N$ ($G_I > G_N$) we denote the budget assigned to schools of type $A_U$ and type $A_T$, respectively. By $S$ we denote the basis salary establish by Governmental authorities, which is independent of the preparation of the professors.

By $B$ denotes the bonus or differentiation payment between professors trained in the use of ICTs and those who are not, schools that support the use of these technologies in class offer a higher payment to professors who have training than to those who do not.

**Table 1** shows the level of utility of the school authorities according to their preferences, $A_U$ or $A_T$, depending on the behavior of their student and professors.

The educational authorities in favor of the use of ICTs impose an $F_I$ tuition fee, higher than $F_N$ the imposed by the authorities of institutions with a contrary position ($F_I > F_N$), we will assume that the fee imposed by each kind of school is the same for both types of students, since this fee only reflects the conditions that the school has.

**Table 1.** Payoff table of authorities.

| | $P_U, S_U$ | $P_U, S_T$ | $P_T, S_U$ | $P_T, S_T$ |
|---|---|---|---|---|
| $A_U$ | $G_I + F_I - I_I$ $-(S+B)$ | $G_I + F_I - I_I$ $-(S+B)$ | $G_I + F_I - I_I$ $-S$ | $G_I + F_I - I_I$ $-S$ |
| $A_T$ | $G_N + F_N - I_N$ $-S$ | $G_N + F_N - I_N$ $-S$ | $G_N + F_N - I_N$ $-S$ | $G_N + F_N - I_N$ $-S$ |

$I_I$ denotes the investment in infrastructure and training offered by schools that favor the use ICTs, this investment is independent of the type of professors and students. So, $I_I$ captures the expenses in technological infrastructure and maintenance, and in addition to costs training of their teaching staff and students. $I_N$ represents the investment in maintenance or creation of spaces made by educational authorities, whose position does not favor the use of ICTs. As it is natural we assume $I_I \geq I_N$.

For exposition purposes we summarize the inequalities presented in the following item

$$G_I > G_N,\ F_N < F_I \text{ and } I_I \geq I_N. \tag{1}$$

Now we consider two scenarios related to schools' investments.

E1) High investment in technological infrastructure: The first scenario is summarized by the following inequality:

$$G_N + F_N - I_N > G_I + F_I - I_I > G_I + F_I - I_I - B > 0. \tag{2}$$

This means that even when the resources collected by the schools with the greatest technological infrastructure are higher than the resources collected by traditional schools, the investment in maintenance and creation of technological spaces, by the first is too high in comparison with the investment made in traditional schools in such a way that the benefits (income minus expenses) of the first is less than that of the schools with less infrastructure.

The situation describe by Equation (2) can occur after long periods in which schools with high technological infrastructure, put aside the investment in updating equipment and teacher training or when new high-quality and modern technologies break into the technological market and make necessary to change the equipment available in the classrooms to continue keeping at the technological forefront. In game theory terms (2) implies that traditional education is a strictly dominant strategy for the school.

E2) Infrastructure investment and bonuses: Consider the second scenario, defined by the inequality:

$$G_I + F_I - I_I > G_N + F_N - I_N > 0 \tag{3}$$

(3) summarize the fact that net benefit, of schools in favor the use of digital technology in classes before the payment of bonuses to $P_U$ professors, is greater than the benefit of schools that follow traditional methods. Once we include bonuses, this brings two completely different possible scenarios.

i) Low bonuses for professors: To this case corresponds the inequality:

$$G_I + F_I - I_I > G_I + F_I - I_I - B > G_N + F_N - I_N > 0 \tag{4}$$

This occurs when the investment in technological infrastructure and the payment of bonuses to professors is very low compared to the investment in traditional infrastructure. Perhaps schools have reached the maximum technological development and they should not invest more in maintaining and updating its technological infrastructure and professors are offered a very small bonus, while schools with a traditional trend invest a higher percentage of their income in infrastructure compared to the investment made by schools with greater infrastructure. Making the strategy of supporting the use of ICTs in class is a strictly dominant strategy for the authorities.

ii) High bonuses for professors:

$$G_I + F_I - I_I > G_N + F_N - I_N > G_I + F_I - I_I - B > 0 \quad (5)$$

In difference with i), the final benefit of the school in favor of the technology (once the bonus has been paid) is less than the benefit of traditional schools.

Once we have presented the payoff table of school authorities and describe the parameter that define it, we continue with the discussion about the election of school authorities towards the use of ICTs. Hence, according to the VNN theory [22], in presence of uncertain about the possible results of behaviors, rational individuals will choose according to the expected value of their strategies. Therefore, whenever $E(A_U) - E(A_T) > 0$ schools authorities will invest in technological infrastructure to offer easy access to ICTs to its students and professors, while if the opposite inequality holds they will prefer supporting traditional teaching-learning methods and if $E(A_U) - E(A_T) = 0$, schools authorities will be indifferent between the strategies. Now note that

$$E(A_U) - E(A_T) = -Bx_{P_U} + (G_I - G_N) + (F_I - F_N) + (I_N - I_I).$$

Hence, setting

$$B_A = -B \text{ and } D_A = (G_I - G_N) + (F_I - F_N) + (I_N - I_I) \quad (6)$$

we have

$$E(A_U) - E(A_T) = B_A x_{P_U} + D_A. \quad (7)$$

Remark 1 *Since* $B > 0$ *then* $B_A < 0$, *while the sign of* $D_A$ *will depend on the scenario E*1) *or E*2), *whenever inequality* (5) *or* (4) *holds* $D_A$ *will be positive*, *while* $D_A < 0$ *for the case in which* (2) *occurs*.

### 3.2. Professors

Professors will choose the way to teach according to their abilities and preferences for the type of teaching, some professors will prefer traditional techniques, intensive use of blackboard and chalk, and others will prefer the use of digital techniques. The level of satisfaction that professors gets from their work will not only depend on their own tastes, but also on the degree of attention they get from their students and the incentives that the school gives to them. **Table 2** shows the level of satisfaction of professors according to their behavior, $P_U$ or $P_T$, depending on the behavior of the students and the position set by the educational authorities.

As is typical in game theory and economics, the level of satisfaction of the different conditions that individuals face is measured through a utility function, which assigns a subjective value to the income of teachers and in the same way an assessment is assigned to the working conditions in which it is immersed. Hence $u_p(S+B)$ and $u_p(S)$, with

**Table 2.** Payoff table of professors.

| | $S_U, A_U$ | $S_U, A_T$ | $S_T, A_U$ | $S_T, A_T$ |
|---|---|---|---|---|
| $P_U$ | $u_p(S+B)+Sl_p(U,U)$ | $u_p(S)+u_p(I_{ICT})+Sl_p(U,T)$ | $u_p(S+B)+Sl(T,U)$ | $u_p(S)+Sl(T,T)$ |
| $P_T$ | $u_p(S)+sl(U,U)$ | $u_p(S)+sl(U,T)$ | $u_p(S)+sl(T,U)$ | $u_p(S)+sl(T,T)$ |

$$u_p(S+B) > u_p(S) > 0 \tag{8}$$

It denotes the value, utility or satisfaction that professor assign to the salaries paid by the different types of schools, according to their degree of preparation in the use of ICTs.

$Sl(S_a, A_b)$ and $sl(S_a, A_b)$, $a,b \in \{U,T\}$, which to save notation we will write as $Sl(a,b)$ and $sl(a,b)$, denote the subjective value or utility that updated and outdated professors in use of ICTs give to the different realities they face. This value represents the assessment that different professors give to their working conditions, then we assume

$$\begin{gathered} Sl_p(T,T) < Sl_p(T,U), Sl_p(U,T) < Sl_p(U,U) \text{ and} \\ sl_p(U,U) < sl(U,T), sl_p(T,U) < sl_p(T,T) \end{gathered} \tag{9}$$

This means that professors who prefer the use of ICTs in the teaching-learning process have greater satisfaction than those who prefer traditional methodologies, when both educational authorities and students promote their use in school. We will assume

$$\begin{gathered} Sl_p(U,U) > sl_p(U,U),\ Sl_p(U,T) < sl_p(U,T), \text{ and} \\ Sl_p(T,U) > sl_p(T,U), Sl_p(T,T) < sl_p(T,T) \end{gathered} \tag{10}$$

These inequalities reflect the level of satisfaction of professors with the conditions in which their work is carried out, this satisfaction depends on their own inclinations or tastes and experiences with digital methods and on the technological infrastructure in schools and on the preferences on the studying techniques followed by their students.

$I_{ICT}$ is the invest in technology, which professors in favor of its use in classes have to assume, whenever they face schools with lack of technological infrastructure and students prefer learning process in which the technologies are part. Whenever school authorities support the use of ICTs $I_{ICT} = 0$. $u_p(I_{ICT})$ is a measure of the disutility of professor trained for the use of technology in class, when they must assume some expenses ($I_{ICT}$) in technology for their work in class. Hence we assume that $u_p(I_{ICT}) < 0$ whenever $I_{ICT} > 0$ and $u(I_{ICT}) = 0$ for $I_{ICT} = 0$.

As consequences of the assumptions the following inequalities holds:

$$\begin{gathered} u_p(S+B) + Sl_p(U,U) > u_p(S) + sl(U,U), \\ u_p(S) + u_p(I_{ICT}) + Sl_p(U,T) < u_p(S) + sl(U,T), \\ u_p(S) + u_p(I_{ICT}) + Sl_p(U,T) > u_p(S) + sl(T,U) \text{ and} \\ u_p(S) + Sl(T,T) < u_p(S) + sl(T,T). \end{gathered}$$

If $E(P_U)-E(P_T)>0$ professors will choose to be updated and apply pedagogical techniques that involves the use of ICTs. If the opposite inequality holds then they will choose to use traditional teaching techniques, while if $E(P_U)-E(P_T)=0$ then professor will be indifferent between be updated or outdated in the use of ICTs as a part of their teaching performance. In order to save notation, we set

$$\begin{aligned} A_P &= Sl(U,U)-sl(U,U)+sl(U,T)-Sl(U,T)+sl(T,U) \\ &\quad -Sl(T,U)+Sl(T,T)-sl(T,T)-u(I_{ICT}) \\ B_P &= Sl(U,T)-sl(U,T)+sl(T,T)-Sl(T,T)+u(I_{ICT}) \\ C_P &= u(S+B)-u(S)+Sl(T,U)-sl(T,U)+sl(T,T)-Sl(T,T)>0, \\ D_P &= Sl(T,T)-sl(T,T)<0. \end{aligned} \tag{11}$$

Hence from a direct computation we observe that

$$E(P_U)-E(P_T)=A_P x_{S_U} x_{A_U}+B_P x_{S_u}+C_P x_{A_U}+D_P. \tag{12}$$

Inequalities in (11) are a consequence of assumptions along this section. Now we make the following remark, which will be important in the stability analysis of the Nash equilibria made in section 3.

**Remark 2** *Since we assume u to be a decreasing function of investment, we can conclude that for very low investments* (*close to* 0) $I_{ICT}$, $u(I_{ICT})$ *is close enough to* 0 *in such a way that* $B_P>0$.

### 3.3. Students

Table 3 shows the level of satisfaction (or utility) of a student according to his behavior depending on the behavior of the teacher of the class and the preferences of the school in which he participates as a student.

Once again as we did in the case of students, we identify the profile $(P_a,A_b)\in P\times A$, $a,b\in\{U,T\}$ with its subscript. Then $v_{S_U}(a,b)$ and $v_{S_T}(a,b)$, $a,b\in\{U,T\}$, denotes respectively the value or utility than the students give to the different school circumstances they face. To be explicit, we will assume that these utilities are the sum of the level of student satisfaction towards the work carried out by the professors in the classroom and their assessment of the infrastructure available at the school. Hence

$$v_{S_U}(a,b)=w_{S_U}(a)+Sl_{S_U}(b) \text{ and } v_{S_T}(a,b)=w_{S_T}(a)+Sl_{S_T}(b),\ a,b\in\{U,T\}.$$

where $w_{S_U}(a)$ and $w_{S_T}(a)$ denotes respectively, the satisfaction or utility level of students according with their preference to the use of ICTs in class. In the same way $Sl_{S_U}(b)$ denotes the satisfaction of students with preference to the use of ICTs in their learning process when they are in an educational environment of type $b=U$ or $b=T$. As is natural we set

Table 3. Payoff table of students.

| | $P_U, A_U$ | $P_U, A_T$ | $P_T, A_U$ | $P_T, A_T$ |
|---|---|---|---|---|
| $S_U$ | $v_{s_U}(U,U)$ | $v_{s_U}(U,T)$ | $v_{s_U}(T,U)$ | $v_{s_U}(T,T)$ |
| $S_T$ | $v_{s_T}(U,U)$ | $v_{s_T}(U,T)$ | $v_{s_T}(T,U)$ | $v_{s_T}(T,T)$ |

$$w_U(U) > w_T(U) \text{ and } w_T(T) > w_U(T). \tag{13}$$

These inequalities summarize the assessment that different types of students give to the teaching techniques applied by professors. First inequality means that students whose preferences are on the side of the use of ICTs will have greater satisfaction, when professors carry out activities based on the its use, than the utility of students with preferences towards the traditional learning methodologies, while the second denotes that the level of satisfaction of students who prefer traditional techniques is higher than those who do not, when the professors implements traditional teaching techniques. To reflect the satisfaction level of students towards the educational policies of their study centers, we consider the following inequalities

$$\begin{gathered} Sl_{S_U}(U) > Sl_{S_T}(U), Sl_{S_U}(T) < Sl_{S_T}(T), \\ Sl_{S_U}(U) > Sl_{S_U}(T) \text{ and } Sl_{S_T}(T) > Sl_{s_T}(U), \end{gathered} \tag{14}$$

which are not only an assessment of the facilities and technological infrastructure or not of the schools, but are also an assessment of how the different types of educational authorities promote or not the use of ICTs.

The level of satisfaction of the students who face one or another situation, according to their preferences, can be summarized in the following set of inequalities.

$$\begin{gathered} v_{s_U}(U,U) = w_U(S_U) + Sl_U(S_U) > w_T(S_U) + Sl_{S_T}(U) = v_{s_T}(U,U), \\ v_{s_T}(T,T) = w_T(T) + Sl_{S_T}(T) > w_{s_U}(T) + Sl_U(S_T) = v_{s_U}(T,T) \end{gathered}$$

The following scenarios, reflect the perception of students when they face situations where professors prefer the use of ICTs, but the resources are not available in the classroom, or vice versa.

S1) Absence of personnel trained in the use of ICTs—Return to traditional education: In the absence of professors trained in the use of ITCs, students perceive education as traditional, since although the schools have technological resources, these are not available for them, professors prefer to continue with traditional methods of teaching, so the teaching process is carried out in an environment similar to the traditional, which leads students with preferences towards the absence of ICTs in class to have greater satisfaction than those who prefer their use. Hence we assume:

$$u_{S_U}(T,U) < u_{S_T}(T,U) \tag{15}$$

S2) Lack of technological infrastructure a barrier to its implementation: Even with teachers trained in the use of ICTs, the teaching process based on these could fail. Because if the school does not have adequate technology, the lack of technological infrastructure can be perceived as a barrier to its implementation. In this case, students feel closer to a traditional school environment than to a technological one. A fact that can be perceived in the same way by students with preferences towards traditional learning techniques, this can be translated into the following inequality.

$$u_{S_U}(U,T) < u_{S_T}(U,T) \quad (16)$$

This means that the effort of professors to use ICTs is not enough to compensate for the lack of ICTs in schools.

S3) Trained educational personnel: A path towards the use of ICTs. On the contrary, inequality in the other direction could occur when the efforts of professors to use ICTs in class development, even with technological deficiencies in the classroom, are perceived by students as a teaching methodology based on the use of ICTs. In this case we have

$$u_{S_U}(U,T) > u_{s_T}(U,T) \quad (17)$$

Which means that even in the face of the lack of technological infrastructure, the work of professors adapting to these challenges and implementing the use of ICTs as far as possible in classes is enough motivation for students who like their use in their learning process. Hence in any case the orientation of the inequality is related to the perception of the students' reality.

Students will base their learning process in the use of the ICTs, whenever $E(S_U)-E(S_T)>0$, conversely, when the opposite inequality occurs student will prefer traditional learning methods, while if $E(S_U)-E(S_T)=0$, they will be indifferent between the use or not of technology in their studying methods. We observe that

$$E(S_U)-E(S_T) = A_S x_{P_U} x_{A_U} + B_S x_{P_U} + C_S x_{A_U} + D_S \quad (18)$$

Where

$$\begin{aligned} A_S &= v_{S_U}(U,U) - v_{S_T}(U,U) + v_{S_T}(U,T) - v_{S_U}(U,T) \\ &\quad + v_{S_T}(T,U) - v_{S_U}(T,U) + v_{S_U}(T,T) - v_{S_T}(T,T) = 0, \\ B_S &= v_{S_U}(U,T) - v_{S_T}(U,T) + v_{S_T}(T,T) - v_{S_U}(T,T) > 0, \\ C_S &= v_{S_U}(T,U) - v_{S_T}(T,U) + v_{S_T}(T,T) - v_{S_U}(T,T) > 0, \\ D_S &= v_{S_U}(T,T) - v_{S_T}(T,T) < 0, \end{aligned} \quad (19)$$

last inequalities are a direct consequence of (13), (14) and (15) and the definition of $v_{S_U}$ and $v_{S_T}$.

## 4. The Nash Equilibria of the *U* & *T*-Game

A Nash equilibrium for the $U$ & $T$-game occurs, whenever there exist positive real numbers $x^*_{S_U}, x^*_{A_U}, x^*_{P_U} \in [0,1]$ such that if the distribution of each population $h \in \{P,S,A\}$ is given by $(x^*_{h_U}, x^*_{h_T})$ with $x^*_{h_T} = 1 - x^*_{h_U}$, then the expected value associate with this distribution, turns out to be greater than or equal than the expected value associated with any other possible distribution of the behavior of the population over its pure strategies. If $0 < x^*_{h_U}, x^*_{h_T} < 1$, for all $h \in \{P,S,A\}$ we say that the Nash equilibrium is strictly mixed. If $x^*_{h_U} = 0$ or 1 for all $h \in \{P,S,A\}$ the Nash equilibrium corresponds with a Nash equilibrium in pure strategies. Since the distributions $(x^*_{h_U}, 1 - x^*_{h_U}), h \in \{P,S,A\}$ are perfectly define once we know the values of $x^*_{P_U}, x^*_{S_U}$ and $x^*_{A_U}$, then we will identify a strategy profile

$\left(\left(x_{P_U}^*,1-x_{P_U}^*\right),\left(x_{S_U}^* 1-x_{S_U}^*\right),\left(x_{A_U}^*,1-x_{A_U}^*\right)\right)$ with $\left(x_{P_U}^*,x_{S_U}^*,x_{A_U}^*\right)$.

The following theorems give us conditions on the existence of Nash equilibria in pure and mixed strategies.

**Theorem 1** (*Existence of pure Nash equilibria*) *Under the assumptions* (8), (9), (10) *for Professors*, (13), (14), (15) *and* (16) *or* (17) *for Students*, *and* (1) *for educational authorities*, *in the scenario*

a) *High investment in technological infrastructure E*1) *describe by* (2), *the unique Nash equilibria of the game is* $\left(x_{P_U}^*,x_{S_U}^* x_{A_U}^*\right)=(0,0,0)$.

b) *Infrastructure investment and low bonuses for professors E*2)-*i*) (*Equation* (4)) *the unique Nash equilibria of the U&T-game is the pure equilibrium* $\left(x_{P_U}^*,x_{S_U}^* x_{A_U}^*\right)=(1,1,1)$.

*Proof. We postpone the proof of this theorem until the subsection* 0.0.1.

In a) the level of use of ICTs by professors and students is low, hence they have returned to traditional teaching techniques such as those used in the periods prior to the COVID-19 pandemic. This can be triggered by low investment in technology and training by educational authorities, which highlights the importance of the implementation of educative policies that favor their use, if a development of technological skills is desired.

In b) we observe that when educational authorities take policies in favor of the technological development, the best response from students and professors is to correspond to this support by using this tools, thus installing the Nash equilibrium in the scenario where the use of ICTs is high.

For our *U&T*-game model, inequalities (1), (8), (9), (10), (13), (14) and (15) guarantee the existence of an strictly mixed Nash equilibrium in the E2)-ii) describe by (5), this fact is stated in the following theorem.

**Theorem 2** (*Existence of a unique Nash equilibria*) *The U&T-game has a unique Nash equilibria, which is strictly mixed, given by the expressions*

$$x_{S_U}^* = -\frac{D_P + C_P x_{A_U}^*}{B_P + A_P x_{A_U}^*},\ x_{A_U}^* = -\frac{D_S + B_S x_{P_U}^*}{C_S + A_S x_{P_U}^*} \text{ and } x_{P_U}^* = -\frac{D_A}{B_A} \tag{20}$$

*whenever* (1), (8), (9), (10), (13), (14) *and* (15) *together with* (5) *holds. Proof. For the proof of the theorem see subsection* 0.0.2.

Constants $D_A, B_A, A_P, B_P, C_P, D_P, A_S, B_S, C_P$ and $D_S$ are defined in (6), (11) and (19). It is important to remark that this theorem holds independently of which inequality (16) or (17) holds.

Note that a game can have more than one (pure or mixed) Nash equilibrium. In classical game theory it is not possible to indicate which of the Nash equilibria of a game, if there are several, ends up prevailing. However, the study of the stability of the equilibria of replicating dynamics, introduce below, will allow us to define which of them will end up prevailing.

## 5. The Replicator Dynamics

The replicator dynamics models the evolution of an entity called a replicator that manages to make more or less precise copies of itself. The replicator can be a game strategy, a behavior, a technique or cultural form. In our case, it will model the evolution of teaching-learning techniques, according to the VNM theorem, that is, the behavior that obtains the best results will be the one that is replicated.

The following differential equations represent, in our case, the replicator dynamics:

$$\begin{cases} \dot{x}_{P_U} = x_{P_U}\left(1-x_{P_U}\right)\left(A_P x_{S_U} x_{A_U} + B_P x_{S_U} + C_P x_{A_U} + D_P\right) \\ \dot{x}_{S_U} = x_{S_U}\left(1-x_{S_U}\right)\left(A_S x_{P_U} x_{A_U} + B_S x_{P_U} + C_S x_{A_U} + D_S\right) \\ \dot{x}_{A_U} = x_{A_U}\left(1-x_{A_U}\right)\left(B_A x_{P_U} + D_A\right) \end{cases} \tag{21}$$

Using (6), (11) and (19) we can see that the system reflects the fact that the behavior that presents a better performance at a given moment, or that has a higher expected value, will grow, while the opposite would occur if the inequality is reversed.

The system is coupled, which makes it difficult to obtain an analytical solution however, we can obtain numerical solutions. Note that Nash equilibria correspond to stationary states of the system, the converse is not necessarily true. The study of stability according to Liapunov, will help us to distinguish between the different possible equilibria of the model, which one will end up prevailing. Taking account the restrictions of the model the spaces of phases for the system (21) is the cube $\mathcal{C} = [0,1]\times[0,1]\times[0,1]$.

## 5.1. The Stability Analysis of (0, 0, 0) and (1, 1, 1)

The vertices of the cube correspond to dynamic equilibria for (21), but not necessarily to Nash equilibria for the game. However, some of these vertices can verify both properties. The theorem 1 gives us conditions that must verify the states of the system to be Nash equilibria. In this case we are interested in the stationary states $(0,0,0)$ and $(1,1,1)$ which, under certain conditions, are not only dynamic equilibria, but also Nash equilibria of the $U\&T$-game. Hence it becomes relevant to study the stability of these equilibria and analyze their repercussions, which we will do below.

In the remainder of this section we will assume that inequalities (1), (8), (9), (10), (13), (14) and (15) hold and similar as we did for theorem 1 and theorem 2, we will distinguish between scenarios E1) and E2).

The arrows in the edge of cube in **Figure 1**, indicate the direction of evolution of the system (21) once that the variables $\left(x_{P_U}(t), x_{S_u}(t), x_{A_U}(t)\right)$ are replaced by the values they take on each of the edges of the cube $\mathcal{C}$. In each edge, only one of these variables will take values in $(0,1)$ the other two variables will be fixed in 0 or 1.

### 5.1.1. (0,0,0) Nash Equilibrium: Scenario E1)

We begin our discussion about the edges dynamics assuming (2), that is, we assume the context of the scenario *High investment in technological infrastructure E*1). Under these assumptions the cube (a) in **Figure 1** represents the evolution of the replicator dynamics along the edges of the cube.

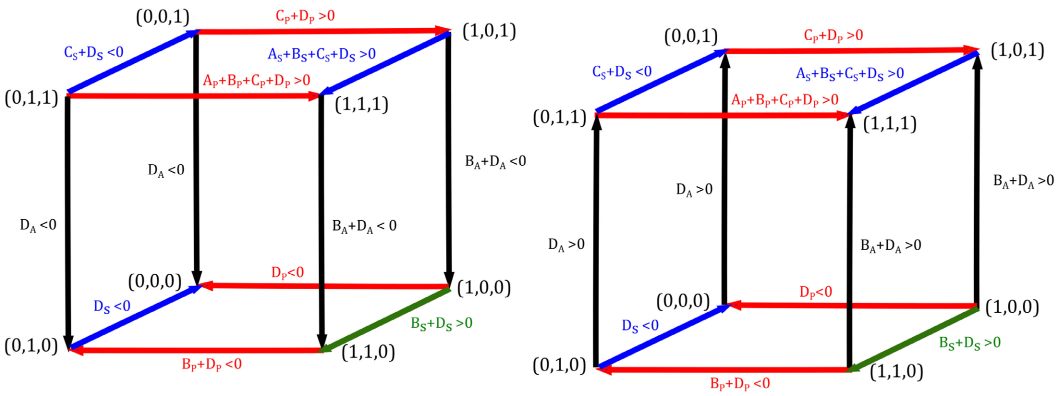


**Figure 1.** Dynamics on the edges of the cube: red Professor, blue and green Students, black Educational Authorities. (a) Corresponds with scenario E1), while (b) with scenario E2)-i). The green arrow points forward as long as (16) holds, and points in the other direction if (17) occurs.

Since an edge $v=\left(x_{P_U},x_{S_U},x_{A_U}\right), x_{P_U},x_{P_U},x_{P_U}\in\{0,1\}$ of the unite cube is an asymptotically stable equilibrium for the replicator dynamics if and if the three edges that are incident to vertex $v$ point toward the vertex $v$ (see [23]) and asymptotically stable states correspond with strict Nash equilibria (see proposition 2.6 in [24]), we can conclude that in this context the vertex $(0,0,0)$ is the only asymptotically stable equilibrium of the $U\&T$-game and hence the only Nash equilibria in pure strategies. Moreover, since inequalities in each edge of the unit cube (a) in **Figure 1** are strict, there is no a steady state for the replicator dynamics (21) in the interior of the edges and therefore does not exists Nash equilibria for the game in the interior of the edges, and since, each face of the unit cube has two parallel arrows pointing in the same direction, there cannot be stationary states for the dynamics of the replicator on the faces of the cube and therefore there cannot be Nash equilibria on the faces either (see [16]). Then we conclude that (1), (8), (9), (10), (13), (14) and (15) together with (2), implies that $(0,0,0)$ is the unique Nash equilibria of the $U\&T$-game.

In this conditions $D_A<0, D_P<0$ and $D_S<0$, then for all $\left(x_{P_u},x_{S_u},x_{A_U}\right)$ close enough to $(0,0,0)$ the following inequalities $\dot{x}_{P_U}<0, \dot{x}_{S_U}<0, \dot{x}_{A_U}<0$ in (21) holds, hence if the initial conditions of the system are closed enough to $(0,0,0)$, the percentage of individuals in each population that follow the typical techniques of the pandemic era tends to decrease over time. On the other hand, the convergence to the origin is asymptotically stable (see [23]), which implies that leaving it would mean doing great efforts, for example by the educational authorities offering significant financial incentives for professors to change their behavior, which *a priori* it is not guaranteed to happen. This convergence towards the origin can also occur if the conditions for an adequate development of the new technology do not exist, such as poor communications or scarce resources to access computers and Internet services in the area.

#### 5.1.2. (1, 1, 1) Nash Equilibrium: Scenario E2)-*i*)

Now in conditions of the scenario: *Infrastructure investment and low bonuses* *for professors* E2)-i) describe by equation (4) the dynamics on the edges of the cube is represented by cube (b) in **Figure 1**. Hence a similar argument to the one made above guarantees that the unique Nash equilibrium of the *U*&*T*-game, is the vertex $\left(x_{P_U}, x_{S_U}, x_{A_U}\right) = (1,1,1)$.

Since $(1,1,1)$ is a Nash equilibrium if and only if $A_P + B_P + C_P + D_P > 0$, $A_S + B_S + C_S + D_S > 0$ and $B_A + D_A > 0$ and Nash equilibria in the vertex of the unit cube are asymptotically stable [23], we conclude that there exists a neighborhood of this point for which the equations in system (21) are all positive, that is, $\dot{x}_{P_U} > 0, \dot{x}_{S_U} > 0, \dot{x}_{A_U} > 0$, which means that the techniques used during the pandemic tend to be generalized in the post-pandemic. However, it is a long process in which these techniques predominate, but it does not mean the disappearance of traditional techniques. The point $(1,1,1)$ that supposes the absolute disappearance of the pre-covid techniques, is only verified as a trend. Only if the society is in it, in it will remain, instead if the initial conditions are met in a reduced environment of the point, we will witness a process of predominance of the new techniques, but the speed of convergence will decrease over time.

### 5.2. Stability Analysis of the Strictly Mixed Nash Equilibrium: Scenario E2)-*ii*)

*Infrastructure investment and high bonuses for professors scenario* which occurs under inequality (5). The evolution along the edges of the unit cube is represented in **Figure 2**. In this case, from a direct inspection in the cube we can observe that there is no a Nash equilbrium in pure strategies, since the fact that each vertex has at least one arrow leaving it, implies that each vertex of the unit cube is unstable dynamical equilibrium of (21), hence no Nash equilibrium can occurs in a vertex, since pure Nash equilibrium for the replicator dynamics are asymptotically stable (see [17]). A similar argument to the one made in the scenario E1) implies that there is not Nash equilibria in the face of the unite cube, hence the Nash theorem [25] implies the existence of an strictly mixed Nash equilibrium, that is, there exist at least one dynamical Nash equilibrium in the interior the cube [23].

We now proceed to analyze the stability of the strictly mixed Nash equilibrium $\left(x^*_{P_U}, x^*_{S_U}, x^*_{A_U}\right)$ defined in theorem 2, in this context the Jacobian matrix of system (21) at the strictly mixed Nash equilibrium point $\left(x^*_{P_U}, x^*_{S_U}, x^*_{A_U}\right)$ is given by the expression

$$J\left(x^*_{P_U}, x^*_{S_U}, x^*_{A_U}\right) = \begin{pmatrix} 0 & a & b \\ c & 0 & d \\ e & 0 & 0 \end{pmatrix} \tag{22}$$

where $a = \frac{\partial \dot{x}_{P_U}}{\partial x_{S_U}}, b = \frac{\partial \dot{x}_{P_U}}{\partial x_{A_U}}, c = \frac{\partial \dot{x}_{S_U}}{\partial x_{P_U}}, d = \frac{\partial \dot{x}_{S_U}}{\partial x_{A_U}}$ and $e = \frac{\partial \dot{x}_{A_U}}{\partial x_{P_U}}$ at $\left(x^*_{P_U}, x^*_{S_U}, x^*_{A_U}\right)$, the characteristic polynomial of (22) is given by

$$p(\lambda) = -\lambda^3 + \lambda(ac + be) + ade. \tag{23}$$

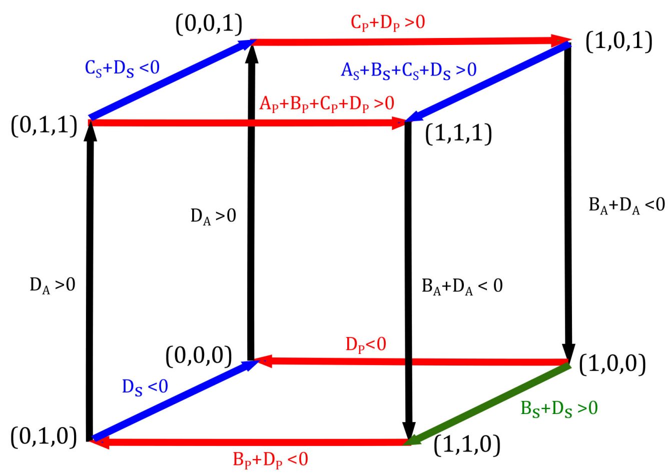


**Figure 2.** Strictly mixed Nash equilibria $\left(x^*_{P_U}, x^*_{S_U}, x^*_{A_U}\right)$, scenario E2)-i).

Note that the fact that $C_S > 0$ and $B_A < 0$, implies $d > 0$ and $e < 0$, using the fact that $B_P > 0$, we have

$$A_P x^*_{A_U} + B_P > \left(Sl(U,U) - sl(U,U) + sl(T,U) - Sl(T,U)\right) x^*_{A_U} > 0,$$

so $a > 0$ and hence $ade < 0$, this observation it is important for point 3) in the following theorem.

**Theorem 3** *Under conditions* (1), (8), (9), (10), (13), (14) *and* (15) *together with* (5) *and* $u(I_{ICT}) = 0$, *let* $K = (ade)^2/4 - (ac+be)^3/27$, *then the characteristic polynomial* $p(\lambda)$ *of J has*

1) One real $\lambda_1$ and two complex conjugate $\lambda_2, \lambda_3$ roots, for $K > 0$,

2) three different real roots $\lambda_1, \lambda_2, \lambda_3$, whenever $K < 0$ and

3) one simple real root $\lambda_1$ and a root $\lambda_2$ of multiplicity 2, in the case in which $K = 0$.

Furthermore, in no case the real eigenvalues and the real part of the complex roots of $p(\lambda)$ are zero.

*Proof.* The assumptions of theorem guarantee that $\left(x^*_{P_U}, x^*_{S_U}, x^*_{A_U}\right) \in (0,1)^3$ and that $p(\lambda)$ is given by expression (23), then the first part of the theorem follows from the Cardano's method for the incomplete cubic equation case.

Since $ade < 0$, then 0 is not a root of $p(\lambda)$, then real roots are not zero, this prove points 2) and 3), and guaranties that $\lambda_1 \neq 0$ in point 1), to conclude this point, note that the Viète's formulas, implies that $\lambda_1 + \lambda_2 + \lambda_3 = 0$, then there is a root whose real part is of opposite sign, and since the two remaining roots can not be a pair of conjugate pure imaginary numbers, they have non-zero real part.

This theorem tells us nothing about the stability of the Nash equilibrium, but it guaranties that we can apply the Hartman-Grobman's theorem to study the stability of the strictly mixed Nash equilibrium $\left(x^*_{S_U}, x^*_{A_U}, x^*_{P_U}\right)$. Hence, we have the following general result

**Theorem 4** *Under the conditions of theorem* 3, *if* $ac + be \leq 0$, *then the strictly mixed Nash equilibrium is a hyperbolic fixed point of system* (21) *with a* 1-*di-*

*mensional stable manifold and a* 2-*dimensional unstable manifold*.

*Proof*. It follows from the Descartes' rules of sing and the Hartman-Grobman's theorem, for more details we recommend to review [16].

Last theorem shows the importance of initial conditions to determine the possible evolution of teaching-learning techniques.

## 6. Technological-Traditional Teaching-Learning Techniques Cycles

In this section we exhibit the existence of heteroclinic cycle of high use of technology in the teaching-learning process followed by a period where traditional teaching-learning techniques prevail and vice-versa.

To show the sequence order at which events describing the cycle occurs we will use the notation $A_U P_U S_U A_T P_T S_T$, hence each letter from left to right denote the event that occurs and the event that follows. So this cycle denote that initially educational authorities decide to encourage the use ICTs, which is followed by the choice of professors to implement teaching techniques that involve the use of technology and subsequently students using it in their learning process. Once in this technological scenario, the educational authorities stop investing in technology to which professors respond by returning to traditional teaching methodologies, leading students to resume study processes where technology is not essential.

In the scenario E2)-ii) inequalities (1), (8), (9), (10), (13), (14), (15) and (5) guaranties the existence of the heteroclinc cycle $A_U P_U S_U A_T P_T S_T$.

When professors and students follow traditional teaching-learning process, educational authorities decide to encourage the use of technology, offering free training to professors and providing of technology to students.

$$\dot{x}_{A_U} = x_{A_U}\left(1 - x_{A_U}\right)D_A > 0.$$

Implies $G_I + F_I - I_I > G_N + F_N - I_N$, which corresponds to situations where the resources captured by those schools interested in the developing of technological infrastructure once discounted the investment in this is greater than that received by schools that follow traditional methodologies once discounted their corresponding inversion in infrastructure.

In this framework, given that the educational authorities promote the use ICTs, professors decide to implement teaching techniques based on this. Since $C_P + D_P > 0$, professors who prefer to continue implementing the knowledge acquired during the pandemic obtain greater utility than those who prefer traditional methods, this is led to the following inequality

$$\dot{x}_{P_U} = x_{P_U}\left(1 - x_{P_U}\right)\left(C_P + D_P\right) > 0.$$

This shows that investment and training decisions in favor of the implementation of technology by educational authorities is a necessary incentive for professors to use ICTs in their teaching work.

Summarizing, in the scenario in which schools promote through their decisions the use of ICTs in the classroom, and professors carry out their work basing their teaching strategies on the use of ICTs, $V_{S_U}(U,U) - S_{S_T}(U,U)$ or equivalently $A_S + B_S + C_S + D_S$ is positive, then students decide to involve these technologies tools in their process of learning. Hence we have

$$\dot{x}_{S_U} = x_{S_U}\left(1 - x_{S_U}\right)\left(A_S + B_S + C_S + D_S\right) > 0.$$

In a school environment with sufficient technology available, professors and students will find favorable conditions to implement the use of ICTs.

Once in this scenario, the educational authorities decide to stop investing in technology infrastructure and payment of bonuses to professors, since having a large number of professors trained in the use of ICTs becomes in an extremely high payment. This fact can be summarize in the following inequality $G_I + F_I - I_I - B < G_N + F_N - I_N$, which implies

$$\dot{x}_{A_U} = x_{A_U}\left(1 - x_{A_U}\right)\left(B_A + D_A\right) < 0.$$

This leads to a decrease in the number of schools that promote the use of ICTs and triggers a drop in the number of professors using them.

The lack of investment in technology and the consequent upgrading of available equipment and the decrease in economic incentives will force to professors to resort to traditional teaching techniques. In this conditions $u(S) + Sl(U,T) + u(I_{ICT}) < u(S) + sl(U,T)$, then the satisfaction of tech-friendly professor about the labor conditions is low in comparison with the utility of traditional professors. Hence we have

$$\dot{x}_{P_U} = x_{P_U}\left(1 - x_{P_U}\right)\left(B_P + D_P\right) < 0$$

In traditional settings, students, even with preference to the use of ICTs, put technology aside and return to traditional study method, in this situation $D_S = v_{S_U}(T,T) - v_{S_T}(T,T) < 0$, and therefore

$$\dot{x}_{S_U} = x_{S_U}\left(1 - x_{S_U}\right)D_S < 0.$$

Without technology at their disposal, nor professors trained in its use, students decide to return to traditional study methods.

Contrary to first scenario, in the second we observe that the lack of investment in technology by school authorities leads to discourage its use. Then to generalize the use of ICTs in the teaching-learning process, schools must permanently invest in developing and updating the necessary equipment and create policies that favor the training of professors, this should allow the use of traditional teaching practices when professors understand that these are necessary for a better development of the students.

Below we present some simulations of the trajectories of the teaching-learning methods, which corresponds to future distribution of the behaviors of the participants in the educational process on their possible behaviors. Once the parameters of the model are known, the evolution will depend solely on the initial conditions.

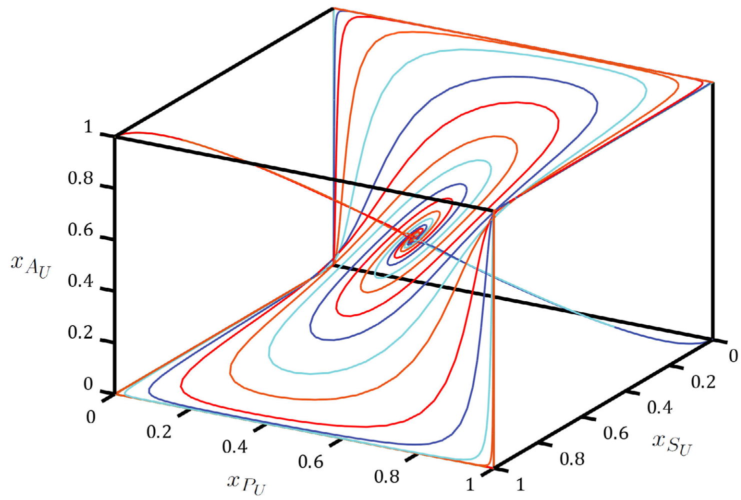


**Figure 3.** Technological-Traditional teaching-learning techniques cycles.

Any initial distribution of the populations in the interior of the unit cube, correspond with a distribution of the participants of the $U\&T$-game that follows teaching-learning strategies based on the use of the ICTs. Figure 3 showed that distributions taken inside the unit cube evolve to the interior, close to the Nash equilibrium $\left(x^*_{P_U}, x^*_{S_U}, x^*_{A_U}\right)$ but once there it follows spiral trajectories to outside until approach the heteroclinic cycle. This suggest that independently where the use of ICTs in the teaching-learning process be located at the beginning, this will evolve until reach period where its use is high, follow by periods where its use is set aside.

## 7. Conclusions

In this work we have shown that the evolution of teaching-learning techniques is a process that depends on the preferences of all those involved. The interaction of professors, students and school authorities determines the evolution of the ways in which knowledge is transmitted and assimilated. It is clear that the development of digital techniques will require an effort on the part of the school authorities, professors and students.

In addition, we have observed, under the different scenarios presented in this work, which the path that education follows in the post-pandemic era could depend on the decisions of the educational authorities on the use of ICTs. Within schools, these decisions will depend to a large extent on the economic resources and the technological conditions available, since the forecast of large investments that are not very profitable for the educational authorities could lead them to take a position towards traditional education returning to offline education models ((0, 0, 0) Nash equilibrium). On the other hand, moderate investments in ICTs could be enough for professors to decide to train and this trigger the appropriate use of these available technologies by students ((1, 1, 1) Nash equilibrium). Although the decisions of the educational authorities have a great weight in the course of education, these are not the only ingredients necessary for the different scenarios to occur, since all the decisions are linked to the preferences of the other participants in the educational system. We have shown that there are possibilities for the emergence of cycles in teaching, the incentives for the development of digital techniques must be maintained for a while because if they are eliminated, even when the use of these techniques is predominant, the process can be reversed and return to the initial situation.

Although the scenarios presented throughout the work are adjusted to the realities experienced by many societies, since these are the main source of inspiration for our model, for future work it is necessary to analyze the results of the teaching-learning process in the post-pandemic era and compare them with those obtained in the times prior to the pandemic. Although digital techniques are an increasingly used tool, and that came to stay, it is not clear what role they should play during the activities carried out in class, that is, the level of use that they should have in the teaching-learning process, so determining the optimal time of use that these must have in these processes is of vital importance to analyze the trajectories that the methodologies based on its use will follow from now on with which we could compare the theoretical results emerged from our work.

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## Conflicts of Interest

The authors declare no conflicts of interest.

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