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Nonlinear 6D Dynamical System with Hidden Attractors and Its Electronic Circuit

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Abstract

In this paper, a six-dimensional model of continuous time dynamical systems is proposed. This system was analyzed by finding equilibrium points, and the stability of the system was also analyzed using different methods, namely the roots of the characteristic equation, Routh's invariance criterion, Hurwitz's invariance criterion, Lyapunov function, and the continued fraction stability criterion. The chaoticity of the system was tested by Lyapunov exponent, the hexagonal system was found to be chaotic. The dissipation and Hopf Bifurcation of the proposed system were also found, and then the system was controlled using the adaptive control technique. Finally, the results and numerical figures before and after control were compared for the system under study. An electronic circuit was created as a six-dimensional system application consisting of twelve resistors, six capacitors, six voltages and six operational amplifiers, where the results were obtained from Multisim12, and it was found that the designed electronic circuit simulates the theoretical results of the six-dimensional dynamical system well.

Subject Areas

Mathematics

Keywords

Lyapunov Function, Stability, Hopf Bifurcation, Lyapunov Dimension, Electronic Circuit

1. Introduction

Research on chaotic phenomena has been increasingly important in recent years because of the growing range of chaotic applications in scientific and technical systems [1]. Chaotic phenomena arise from the reactivity of adversaries to changes in the structural parameters and initial conditions of some types of dynamic systems [2] [3]. The aperiodicity, broad spectrum, and random-like properties of chaotic signals are characteristics of these phenomena [4] [5] [6] [7]. The chaotic orbits must be packed in phase space, it is not a transitional topology, and it is sensitive to perturbations in its initial conditions, all of which should lead to unpredictable behavior over time [8] [9]. Studies claim that some of the produced chaos attractors include Chen's [10], the 4-wing attractor [11], Sundarapandian Pehlivan [12], and the Rabinovich system [13] [14] [15]. The fact that a chaotic system has at least one Lyapunov exponent greater than zero is one of its fundamental properties. A system becomes extremely chaotic and sensitive to even the slightest changes in its dynamics when it has a lot of positive Lyapunov exponents [16] [17]. Researchers are paying more and more attention to chaos management because of its synchronizability and controllability, which suggests that it will be helpful in a range of designs, such as biometric identification, artificial intelligence, and secure communications [18] [19] [20]. Dissipative systems can be settled successfully using one of the Lyapunov stability principles [21] [22] [23] [24]. Owing to the example, some components, such as multi-leveled equations, graphics, and tables are not prescribed, although the various table text styles are provided. The formatter will need to create these components, incorporating the applicable criteria that follow. We presented an engineering application of the six-dimensional chaotic system, such as electronic circuit simulation.

2. Description of the System

The six-dimensional system contains the following six differential equations:

$$\begin{aligned} \dot{x}_1 &= a\left(x_2 - x_1\right) + x_4 \\ \dot{x}_2 &= cx_1 - x_2 - x_1 x_3 + x_5 \\ \dot{x}_3 &= -bx_3 + x_1 x_2 \\ \dot{x}_4 &= dx_4 - x_1 x_3 \\ \dot{x}_5 &= -kx_2 \\ \dot{x}_6 &= lx_2 - hx_6 \end{aligned} \tag{1}$$

$x_1, x_2, x_3, x_4, x_5, x_6$ are state variables and a, b, c, d, k, l, h are constants. Where

$$a = 10, b = 2.6, c = 28, d = 2, k = 8.4, l = h = 1. \tag{2}$$

3. System Analysis

When Equation (1) is set to zero, just one equilibrium point, the origin point, is produced, allowing us to examine a dynamical system's equilibrium points Ŏ = (0, 0, 0, 0, 0, 0).

3.1. Stability Analysis

The characteristic equation's eigenvalues including negative real components are a necessary and sufficient condition for the system to remain stable. Following is the Jacobian matrix for the new system (1) up to $E = (0, 0, 0, 0, 0, 0)$:

$$J = \begin{bmatrix} -10 & 10 & 0 & 1 & 0 & 0 \\ 28 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -2.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & -8.4 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{3}$$

The characteristic equation is:

$$\lambda^6 + 12.5\lambda^5 - 248.6\lambda^4 - 212.76\lambda^3 + 1387.12\lambda^2 + 50.4\lambda - 436.8 = 0, \tag{4}$$

Roots of the characteristic equation:

$$\lambda_1 = -22.823, \lambda_2 = -2.617, \lambda_3 = -0.574, \lambda_4 = 0.581, \lambda_5 = 1.860, \lambda_6 = 10.940$$

Thus, we conclude that it is unstable system.

3.2. Routh Stability Criteria

A system meets the Routh requirement for stability (all poles in the half-loop level), if and only if the entries in the Routh array's first column have values that are entirely positive. The number of sign changes in the first column multiplied by the sum of the non-OLHP columns [25]. Regarding the Roth stability test, see Table 1.

$$a_0 = -436.8, a_1 = 50.4, a_2 = 1387.12, a_3 = -212.76, a_4 = -248.6, a_5 = 12.5, a_6 = 1,$$

$$b_0 = \frac{a_5 a_0 - a_6 a_7}{a_5} = -436.8, \quad c_1 = \frac{b_4 a_1 - a_5 b_0}{b_4} = 26.822,$$

$$b_2 = \frac{a_5 a_2 - a_6 a_1}{a_5} = 1383.088, \quad c_3 = \frac{b_4 a_3 - a_5 b_2}{b_4} = -138.104,$$

$$b_4 = \frac{a_5 a_4 - a_6 a_3}{a_5} = -231.579, \quad e_1 = \frac{c_3 d_0 - c_1 d_2}{d_2} = 18.259,$$

$$d_0 = \frac{b_4 - b_0 c_3}{c_3} = 436.8, \quad d_2 = \frac{b_4 c_1 - b_2 c_3}{c_3} = -1338.111.$$

The system is unstable because the first column has four negative elements.

Table 1. Routh array.

λ^6	1	−248.6	1387.12	−436.8
λ^5	12.5	−212.76	50.4	0
λ^4	−231.579	1383.088	−436.8	0
λ^3	−138.104	26.822	0	0
λ^2	−1338.111	436.8	0	0
λ^1	18.259	0	0	0
λ^0	−436.8	0	0	0

3.3. Hurwitz Stability Criteria

Determinants generated from the coefficients of the characteristic equation are used to implement this criterion. System (1) is stable if the tiny minors of its square matrix J are all positive; if not, it is unstable [25] [26].

If $n=6$,

From Equation (3):

$$\Delta_1 = a_{n-1} = a_5 = 12.5 > 0,$$

$$\Delta_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix} = \begin{vmatrix} a_5 & a_3 \\ a_6 & a_4 \end{vmatrix} = \begin{vmatrix} 12.5 & -212.76 \\ 1 & -248.6 \end{vmatrix} = -2894.74 < 0,$$

$$\Delta_3 = \begin{vmatrix} a_{n-1} & a_{n-3} & a_{n-5} \\ a_n & a_{n-2} & a_{n-4} \\ 0 & a_{n-1} & a_{n-3} \end{vmatrix} = \begin{vmatrix} 12.5 & -212.76 & 50.4 \\ 1 & -248.6 & 1387.12 \\ 0 & 12.5 & -212.76 \end{vmatrix} = 39977.383 > 0,$$

$$\Delta_4 = -56781.350 < 0,$$

$$\Delta_5 = 79321.51 > 0,$$

$$\Delta_6 = 103579.12 < 0.$$

System (1) is unstable because some of the values of the determinants are less than zero.

3.4. Lyapunov Function

Where we assume the Lyapunov function is:

$$V(x_1,x_2,x_3,x_4,x_5,x_6) = 1/2\left(x_1^2 + x_2^2 x_3^2 + x_4^2 + x_5^2 + x_6^2\right)$$

$$\dot{V}(x_1,x_2,x_3,x_4,x_5,x_6) = x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3 + x_5\dot{x}_5 + x_6\dot{x}_6$$

$$\dot{V} = 38x_1x_2 - 10x_1^2 + x_1x_4 - x_2^2 - 7.4x_2x_5 - 2.6x_3^2 + 2x_4^2 - x_1x_3x_4 + x_2x_6^2 - x_6^2. \quad (5)$$

Since $\dot{V}(x_1,x_2,x_3,x_4,x_5,x_6) > 0$, as a result, New system (1) is unstable.

3.5. Continued Fraction Stability Criteria

By creating a continuous fraction from the odd and even parts of the equation, the characteristic equation of a continuous system is subjected to this condition [25]. The distinguishing equation:

$$\lambda^6 + 12.5\lambda^5 - 248.6\lambda^4 - 212.76\lambda^3 + 1387.12\lambda^2 + 50.4\lambda - 436.8 = 0,$$

By taking the even terms and then the odd terms, respectively, we have:

$$Q_1(\lambda) = \lambda^6 - 248.6\lambda^4 + 1387.12\lambda^2 - 436.8, \quad (6)$$

$$Q_2(\lambda) = 12.5\lambda^5 - 212.76\lambda^3 + 50.4\lambda, \quad (7)$$

After dividing the even terms by the odd terms and using algebraic steps, we get the following results:

$$h_1 = 0.08, h_2 = -0.053, h_3 = 0.809, h_4 = -0.21, h_5 = -21.108, h_6 = 0.147.$$

Since some values of h are negative, the equation of the system has some positive real roots, so system (1) is chaotic.

3.6. Dissipativity

Suppose that

$$f_1=\frac{dx_1}{dt},\ f_2=\frac{dx_2}{dt},\ f_3=\frac{dx_3}{dt},\ f_4=\frac{dx_4}{dt},\ f_5=\frac{dx_5}{dt}\ \text{ and }\ f_6=\frac{dx_6}{dt}.$$

The obtained vector field,

$$V(\dot{x}_1,\dot{x}_2,\dot{x}_3,\dot{x}_3,\dot{x}_4,\dot{x}_5,\dot{x}_6)^T=(f_1,f_2,f_3,f_4,f_5,f_6)^T$$

$$\nabla\cdot(\dot{x}_1,\dot{x}_2,\dot{x}_3,\dot{x}_3,\dot{x}_4,\dot{x}_5,\dot{x}_6)^T=\frac{\partial f_1}{\partial x_1}+\frac{\partial f_2}{\partial x_2}+\frac{\partial f_3}{\partial x_3}+\frac{\partial f_4}{\partial x_4}+\frac{\partial f_5}{\partial x_5}+\frac{\partial f_6}{\partial x_6}$$
$$=-(a+1+b-d+h)=f.$$

Note that, $f=-(a+1+b-d+h)=-12.6$, for all (a,b,d,h) values that are positive and greater than zero, the system (1) is dissipative

The exponential rate is:

$$\frac{dV}{dt}=fV\Rightarrow V(t)=V_0e^{ft}=V_0e^{-12.6t}$$

By flowing into ($V_0e^{-12.6}$), the volume element (V_0) from the previous equation is condensed at the time (t).

4. Hopf Bifurcation

One of the types of bifurcation that is recognized in mathematics occurs when a modest modification to one of the initial conditions causes a qualitative change in the behavior of the system at an equilibrium point [7]. We take the Equation (3)

$$\lambda^6+12.5\lambda^5-248.6\lambda^4-212.76\lambda^3+1387.12\lambda^2+50.4\lambda-436.8=0$$

The roots of Equation (3) are:

$$\lambda_1=-22.823,\lambda_2=-2.617,\lambda_3=-0.574,\lambda_4=0.581,\lambda_5=1.860,\lambda_6=10.94$$

Differentiate the Equation (3) and normalize it to zero to find the critical value.

$$6\lambda^5+62.5\lambda^4-994.4\lambda^3-638.28\lambda^2+5548.48\lambda+50.4=0,$$

So, the critical values are $\lambda=-50.4$.

Derivative at one of the eigenvalues of the equation = −8835774.285

$$\frac{\text{The critical values}}{\text{Derivative at the eigenvalue of the equation}}=\frac{-50.4}{-8835774.285}=0.0000057\neq 0$$

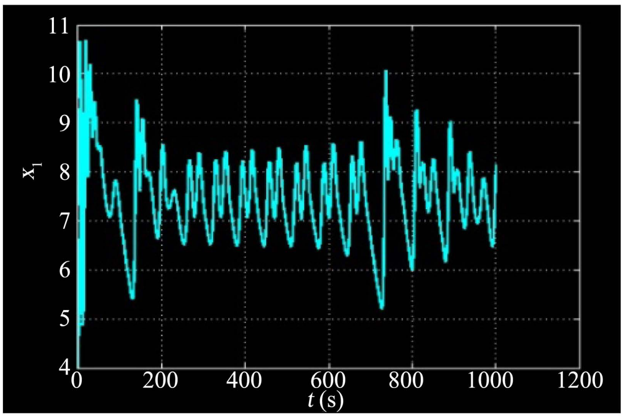
4.1. Numerical and Graphical Analysis

The fifth- and sixth-order Runge-Kutta method is used to solve the system (1). Initial values included

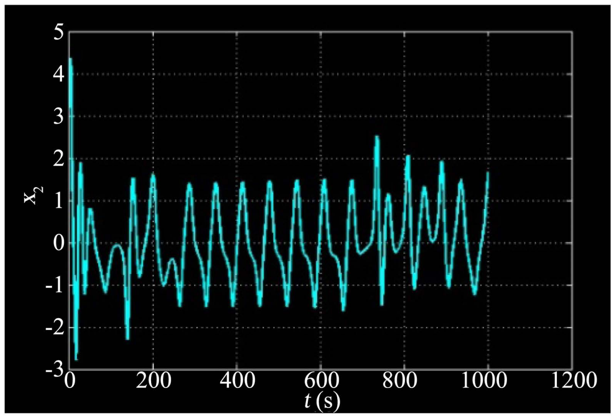
$$x\mid x_{1_{(0)}},x_{2_{(0)}},x_{3_{(0)}},x_{4_{(0)}},x_{5_{(0)}},x_{6_{(0)}}=[3,2,0.5,1,2.5,3.5]$$

4.2. Waveform of the New System (1)

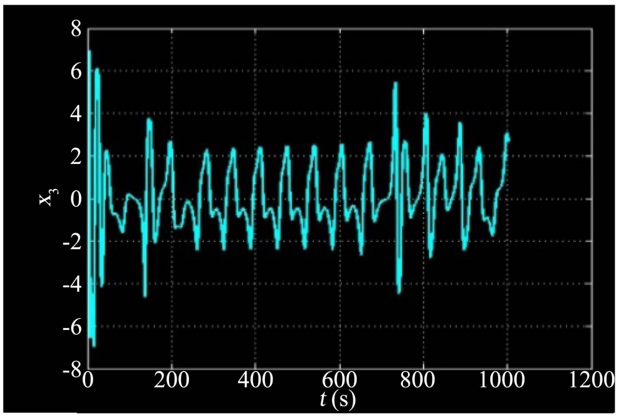
The waveform exhibits aperiodic structure, the primary defining feature of chaotic systems. $x_{1_{(t)}},x_{2_{(t)}},x_{3_{(t)}},x_{4_{(t)}},x_{5_{(t)}}$ and $x_{6_{(t)}}$ for system (1) (**Figure 1**).



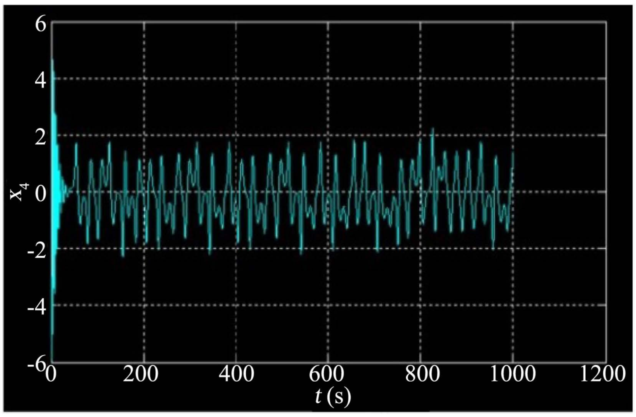
(a)



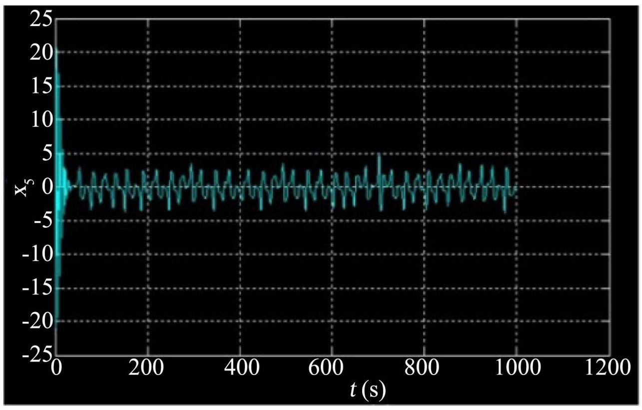
(b)



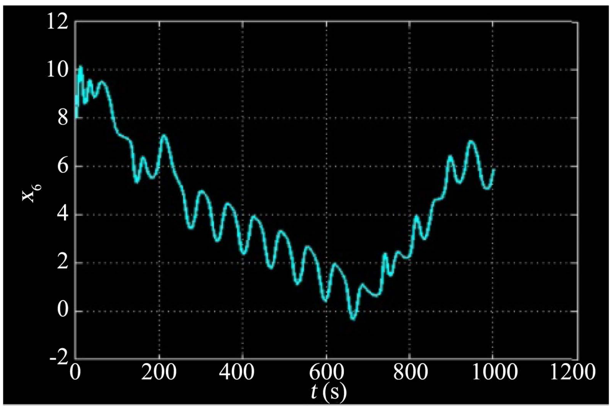
(c)



(d)



(e)



(f)

Figure 1. The waveform of a new system (1). (a): x_1 versus time; (b): x_2 versus time; (c): x_3 versus time; (d): x_4 versus time; (e): x_5 versus time; (f): x_6 versus time.

4.3. Phase Portrait of the System (1)

Figure 2 and **Figure 3** in this paragraph depict the chaotic strange attractor for the system (1) in $(x_1, x_2, \cdots, x_6)$ space and the chaotic strange attractor for the system (1) in (x_1, x_6) the plane, respectively.

Since the orbit in each graph looks to be dense, the new system exhibits a chaotic attractor.

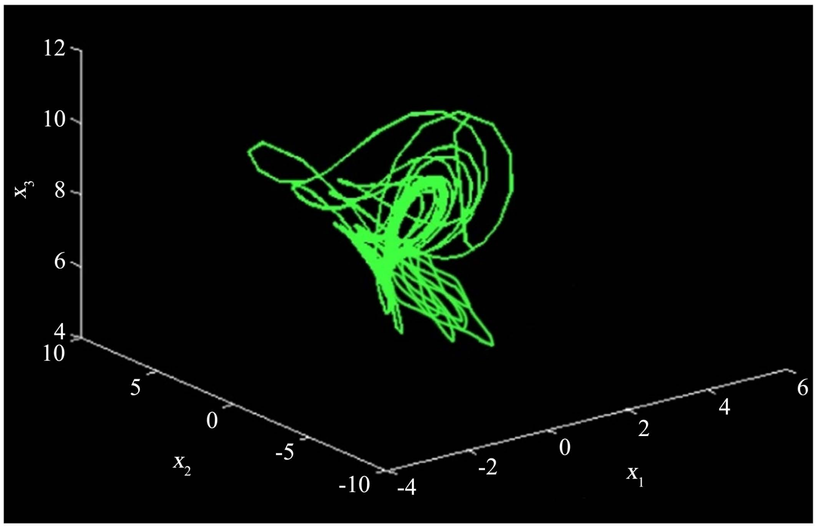


Figure 2. The system attractor in (x_1, x_2, x_3).

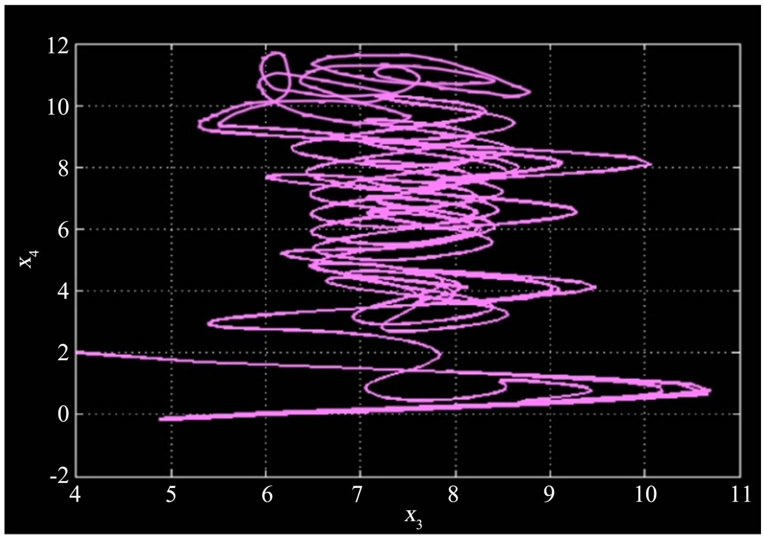


Figure 3. The system attractor in (x_4, x_3).

4.4. Lyapunov Exponent and Lyapunov Dimension

The average exponential rates of almost divergent trajectories in phase space are frequently referred to as the Lyapunov exponent. The new system is regarded as chaotic if it has at least one positive Lyapunov exponent. Values of the Lyapunov exponent are:

$$L_1 = 0.524, L_2 = -0.279, L_3 = 2.987, L_4 = -5.435, L_5 = -9.21, L_6 = -12.889.$$

As a result, the system's "Kaplan-Yorke dimension" or Lyapunov dimension is as follows:

$$D_L = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3.59466$$

Figure 4 show that system (1) is very Chaotic.

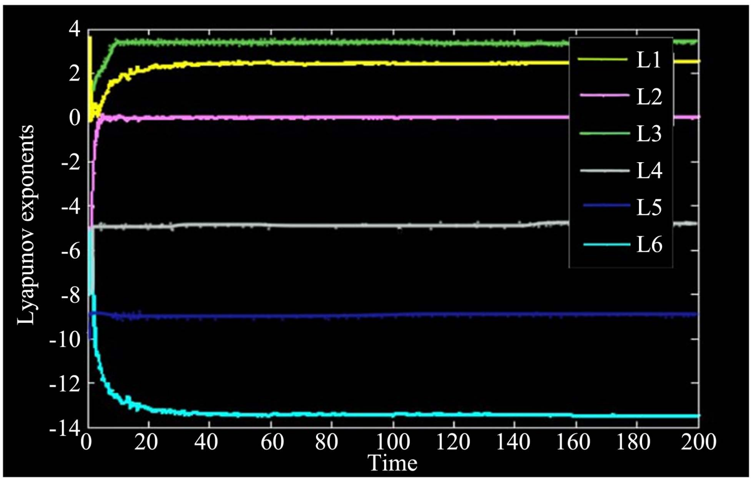


Figure 4. Lyapunov exponent of the system (1).

5. Adaptive Controller Technique

5.1. Theoretical Results

To stabilize a chaotic system (1) use the sufficiency control law generalized with an unknown parameter c as follows:

$$\begin{aligned}\dot{x}_1 &= 10(x_2 - x_1) + x_4 + u_1 \\ \dot{x}_2 &= cx_1 - x_2 - x_1x_3 + x_5 + u_2 \\ \dot{x}_3 &= -2.6x_3 + x_1x_2 + u_3 \\ \dot{x}_4 &= 2x_4 - x_1x_3 + u_4 \\ \dot{x}_5 &= -8.4x_2 + u_5 \\ \dot{x}_6 &= x_2 - x_6 + u_6\end{aligned} \tag{8}$$

where $[u_1, u_2, u_3, u_4, u_5, u_6]^T$ are feedback controllers.

We now consider the following adaptive control procedures to make sure the managed system (8) converges asymptotically to the origin.

$$\begin{aligned}u_1 &= -10(x_2 - x_1) + x_4 - \mu_1 x_1 \\ u_2 &= -\hat{c}x_1 + x_2 + x_1x_3 - x_5 - \mu_2 x_2 \\ u_3 &= 2.6x_3 - x_1x_2 - \mu_3 x_3 \\ u_4 &= -2x_4 + x_1x_3 - \mu_4 x_4 \\ u_5 &= 8.4x_2 - \mu_5 x_5 \\ u_6 &= -x_2 + x_6 - \mu_6 x_6\end{aligned} \tag{9}$$

where $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6$ are constants, $\hat{c}$ is an estimator of the parameter *c*.

Substituting (9) into (8), we get:

$$\begin{aligned}\dot{x}_1 &= -\mu_1 x_1 \\ \dot{x}_2 &= (c - \hat{c})x_1 - \mu_2 x_2 \\ \dot{x}_3 &= -\mu_3 x_3 \\ \dot{x}_4 &= -\mu_4 x_4 \\ \dot{x}_5 &= -\mu_5 x_5 \\ \dot{x}_6 &= -\mu_6 x_6\end{aligned} \tag{10}$$

Let the error of estimating parameter is:

$$e_c = c - \hat{c} \tag{11}$$

Using (11), system (10) can be written as:

$$\begin{aligned} \dot{x}_1 &= -\mu_1 x_1 \\ \dot{x}_2 &= e_c x_1 - \mu_2 x_2 \\ \dot{x}_3 &= -\mu_3 x_3 \\ \dot{x}_4 &= -\mu_4 x_4 \\ \dot{x}_5 &= -\mu_5 x_5 \\ \dot{x}_6 &= -\mu_6 x_6 \end{aligned} \tag{12}$$

The parameter estimate $\hat{c}$ is changed using the Lyapunov method of obtaining the updated law. It is thought that the quadratic Lyapunov function:

$$V(x_1, x_2, x_3, x_4, x_5, x_6, e_c) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 + e_c^2\right), \tag{13}$$

Which definite, positive-in $\mathbb{R}^7$.

Also

$$\dot{e}_c = -\dot{\hat{c}}. \tag{14}$$

Differentiate V & substituting (11) and (13), we get:

$$\dot{V} = -\mu_1 x_1^2 - \mu_2 x_2^2 - \mu_3 x_3^2 - \mu_2 x_4^2 - \mu_5 x_5^2 - \mu_6 x_6^2 + e_c\left(x_1 x_2 - \dot{\hat{c}}\right).$$

Assume that:

$$\dot{\hat{c}} = x_1 x_2 + \mu_7 e_c. \tag{15}$$

where μ_7 is greater than zero.

Substitute (15) into $\dot{V}$, we get:

$$\dot{V} = -\mu_1 x_1^2 - \mu_2 x_2^2 - \mu_3 x_3^2 - \mu_4 x_4^2 - \mu_5 x_5^2 - \mu_6 x_6^2 - \mu_7 e_c^2. \tag{16}$$

Which is negative-definite on $\mathbb{R}^7$.

The outcome is as follows because of Lyapunov stability, Eigenvalues, and the Routh array criteria.

Proposition 1. By adaptive control (9), where $\dot{\hat{c}} = x_1 x_2 + \mu_7 e_c$ and $\mu_1, \mu_2, -\mu_3, -\mu_4, -\mu_5, \mu_7$ are positive constants, The chaotic system (8) is stabilized for $x(0) \in \mathbb{R}^6$.

5.2. Simulation and Numerical Results

The controlled extremely chaotic system (10) was simulated using

$$x \mid x_{1_{(0)}}, x_{2_{(0)}}, x_{3_{(0)}}, x_{4_{(0)}}, x_{5_{(0)}}, x_{6_{(0)}} = [-4, 5, 2, 1, -2.5, 3]$$

$$\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6 = [10, 6, 4, 7, 5, 8, 3] \text{ and } e_c = 19.$$

The new system (1)'s-controlled state trajectories are displayed in Figure 5.

6. A Comparison Table before and Following the Control

Comparison of the Routh array criterion in Table 3 and the new system (1) eigenvalues in Table 2 before and after control at the equilibrium point (0, 0, 0, 0, 0, 0) (Tables 2-5).

Table 2. Eigenvalues of a new system (1).

Equilibrium point	Before Control	After Control
$(0,0,0,0,0,0)$	$\lambda_1 = -22.823$	$\lambda_1 = -10$
	$\lambda_2 = -2.617$	$\lambda_2 = -8$
	$\lambda_3 = -0.574$	$\lambda_3 = -7$
	$\lambda_4 = 0.581$	$\lambda_4 = -6$
	$\lambda_5 = 1.86$	$\lambda_5 = -5$
	$\lambda_6 = 10.94$	$\lambda_6 = -4$

Table 3. Calculated values of Routh array criteria of a new system (1).

Equilibrium point	λ	Before Control				After Control			
$(0,0,0,0,0,0)$	λ^6	1	−248.6	1387.12	−436.8	1	655	26,644	0
	λ^5	12.5	−212.76	50.4	0	40	5620	66,160	0
	λ^4	−231.579	1383.088	−436.8	0	514.5	26478.6	67,200	0
	λ^3	−138.104	26.822	0	0	3561.411	60635.51	0	0
	λ^2	−1338.11	436.8	0	0	6153.194	6,725,092	0	0
	λ^1	18.259	0	0	0	3124.56	0	0	0
	λ^0	−436.8	0	0	0	67,200	0	0	0

Table 4. Calculated values of Hurwitz stability criteria of a new system (1).

Equilibrium point	Before Before Control	After Control
$(0,0,0,0,0,0)$	$\Delta_1 = 12.5$	$\Delta_1 = 40$
	$\Delta_2 = -2894.74$	$\Delta_2 = 20580$
	$\Delta_3 = 39977.383$	$\Delta_3 = 75675$
	$\Delta_4 = -56781.35$	$\Delta_4 = 300274$
	$\Delta_5 = 79321.51$	$\Delta_5 = 933521$
	$\Delta_6 = 103579.12$	$\Delta_6 = 2513222$

Table 5. Calculated values of continued fraction stability criteria of new.

Equilibrium point	Before Control	After Control
$(0,0,0,0,0,0)$	$h_1 = 0.08$	$h_1 = 0.025$
	$h_2 = -0.053$	$h_2 = 0.077$
	$h_3 = 0.809$	$h_3 = 0.139$
	$h_4 = -0.21$	$h_4 = 0.16$
	$h_5 = -21.108$	$h_5 = 6.275$
	$h_6 = 0.147$	$h_6 = 0.054$

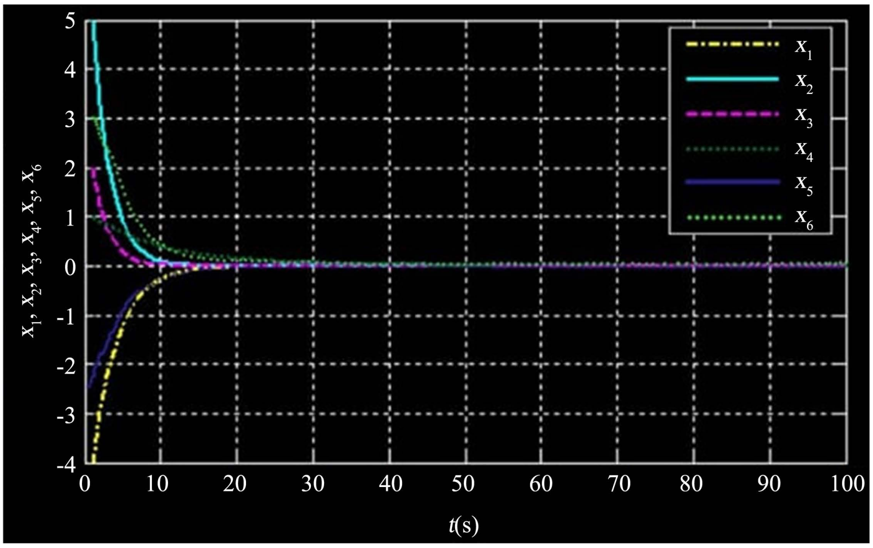


Figure 5. The behavior of state variables $x_1, x_2, x_3, x_4, x_5, x_6$ for the controlled (10).

7. Electronic Circuit Proposed

This section outlines the electronic circuit design for the 6-D chaotic system (1). Resistors, capacitors, multipliers, and operational amplifiers TL034CN are the electronic components that make up this device. Using Kirchhoff's laws [25], we may arrive to the following equations for the analogous circuit

$$\begin{aligned}
\frac{dV_{x_1}}{dt} &= \frac{1}{R_1C_1}\left(V_{x_1} - V_{x_2}\right) + \frac{1}{R_2C_1}V_{x_4} \\
\frac{dV_{x_2}}{dt} &= \frac{1}{R_3C_2}V_{x_1} + \frac{1}{R_4C_2}\left(V_{x_5} - V_{x_2}\right) - \frac{1}{R_5C_2}V_{x_1}V_{x_3} \\
\frac{dV_{x_3}}{dt} &= -\frac{1}{R_6C_3}V_{x_3} + \frac{1}{R_7C_3}V_{x_1}V_{x_2} \\
\frac{dV_{x_4}}{dt} &= \frac{1}{R_8C_4}V_{x_4} - \frac{1}{R_9C_4}V_{x_1}V_{x_3} \\
\frac{dV_{x_5}}{dt} &= -\frac{1}{R_{10}C_5}V_{x_2} \\
\frac{dV_{x_6}}{dt} &= \frac{1}{R_{11}C_6}V_{x_2} + \frac{1}{R_{12}C_6}V_{x_6}
\end{aligned} \tag{17}$$

where the output voltages are $V_{x_1}, V_{x_2}, V_{x_3}, V_{x_4}, V_{x_5}$ and V_{x_6}, and the fixed multipliers constant is $k_m = 10v$, the outputs are

$$V_{x_1x_2} = \frac{V_{x_1}V_{x_2}}{k_m}, \cdots, V_{x_1x_6} = \frac{V_{x_1}V_{x_6}}{k_m}$$

dimensionless state variables were used to control voltage and time

$$V_{x_1} = 1\,\text{V} \cdot x_1, \cdots, V_{x_6} = 1\,\text{V} \cdot x_6, t' = \tau \cdot t = 100\ \mu\text{s} \cdot t \tag{18}$$

when we replace (18) in the system (17) equations, we get:

$$
\begin{aligned}
\frac{\mathrm{d}x_1}{\mathrm{d}t'} &= \frac{I}{R_1C_1}(x_1 - x_2) + \frac{I}{R_2C_1}x_4 \\
\frac{\mathrm{d}x_2}{\mathrm{d}t'} &= \frac{I}{R_3C_2}x_1 + \frac{I}{R_4C_2}(x_5 - x_2) - \frac{I}{R_5C_2}x_1x_3 \\
\frac{\mathrm{d}x_6}{\mathrm{d}t'} &= -\frac{I}{R_6C_3}x_3 + \frac{I}{R_7C_3}x_1x_2 \\
\frac{\mathrm{d}x_4}{\mathrm{d}t'} &= \frac{I}{R_8C_4}x_4 - \frac{I}{R_9C_4}x_1x_3 \\
\frac{\mathrm{d}x_5}{\mathrm{d}t'} &= -\frac{I}{R_{10}C_5}x_2 \\
\frac{\mathrm{d}x_6}{\mathrm{d}t'} &= \frac{I}{R_{11}C_6}x_2 + \frac{I}{R_{12}C_6}x_6
\end{aligned}
\tag{19}
$$

Systems (1) and (19) compared side by side produce the following posterior conditions:

$$
\begin{aligned}
&\frac{I}{R_1C_1} = a, \frac{I}{R_3C_2} = c, \frac{I}{R_6C_3} = b, \frac{I}{R_8C_4} = d, \frac{I}{R_{10}C_5} = k, \frac{I}{R_{11}C_6} = I, \\
&\frac{I}{R_{12}C_6} = h, \frac{I}{R_2C_1} = \frac{I}{R_4C_2} = \frac{I}{R_5C_2} = \frac{I}{R_7C_3} = \frac{I}{R_9C_4} = 1
\end{aligned}
\tag{20}
$$

with the following parameters: $a = 10$, $b = 2.6$, $c = 28$, $d = 2$, $k = 8.4$, $l = h = 1$, we obtained the empirical electrical circuit (19) for system (1).

8. Results of the Simulation

This part uses MultiSIM12 to simulate the circuit created to electronically implement the chaotic system (1). Figure 6 shows the chaotic system (1)'s circuit diagram. Figure 7 shows the phase diagrams of an electronic circuit and the output voltage signals $V_{x_1}, V_{x_2}, V_{x_3}, V_{x_4}, V_{x_5}$ and V_{x_6} versus time. Comparing Figure 7 from MultiSIM 12 to Figure 1, Figure 3 and Figure 5, which were obtained from MATLAB, we can see that there is a strong qualitative agreement between experimental successes and numerical simulation.

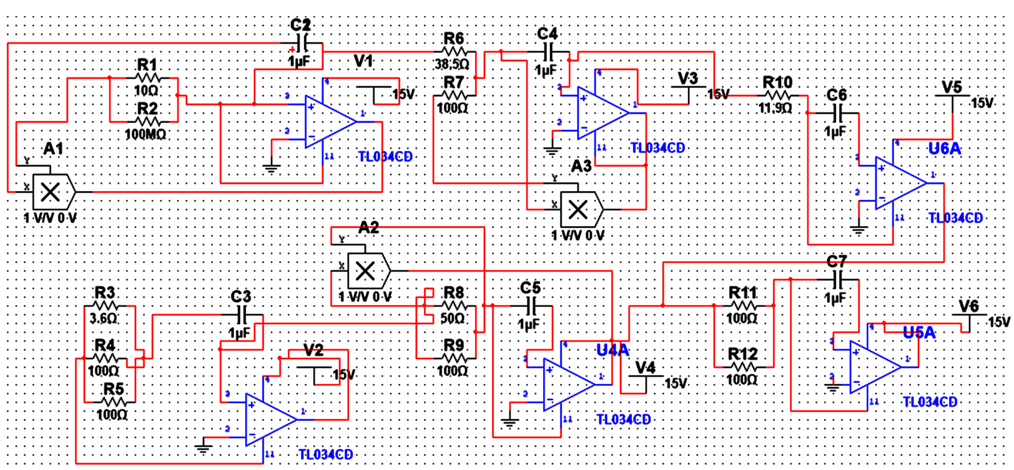
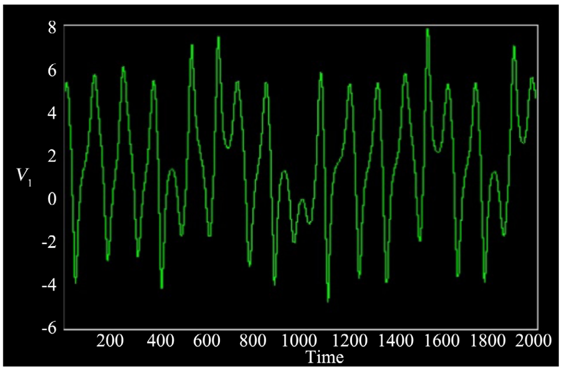
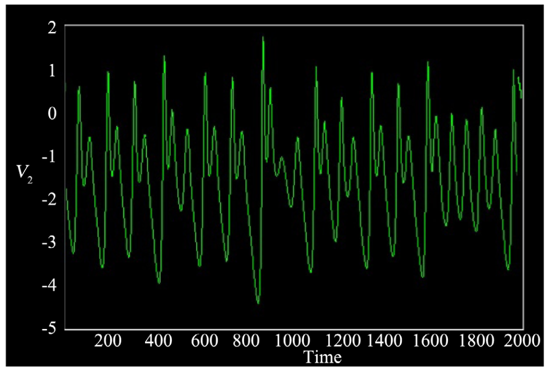


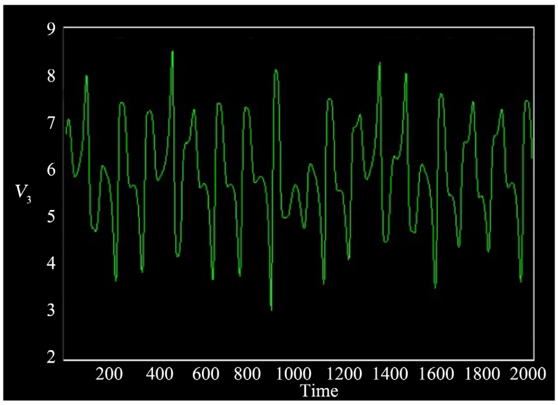
Figure 6. A schematic for an electronic circuit created for a chaotic (1.1) system.



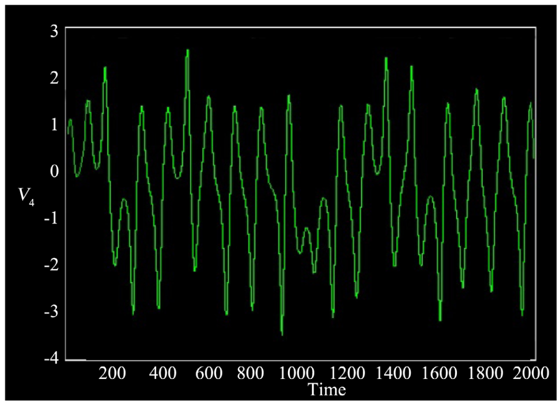
(a)



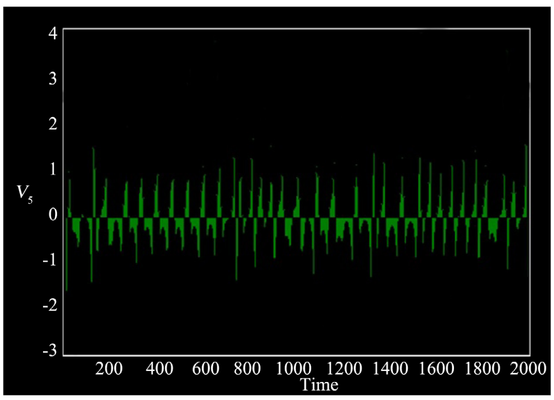
(b)



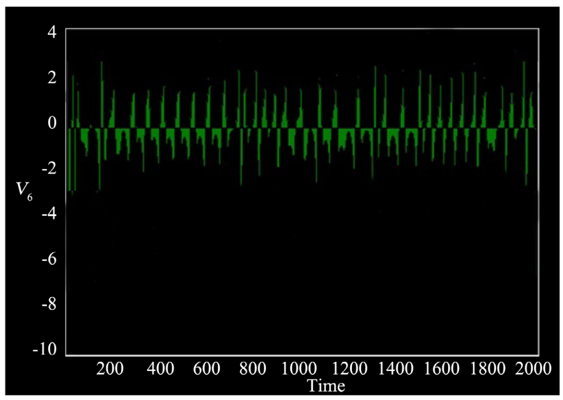
(c)



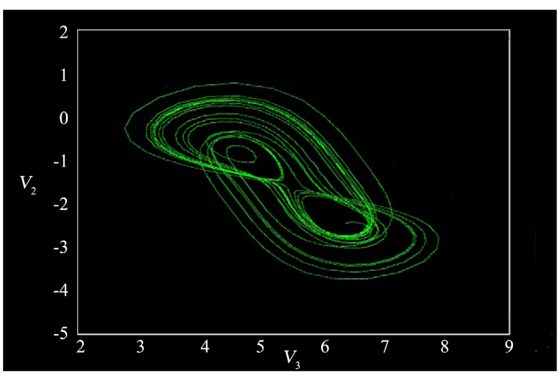
(d)



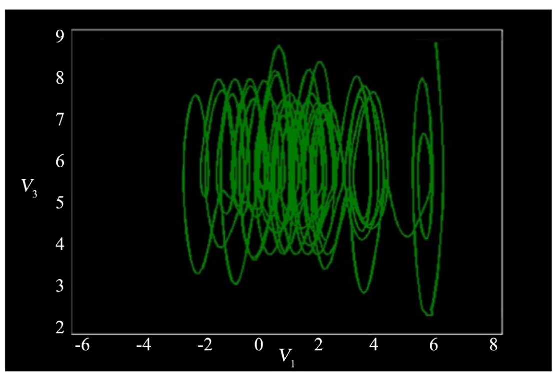
(e)



(f)



(g)



(h)

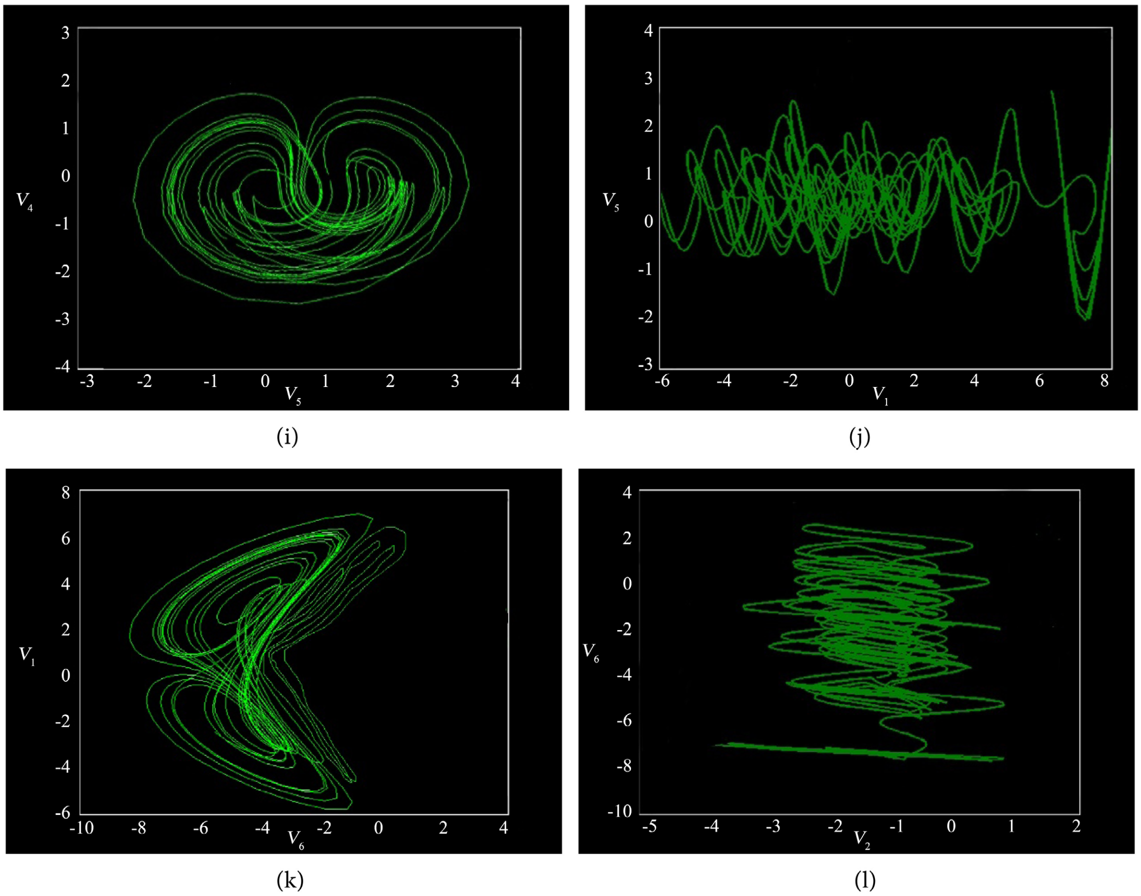


Figure 7. The right side displays phase portraits of the proposed electronic circuit in A (V_1 versus time), B (V_2 versus time), C (V_3 versus time), D (V_4 versus time), E (V_5 versus time), F (V_6 versus time) While The output voltage signals are displayed on the left as seen in G (V_2, V_3), H (V_2, V_3), I (V_4, V_5), J (V_5, V_1), K (V_1, V_6), L (V_6, V_2).

9. Conclusion

In this study, a six-dimensional model of continuous dynamical systems was taken. The permissible equilibrium points for the analysis of this system were found, and the parameters of stability were evaluated in various ways, which are:

- Roots of the characteristic equation.
- Routh stability criterion.
- The criterion of the stability of Hurwitz.
- Lyapunov function.
- Fractional stability criterion.
- The Lyapunov exponentially was examined and the hexagonal system was found to be chaotic. The proposed system dissipation detected Hopf bifurcation, and then the system was regulated using an adaptive control approach. Finally, for the system under study, the numerical and morphological results before and after the control were compared.
- Finally, an electric circuit was designed as an application on a hexagonal chaotic system, and was analyzed with the same methods of analyzing the hexagonal system, as the results obtained showed good agreement that the

designed circuit simulated the theoretical model.

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Conflicts of Interest

The authors declare no conflicts of interest.

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