



The Gravitational and Electromagnetic Field

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Abstract

We introduce the concept of equivalent electromagnetic (EEM) field, on the basis of which we describe the gravitational field (G-field). We determine this EEM field based on the principle of equivalence (PE), *i.e.* by direct application of PE in the field equations and in the equations of motion, by introducing the EEM field potential. In this way, we introduce the mathematical formalism of the electromagnetic (EM) field into the G-field equations. This procedure of describing the G-field has a limitation and can be applied to static, stationary and quasi-stationary fields. Obtained equations and solutions are compared with the General Theory of Relativity and the differences are analyzed.

Subject Areas

Theoretical Physics, General Theory of Relativity, Astronomy, Gravitation and Cosmology

Keywords

Gravitational Field, Theory of Gravity, Theory of Relativity, Principle of Equivalence, Time and Space

1. Introduction

Solving the gravitational field problem is approached in the simplest possible way. In this approach the gravitational field (G-field) is described indirectly through some other field, which we named the equivalent electromagnetic (EEM) field. This EEM field is found from the Principle of equivalence (PE), *i.e.* through the movement of elementary particles in a given medium of the field source. Name of the electromagnetic in EEM comes from the fact that PE can be observed up to the level of elementary particles in the environment dominated by EM force.

An action is proposed to indirectly describe the G-field, thus the G-field equations can be obtained without action to directly describe the G-field, which in

the General Theory of Relativity contains a scalar curve [1]. The resulting field equations differ from the equations of the General theory of relativity by the terms that represent the correction of the second and higher order. Thus, in a centrally symmetric field, by taking into account the energy of the gravitational field, we obtain an additional field as a correction of the second order in the field equations. The results that follow from here differ by small values from the case of treating the G-field as empty space.

A particularly interesting solution is for the case of a centrally symmetric static field in which clock synchronization can be established and observers can be introduced at some distance from the center of symmetry. The concept of measuring length and time is based on the observation of a light signal by a given observer. Since each observer has its own standards of length and time; they are compared to each other and thus we can completely describe the field.

2. Principle of Equivalence

We observe the source of G-fields (continuous medium or set of bodies) and the element of continuous medium density ρ ; it interacts with the G-field with other particles of matter in the massive body of the field source. However, this particle is not free but is in the environment of other matter with which it interacts; another interaction is established that is equivalent to the gravitational interaction between the observed elements (equivalent in intensity and opposite in direction). Therefore, this interaction has a repulsive character and is opposed to gravitational attraction, otherwise there would be a gravitational collapse of the body; which is analogous to repulse charges of the same sign that form the source of an EM field, only here does an external force hold the charges together. In this view, the active force is gravitational, and the EEM field force is taken as the generalized reaction force.

In the language of theoretical mechanics, equations are found by the direct application of PE, which consists in the fact that the active (gravitational) force is equal to the reaction force (the equivalent EM force named here) with the opposite direction. Introducing a modified Lagrangeian which, among other terms, also contains terms related to the generalized reaction force by appropriate selection of generalized coordinates. By varying such a modified action, we obtain the field and motion equations, which contain these reaction forces and the corresponding field tensors.

Since we have introduced the EEM field based on PE, we can use the mathematical formalism of the EM field in describing the G-field.

Now we can introduce the potentials of the EEM field, then the current density vector of matter and other quantities that characterize that field.

Let's assume that the potentials EEM field are composed by coefficients of metrics and let them be¹:

¹Here we have introduced designation $h = g_{00}, g_{\alpha} = -\frac{g_{0\alpha}}{g_{00}}$ for scalar and vector in the space with three-dimensional tensor $\gamma^{\alpha\beta}$.

$$A_0 = \sqrt{h}, \quad A_\alpha = -\sqrt{h}g_\alpha, \quad \text{or} \quad A_i = \frac{g_{0i}}{\sqrt{g_{00}}} \quad (1.1)$$

This choice of the vector A_i ensures compliance with PE above defined and it is applicable in the case of the static and stationary fields, and there is a limitation for variable field, so that valid in the case of quasi-stationary motion, this definition of PE cannot be applied in other cases of the variable field, nor the corresponding quantity A_s , as can be seen from the equations that follow.

This can also be seen from the definition of when a quantity is a vector. So A_i is a vector in a stationary field, because we have:

$$ds = \left(\sqrt{h} dx^0 - \sqrt{h} g_\alpha dx^\alpha \right) \sqrt{1 - v^2/c^2} \quad (1.2)$$

where v^α is the three-dimensional velocity vector in the space in which the metric defined by the tensor $\gamma^{\alpha\beta}$:

$$v^\alpha = \frac{c dx^\alpha}{\sqrt{h} (dx^0 - g_\alpha dx^\alpha)} \quad (1.3)$$

It follows that the A_i behaves as a vector in relation to the transformation, which do not change the stationarity of the field:

$$x^\alpha \rightarrow x'^\alpha, \quad x^0 \rightarrow x'^0 + f(x^\alpha) \quad (1.4)$$

In such a transformation:

$$\sqrt{h} \rightarrow \sqrt{h'}, \quad g_{0i} = \frac{\partial x'^k}{\partial x^i} g'_{0k} \quad (1.5)$$

Field Equations

If we assign the potentials A_s try to derive the equation for the equivalent electromagnetic (EEM) field; they get from the principle of least action, $\delta S = 0$, *i.e.* variations of the action of S . The total action for the interaction of matter with equivalent electromagnetic field has the equation form:

$$S = S_E + S_m + S_I = \int \left(\Lambda^E \sqrt{-g} - \sqrt{\mathfrak{F}_i \mathfrak{F}^i} + A_i j^i \sqrt{-g} \right) d\Omega \quad (2.1-i)$$

The density of Lagrangian EEM field (Λ^E) are used in the form $\Lambda^E = -(16\pi\gamma)^{-1} F_{ik} F^{ik}$, with a minus sign, because the EEM force between the particles of matter that generates G-field is always repulsive, because the system spontaneously tends towards minimum gravitational energy, condensation and collapse of the system occurred;

S_m is the term of continuous distribution of matter and it is obtained from the action of isolated particle of mass m : $-m \int ds$, substituting m for $\rho \sqrt{g} u^0 dx^1 dx^2 dx^3$, then the label was introduced $\mathfrak{F}^i = \rho u^i \sqrt{g}$; the interactional term S_I is part of the action is conditioned by the presence of matter density ρ , *i.e.* interaction of matter and EEM field, because the matter establishes EEM interaction between its elementary particles which is equivalent to gravitational interaction between these elements, this interaction has repulsive character as the force between the

charge of the same sign, and because the sign of gravitational potential we put a minus sign in front Lagrangian this term.

In order varying by g_{ik} which directly describe the G-field in the action S we introduce a term $S_g = \int -R\sqrt{-g}d\Omega$, where the geometric a term R refers to the gravitational field and it consists of g_{ik} and its derivatives.

$$S = S_g + S_E + S_m + S_I = \int \left(-R\sqrt{-g} + \Lambda^E \sqrt{-g} - \sqrt{\mathfrak{S}_i \mathfrak{S}^i} + A_i j^i \sqrt{-g} \right) d\Omega \quad (2.1-ii)$$

By varying the total action S quantities that characterize the observed fields and their interactions, we have:

$$\delta S = \int \left[-\frac{c^3}{16\pi\gamma} \left(R_{ik} - \frac{1}{2} g_{ik} R - \frac{8\pi\gamma}{c^4} T_{ik}^E - \frac{8\pi\gamma}{c^4} \rho u_i u_k \right) \delta g^{ik} \sqrt{-g} - \left(\frac{c^4}{4\pi\gamma} F^{ik}{}_{;k} - j^i \right) \delta A_i \sqrt{-g} + u_i \delta \mathfrak{S}^i + A_i \delta \left(j^i \sqrt{-g} \right) \right] d\Omega = 0 \quad (2.2)$$

For complete arbitrariness A_i coefficient we equal to zero before the δA_i and find an expression for the field EEM²:

$$F^{ik}{}_{;k} = \frac{4\pi\gamma}{c^4} j^i, \quad (2.3)$$

or components taking potential (1.1), Equation (2.3) is transformed into form:

$$F_{0k}{}^{;k} = \left(\sqrt{h} \right)_{;\alpha}{}^{;\alpha} - \frac{1}{\sqrt{h}} \left(\sqrt{h} \right)_{;\alpha} \left(\sqrt{h} \right)_{;\alpha}{}^{;\alpha} + \frac{h\sqrt{h}}{2} f^{\alpha\beta} f_{\alpha\beta} = \frac{4\pi\gamma}{c^4} j_0; \quad (2.3-i)$$

$$F_{\alpha k}{}^{;k} = \sqrt{h} f_{\alpha\beta}{}^{;\beta} + 2 \left(\sqrt{h} \right)_{;\beta}{}^{;\beta} f_{\alpha\beta} = \frac{4\pi\gamma}{c^4} j_\alpha. \quad (2.3-ii)$$

Coefficient of δg_{ik} provides:

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi\gamma}{c^4} T_{ik}^E + \frac{8\pi\gamma}{c^4} \rho u_i u_k \quad (2.4)$$

where

²Current four-vector in constant field takes the form $j^k = \rho / \sqrt{1-(v/c)^2} u^k$. For the point system of the body, density of matter we can write as $\rho = \sum_a \frac{m_a}{\sqrt{\gamma}} \delta(\mathbf{r} - \mathbf{r}_a)$. Using the equation of continuity $j^i{}_{;i} = 0$, provided $(\rho u^k)_{;k} = 0$, we find the following equation:

$\rho u^k \frac{\partial}{\partial x^k} \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{(1-v^2/c^2)^{3/2}} \frac{\rho u^\alpha}{c^2} v_{\alpha;\beta} v^\beta = 0$. (1) So, as expected in a stationary field strain rate tensor components are equal to zero, i.e. no shear or dilatation of a given environment. Condition (1) shall be in accordance with the equations of motion of the element of matter (2.9). In general, from Equation (2.9) we find: $\sqrt{h} \frac{d}{ds} \frac{1}{\sqrt{1-v^2/c^2}} = -\frac{1}{2} \frac{\partial \gamma_{\alpha\beta}}{\partial x^\alpha} u^\alpha u^\beta$. (2) Here we see the concurrence of Equations (1) and (2) in the stationary field; while the time-varying fields appears the problem of defining the velocity v , so Equation (2) resolves to a certain degree of accuracy, neglecting the terms of the Order $1/c^5$, where it gets approximate relation: $v^\alpha \frac{dv_\alpha}{dt} = v^\alpha \left(\frac{\partial v_\alpha}{\partial t} + \frac{\partial v_\alpha}{\partial x^\beta} v^\beta \right) \approx 0$, (3) i.e. condition that mater velocity flow should meet v in Equation (2.3), where v is the ordinary three-dimensional velocity ($v^\alpha = dx^\alpha/dt$).

$$T^{Ei}_k = \frac{1}{4\pi} \left(-F^{il} F_{kl} + \frac{1}{4} \delta^i_k F_{lm} F^{lm} \right) \tag{2.5}$$

term which with its energy contributes to G-field.

However, in this case (2.4) there is a limit, as a result of the election S_g in (2.1-ii). In order that the Equation (2.4) which describes the gravitational field be in accordance with the equation that describes the EEM field (2.3), condition needs to be fulfilled $g_\alpha = 0$ in (2.4). This is fulfilled in the static field.

For the remaining action in (2.2) we take into account the conservation of matter $\mathfrak{T}^i_{;i} = (\rho u^i)_{;i} = 0$ and the equation of continuity $j^i_{;i} = 0$. Suppose that each element of the matter moved to the small size b^i , in this to determine the result of changes \mathfrak{T}^i . From the literature, we find:

$$\delta \mathfrak{T}^i = (\mathfrak{T}^k b^i - \mathfrak{T}^i b^k)_{;k} \tag{2.6}$$

In the same way we change the vector density and flow of matter $j^i \sqrt{g}$ and we get:

$$\delta (j^i \sqrt{g}) = (j^k \sqrt{g} b^i - j^i \sqrt{g} b^k)_{;k} \tag{2.7}$$

When you make a replacement (2.6) and (2.7) in (2.2), we calculated:

$$\int [u_i \delta \mathfrak{T}^i + A_i \delta (j^i \sqrt{-g})] d\Omega = \int [-u_{i;k} \rho u^k b^i \sqrt{-g} - F_{ik} j^k b^i \sqrt{-g}] d\Omega = 0 \tag{2.8}$$

Finally the coefficient of b^i gives:

$$\rho u_{i;k} u^k = -F_{ik} j^k \tag{2.9}$$

Skip to calculate the equation of conservation in the field that describes the action of S in (2.1-ii). Return to the integral of δg_{ik} and δg_{ik} present as a result of the transformation of coordinates. Based on the formula for the transformation tensor, the small changes of coordinates x^i in coordinates $x^i = x^i + \xi^i$, we find transformation of the components g^{ik} in the form:

$$g^{ik} = g^{ik} + \delta g^{ik}; \quad \delta g^{ik} = \xi^{i;k} + \xi^{k;i} \tag{2.10}$$

In Equation (2.2) put δg^{ik} in (2.10); after appropriate transformations we find the following expression:

$$\int \left[-\frac{c^3}{16\pi\gamma} \left(R^k_i - \frac{1}{2} \delta^k_i R - \frac{8\pi\gamma}{c^4} T^{Ei}_k - \frac{8\pi\gamma}{c^4} \rho u_i u^k \right) \xi^i \sqrt{-g} \right]_{;k} d\Omega = 0 \tag{2.11}$$

Due to the complete arbitrariness ξ^i we can write:

$$T^k_{i;k} + \rho \frac{Du_i}{ds} + (\rho u^k)_{;k} u_i = 0 \tag{2.12}$$

So we get the equation of motion element of continuous medium density ρ . After replacing (2.3) in (2.12) we find:

$$F_{il} j^l + \rho \frac{Du_i}{ds} + (\rho u^k)_{;k} u_i = 0, \tag{2.13}$$

Multiplying by u^i gives the equation of conservation of matter $(\rho u^i)_{;i} = 0$, and (2.13) is reduced to (2.9).

Taking divergence Equation (2.3) and the conditions,² we obtain an additional relation, which joins the Equation (3.5):

$$\frac{1}{2} F^{\alpha\beta} \frac{\partial \gamma_{\gamma\delta}}{\partial x^\alpha} \frac{\partial \gamma^{\gamma\delta}}{\partial x^\beta} + F^{0\alpha} \frac{\partial \gamma_{\gamma\delta}}{\partial x^\alpha} \frac{\partial \gamma^{\gamma\delta}}{\partial x^0} = 0 \quad (2.14)$$

We arrive at the solution of Equations (2.3) by applying an approximate calculation. Developing in the order of the component g_{ik} and then solving Equations (2.3) approximatively to some degree of accuracy.

The calculations of the g_{ik} coefficients, for weak fields, are done by approximations. We decompose the covariant components of the metric tensor into small first order corrections:

$$g_{ik} = g_{ik}^{(0)} + h_{ik}^{(1)}. \quad (2.15)$$

Of course, at zero approximation, the metric $g_{ik}^{(0)}$ is Euclidean and takes Galileo values. We consider the stationary case and Equation (2.3-*i*). In the second approximation, we omit the third term, as a small higher order size ($\sim v^2/c^2$). After easy calculations neglecting the terms of order $1/c^6$ we arrive at the results:

$$h_\alpha^\beta = -\frac{2}{c^2} \varphi \delta_\alpha^\beta, \quad h_{00} = \frac{2}{c^2} \varphi + o\left(\frac{1}{c^4}\right); \quad (2.16)$$

$$\frac{1}{2} \Delta h_{00} + \frac{2}{c^4} \varphi \Delta \varphi - \frac{3}{c^4} (\nabla \varphi)^2 = \frac{4\pi\gamma}{c^2} \sum_\alpha m_\alpha \left(1 + \frac{5\varphi_\alpha}{c^2} + \frac{v_\alpha^2}{c^2} \right) \delta(\mathbf{r} - \mathbf{r}_\alpha) \quad (2.17)$$

We find the final solution in the form:

$$h_{00} = \frac{2\varphi}{c^2} + \frac{3\varphi^2}{c^4} - \frac{2\gamma}{c^4} \sum_\alpha \frac{m_\alpha v_\alpha^2}{|\mathbf{r} - \mathbf{r}_\alpha|}. \quad (2.18)$$

We can look for the solution of Equation (2.3-*ii*) only in approximate form neglecting the terms of order $1/c^5$:

$$\Delta h_{0\beta} = -\frac{4\pi\gamma}{c^3} \sum_a m_a v_{a\beta} \delta(\mathbf{r} - \mathbf{r}_a). \quad (2.19)$$

From here we get

$$h_{0\beta} = \frac{\gamma}{c^3} \sum_\beta \frac{m_\beta v_{\beta\alpha}}{|\mathbf{r} - \mathbf{r}_\beta|}. \quad (2.20)$$

In the case of a quasi-stationary field, the time-dependent Equation (2.3-i) with the addition (2.23-i), and with appropriate supplementary condition, is as follow $\partial g^\beta/\partial x^\beta = 0$; can be solved in the approximation of the given densities to a certain degree of accuracy as in the case of a stationary field, and the solution is of the same form as (2.18) and is:

$$h_{00} = \frac{2\varphi}{c^2} + \frac{3\varphi^2}{c^4} - \frac{2\gamma}{c^4} \sum_a \frac{m_a v_a^2}{|\mathbf{r} - \mathbf{r}_a|}. \quad (2.21)$$

Only now the movement of matter has a quasi-stationary character. Here the velocity v is defined as $v^\alpha = dx^\alpha/dt$. The following Equation (2.3-*ii*) with the addition of (2.23-*ii*) in a given approximation goes into a form in which only one term of order $1/c^3$ appears, that is $\partial^2 \sqrt{h}/\partial x^0 \partial x^\alpha$, a term. With the calibration

condition $\partial g^\beta/\partial x^\beta = 0$ and the corresponding substitutions we get:

$$h_{0\alpha} = \frac{\gamma}{2c^3} \sum_b \frac{m_b}{|\mathbf{r} - \mathbf{r}_b|} (v_{b\alpha} + (\mathbf{v}_b \mathbf{n}_b) n_{b\alpha}) \tag{2.22}$$

where \mathbf{n}_b is the unit vector in the direction of the vector $\mathbf{r} - \mathbf{r}_b$.

Go back to the time-variable fields and display the Equation (2.3) in the general form³:

$$F_{0k;l} g^{lk} = \sqrt{h} \left(\frac{\partial g_\beta}{\partial x^0} \right)^{;\beta} + \frac{\partial g^\alpha}{\partial x^\alpha} \frac{\partial \sqrt{h}}{\partial x^0}, \tag{2.23-i}$$

$$\begin{aligned} F_{\alpha k;l} g^{lk} &= \frac{1}{h} \frac{\partial^2 \sqrt{h}}{\partial x^0 \partial x^\alpha} - \frac{1}{h \sqrt{h}} \frac{\partial \sqrt{h}}{\partial x^0} (\sqrt{h})_{;\alpha} - \frac{1}{h} (\sqrt{h})^{;\beta} \frac{\partial \gamma_{\alpha\beta}}{\partial x^0} \\ &+ \frac{1}{\sqrt{h}} \frac{\partial^2 g_\alpha}{\partial x^{02}} + \frac{1}{h} \frac{\partial \sqrt{h}}{\partial x^0} \frac{\partial g_\alpha}{\partial x^0} - \frac{1}{\sqrt{h}} \frac{\partial g^\beta}{\partial x^0} \frac{\partial \gamma_{\alpha\beta}}{\partial x^0} + \sqrt{h} \frac{\partial g^\beta}{\partial x^0} \frac{\partial g_\beta}{\partial x^\alpha} \\ &- \sqrt{h} \frac{\partial g_\alpha}{\partial x^0} \frac{\partial g^\beta}{\partial x^\beta} + \left(\gamma^{\beta\gamma} \frac{\partial \gamma_{\beta\gamma}}{\partial x^0} \right) \left(\frac{1}{2h} (\sqrt{h})_{;\alpha} + \frac{1}{2\sqrt{h}} \frac{\partial g_\alpha}{\partial x^0} \right) \end{aligned} \tag{2.23-ii}$$

For comparison, we find the R_{ik} tensor for the general case depending on x^0 :

$$\begin{aligned} R_{00} &= h \left(\frac{\partial g_\beta}{\partial x^0} \right)^{;\beta} + \sqrt{h} \frac{\partial g^\alpha}{\partial x^\alpha} \frac{\partial \sqrt{h}}{\partial x^0} + 2\sqrt{h} (\sqrt{h})^{;\beta} \frac{\partial g_\beta}{\partial x^0} + h \frac{\partial g_\beta}{\partial x^0} \frac{\partial g^\beta}{\partial x^0} \\ &- \frac{\partial}{\partial x^0} \left(\frac{1}{2} \gamma^{\beta\gamma} \frac{\partial \gamma_{\beta\gamma}}{\partial x^0} \right) + \frac{1}{2} \frac{1}{\sqrt{h}} \frac{\partial \sqrt{h}}{\partial x^0} \gamma^{\beta\gamma} \frac{\partial \gamma_{\beta\gamma}}{\partial x^0} - \frac{1}{4} \gamma^{\beta\gamma} \frac{\partial \gamma_{\beta\gamma}}{\partial x^0} \gamma^{\alpha\delta} \frac{\partial \gamma_{\alpha\delta}}{\partial x^0} \end{aligned} \tag{2.24-i}$$

$$\begin{aligned} R_{0\alpha} &= \frac{1}{2} \left(\frac{\partial \gamma_{\alpha\beta}}{\partial x^0} \right)^{;\beta} + \frac{1}{h} \frac{\partial \sqrt{h}}{\partial x^0} (\sqrt{h})_{;\alpha} - \frac{1}{2} \frac{1}{\sqrt{h}} (\sqrt{h})^{;\beta} \frac{\partial \gamma_{\alpha\beta}}{\partial x^0} - \frac{1}{2} h \frac{\partial g_\alpha}{\partial x^0} \frac{\partial g^\beta}{\partial x^\beta} \\ &+ \frac{1}{2} h \frac{\partial g^\beta}{\partial x^0} \frac{\partial g_\beta}{\partial x^\alpha} - \frac{1}{2} \frac{\partial}{\partial x^\alpha} \left(\gamma^{\beta\gamma} \frac{\partial \gamma_{\beta\gamma}}{\partial x^0} \right) + \frac{1}{2\sqrt{h}} (\sqrt{h})_{;\alpha} \gamma^{\beta\gamma} \frac{\partial \gamma_{\beta\gamma}}{\partial x^0} \end{aligned} \tag{2.24-ii}$$

3. Static Field

3.1. The Case of Central Symmetric Gravitational Field

Our starting point is Equation (2.4), and consider six additional conditions for the central symmetric field: let all three components $g_{0\alpha}$ be equal to zero and $g_{12} = g_{13} = g_{23} = 0$. Now we define the coordinate system in this form:

$$ds^2 = g_{00} c^2 dt^2 - \gamma_{11} dr^2 - r^2 (\sin^2 \theta d\phi^2 + d\theta^2) \tag{3.1}$$

From (2.4) and (3.1) for fields outside the source ($\rho = 0$) solution is form $g_{00} \equiv h = \gamma^{11} = 1 + \frac{c_1}{r} + \frac{c_2}{r^2}$. The same solution with two constants we get from the conditions $R = 0$.

Now the components of tensor EEM field (2.5) are:

$$T_0^0 = T_1^1 = -T_2^2 = -T_3^3 = \frac{1}{8\pi\gamma} \left(\frac{\partial \sqrt{h}}{\partial r} \right)^2 \frac{\gamma^{11}}{g_{00}} = \frac{1}{8\pi\gamma} \frac{\gamma^2 m^2}{r^4}. \tag{3.2}$$

From here we find the metric space time of the static field:

³In the following equations, only the terms containing the derivative of x^0 are shown, to which the other terms in (2.3-i) and (2.3-ii) for the constant field should be added.

$$ds^2 = \left(1 - \frac{\gamma m}{c^2 r}\right)^2 c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{\gamma m}{c^2 r}\right)^2} - r^2 (\sin^2 \theta d\phi^2 + d\theta^2) \quad (3.3)$$

We transform metric (3.3) into new coordinates (t', θ, ϕ', t') :

$$t = t', \quad r = r' + \frac{\gamma m}{c^2}, \quad \theta = \theta', \quad \phi = \phi', \quad (3.4-i)$$

and we get:

$$ds = \frac{c^2 dt'^2}{\left(1 + \frac{\gamma m}{c^2 r'}\right)^2} - \left(1 + \frac{\gamma m}{c^2 r'}\right)^2 (dr'^2 + r'^2 d\theta'^2 + r'^2 \sin^2 \theta' d\phi'^2) \quad (3.4-ii)$$

3.2. The General Form of Metrics

The above Equation (3.4-ii) is a special case of a general spherically symmetric metric form given in the form:

$$ds^2 = e^\nu c^2 dt'^2 - e^\mu dr'^2 - e^\mu (\sin^2 \theta' d\phi'^2 + d\theta'^2). \quad (3.5)$$

By applying the same procedure; by solving the equation $R_{ik} = T_{ik}$, where T_{ik} is given by Equation (2.2), we obtain the following solution:

$$ds^2 = \frac{1}{\left(1 + \frac{\gamma m}{c^2 r'}\right)^2} c^2 dt'^2 - \left(1 + \frac{\gamma m}{c^2 r'}\right)^2 [dr'^2 + r'^2 (\sin^2 \theta' d\phi'^2 + d\theta'^2)]. \quad (3.6)$$

We get to the same solution (3.6) by using the metrics (3.5) and by solving the equations for the EEM field (2.3). To this static equation, we should also add the relation (3.15) that follows from the equations of motion, on the basis of the equivalence principle, in order to complete the number of unknown quantities.

Here the coordinate r' is calibrated (standardized) in the absolute reference system (the observer is located at infinite distance from the center of symmetry).

Metrics (3.3) and (3.6) refer to observers who measure from an absolute reference system ($R \rightarrow \infty$) using different length standards.

Measurements can also be made by observers that are stationary at the distance R from the center, then we introduce a shift: $r_R = r'(1 + \gamma m/c^2 R)$ and $t = t_R(1 + \gamma m/c^2 R)$, and we get the equation:

$$ds^2 = \left(\frac{1 + \frac{\gamma m}{c^2 R}}{1 + \frac{\gamma m}{c^2 r'}}\right)^2 c^2 dt_R^2 - \left(\frac{1 + \frac{\gamma m}{c^2 r'}}{1 + \frac{\gamma m}{c^2 R}}\right)^2 [dr_R^2 + r_R^2 (\sin^2 \theta d\phi^2 + d\theta^2)] \quad (3.7)$$

$$ds^2 = \frac{1}{\left(1 + \frac{\gamma m}{c^2 r_R} - \frac{\gamma m}{c^2 R}\right)^2} c^2 dt_R^2 - \left(1 + \frac{\gamma m}{c^2 r_R} - \frac{\gamma m}{c^2 R}\right)^2 [dr_R^2 + r_R^2 (\sin^2 \theta d\phi^2 + d\theta^2)] \quad (3.7-i)$$

From here it follows that the observers at some distance R can be attributed to

the corresponding standards of length and time and make calculations with them.

According to the shape of the spherical part of metric (3.3), it can be concluded that the coordinate r is determined by its proper systems, *i.e.* for each distance r , the observer at that distance defines its standard of length.

In the metric form (2.6) the coordinate r' is determined by the absolute observer, *i.e.* the length standard was measured at a great distance from the field source; and using only this length standard, we measure the distance in the field.

In metric (3.3), the length standards are obtained by measuring the length in their proper reference system, *i.e.* at points of the field at the same distance r , which are spherical shells $ds^2 = r^2(\sin^2\theta d\varphi^2 + d\theta^2)$, or $ds = dl_0 = rd\varphi$. In the next step, we determine the length of the measuring rod at the point of the field, at the distance r' (measured in units of the absolute observer, using metric (3.6)), or at the distance r (measured in its own system using metric (3.3)). Under measuring stick we denote a certain length of the stick made of solid materials, so that it cannot deform. For the same point in the field, only in two different coordinate systems we have:

$$ds = dl_0 = rd\varphi = r'd\phi \left(1 + \frac{\gamma m}{c^2 r'}\right) = dl' \left(1 + \frac{\gamma m}{c^2 r'}\right) = dl_r. \quad (3.8)$$

Because fixed RS at a distance r' ($r' = \text{const}$), the formula above changes in the form of: $\Delta l_0 = \Delta l' \left(1 + \frac{\gamma m}{c^2 r'}\right) = \Delta l_r$. This formula compares the length of a given measuring stick in proper system of observers at the distance r (Δl_0), *i.e.* at the distance r' (Δl_r), the length of that same stick Δl in units of absolute observer in absolute RS.

The connection between the coordinates r and r' in the two metrics (2.3) and (2.7) as we saw in (2.4-*i*) is $r = r' + \gamma m/c^2$, and the length between two points on a radius equal $\Delta r = r_2 - r_1 = r_2' - r_1' = \Delta r'$, *i.e.* length between two points on the radius: r_2 and r_1 , where r_1 and r_2 are given in units of observers at these distances, is equal to the length between the two points measured standard absolute observer.

These relations allow clock synchronization in the static field. For example, we notice two fixed internal and external observers at a distance R_A and R_B (in measurement units absolute observer). Let proper length of measuring stick in B be Δl_B , as length of the same stick in A is Δl_A . Connection between these two lengths we find by (3.8) and thus we get:

$$\frac{\Delta l_B}{\Delta l_A} = \frac{1 + \frac{\gamma m}{c^2 R'_B}}{1 + \frac{\gamma m}{c^2 R'_A}}. \quad (3.8-i)$$

3.3. Analysis of Space-Time Static Field

Consider the movement of the light signal in the gravitational field, because the measuring process of time and length based on the observation of light in the

selected reference system. Let us remember that the SR of the Lorentz type is one in which the speed of light is equal to c , and for it the metric is the same as in the special theory of relativity, which is the Minkowski metric: $ds^2 = c^2 dt_0^2 - dl_0^2$.

3.3.1. Propertie of Metric 1

Based on the metrics (3.3) it follows that the propagation of light ($ds = 0$) in the radial direction we have the following relationship:

$$c' = \frac{dr}{dt} = c\sqrt{h}\sqrt{\gamma^{11}} = c\left(1 - \frac{\gamma m}{c^2 r}\right)^2. \quad (3.9)$$

Here c' is the speed of light in point of distance r from the center of the sphere, measured by observers located at the large distance (*i.e.*, ∞) of a given massive body⁴. It is easily observable that there is a boundary radius r_g which stop photon, or any other particle in the field. In order to fulfill this, it is necessary to compress the total body mass into the radius volume $r_g = \gamma M/c^2$ which is known as black hole. Here r_g is given in units of observer at this distance, which translated into units of absolute observer is $r_g^1 = 0$.

Let's find the time needed to reach the gravitational radius. Everything is measured by the absolute observer:

$$cdt = \frac{dr}{\left(1 - \frac{\gamma m}{c^2 r}\right)^2} \quad (3.9-i)$$

$$c\Delta t = \left(r - \frac{\gamma m}{c^2}\right) \Big|_{r_1}^{r_2} + 2 \frac{\gamma m}{c^2} \ln \left(r - \frac{\gamma m}{c^2}\right) \Big|_{r_1}^{r_2} - \frac{\gamma^2 m^2}{c^4 \left(r - \frac{\gamma m}{c^2}\right)}$$

Schwartzschild's solution takes the form of:

$$Sh: c\Delta t = r \Big|_{r_1}^{r_2} + 2 \frac{\gamma m}{c^2} \ln \left(r - 2 \frac{\gamma m}{c^2}\right) \Big|_{r_1}^{r_2} \quad (3.9-ii)$$

In the case of a metric form that also takes into account the gravitational field energy density, (for an outside observer) light tends towards boundary radius r_g which is never actually achieved, but infinitely slowly approaching. In a system moving together with a particle, near the gravitational radius, time practically ceases to flow, so one should expect a finite time of reaching r_g .

3.3.2. Application of PE in the Equations of Motion

Now let a light signal moving in a centrally symmetric field be measured by an observer freely falling from an absolute reference system ($\sqrt{h} = 1$). To describe the measurements, we use the equations for the motion of light ($ds = 0$) and particle:

$$hd t'^2 = \gamma_{11} dr'^2 + \gamma_{22} dl'^2_{\theta,\phi}; \quad (3.10)$$

$$dt' = \frac{d\tau}{\sqrt{h}\sqrt{1 - v^2/c^2}}$$

⁴That is, the speed of light c' was measured from an absolute reference system.

From here we find an element of proper time for observer who fall freely and measured light signal in the element of spacetime $(t + dt, \theta + d\theta, \phi + d\phi, t + dt)$, it is all expressed in the absolute coordinates of the observer:

$$d\tau = \sqrt{1 - v^2/c^2} \sqrt{\gamma_{11} dr'^2 + \gamma_{22} dl_{\theta,\phi}^2} \tag{3.10-i}$$

From the upper equations and it follows⁵:

$$d\tau = \sqrt{1 - v_{pA}^2/c^2} \sqrt{h\gamma_{11} dr'^2 + h\gamma_{22} dl_{\theta,\phi}^2} \tag{3.10-ii}$$

When we follow that same light signal from the absolute observer system and imagine that there is no G-field, *i.e.* in the system AO ($R \rightarrow \infty$) we have borderline ($\sqrt{h} = 1$) when the G-field disappears, we get:

$$d\tau = \sqrt{1 - v_{pA}^2/c^2} \sqrt{dr'^2 + dl_{\theta,\phi}^2} \tag{3.11}$$

From Equations (3.10) and (3.11) on the basis of the principle of equivalence, when the particle motion through the G-field we annul the field; for isotropic metric (3.6) where the length and time measure etalons of absolute observer, we find:

$$h\gamma_{11} = 1 \quad u \quad h\gamma_{22} = 1 \tag{3.12}$$

$$\sqrt{h} = \sqrt{\gamma^{11}} = \sqrt{\gamma^{22}} \tag{3.13}$$

Otherwise said: Let the light signal which moves in the central symmetric field measure of observers who fall in the absolute reference system ($\sqrt{h} = 1$); They are arranged from point to point along the movement of light and measurements made in the local proper system. It has already been said that in the local proper system, the speed of light is c , so that the total motion time of the signal is obtained when the sum is contributed by all observers: $\Delta\tau$, *i.e.* the length which is measured in doing so $c\Delta\tau$ should be equal to the length Δl which measures the absolute observer in an absolute system ($\sqrt{h} = 1$) for the same time $\Delta\tau$, *i.e.* as that the signal moves in the field without gravity, this follows from the principle of equivalence.

When we measure that same light signal from the reference system at a distance r , *i.e.* each distance r assigns the corresponding value length standard with which we measure, by using the metric (3.1), and measurement performed absolute observer ($R \rightarrow \infty$), we get:

$$d\tau = \sqrt{1 - v_{pA}^2/c^2} \sqrt{h^r \gamma_{11}^r dr'^2 + h^r dl_{\theta,\phi}^2} \tag{3.14}$$

Based on the principle of equivalence, from Equation (3.14) and (3.11), it follows:

⁵We find this equality from the equation of motion of a particle in a static G-field:

$$\frac{du^0}{ds} + 2\Gamma_{0\alpha}^0 u^0 u^\alpha = 0 \Rightarrow \frac{du^0}{ds} + 2 \frac{1}{\sqrt{h}} \frac{d\sqrt{h}}{ds} u^0 = 0 \tag{1}; \text{ From here you get:}$$

$$u^0 = \frac{dx^0}{ds} = \frac{dt}{ds} = \frac{1}{\sqrt{h}\sqrt{1 - v^2/c^2}} = \frac{1}{h} \Rightarrow \sqrt{h} = \sqrt{1 - v^2/c^2} \tag{2}$$

$$\begin{aligned} h^r \gamma_{11}^r dr^2 &= dr'^2 \Rightarrow \sqrt{h^r} \sqrt{\gamma_{11}^r} dr = dr' \\ h^r r^2 dl_{\theta,\phi}^2 &= r'^2 dl_{\theta,\phi}'^2 \end{aligned} \tag{3.15}$$

From $dl_{\theta\phi} = dl'_{\theta'\phi'}$, follows: $\sqrt{h^r} = \frac{r'}{r}$.

If we assume that the $\sqrt{h^r} \sqrt{\gamma_{11}^r} = 1$, we get the equality $dr = dr'$, and from here it follows:

$$\begin{aligned} r' + cl &= r \\ \sqrt{h^r} &= 1 - cl/r \\ r' \sqrt{\gamma_{22}^r} &= r \Rightarrow \sqrt{\gamma_{22}^r} = 1 + cl/r' = \sqrt{\gamma_{11}^r} = 1/\sqrt{h} \end{aligned} \tag{3.16}$$

This is consistent with (3.3) and (3.6).

3.3.3. Time and Length

Since the space-time metric is an invariant quantitie, we come to the following relations:

$$dt = \frac{d\tau}{\sqrt{h} \sqrt{1-v^2/c^2}} = \frac{d\tau}{\left(1 - \frac{\gamma m}{c^2 r}\right) \sqrt{1-v^2/c^2}}; \tag{3.17-i}$$

$$dr = dr_0 \frac{1}{\sqrt{\gamma_{11}^r} \sqrt{1-v^2/c^2}} = dr_0 \frac{1 - \frac{\gamma m}{c^2 r}}{\sqrt{1-v^2/c^2}} = dr_0 \frac{1}{\left(1 + \frac{\gamma m}{c^2 r'}\right) \sqrt{1-v^2/c^2}}; \tag{3.17-ii}$$

$$cd\tau = dr_0$$

The equation of time is obtained directly from the metric form, and the equation for measuring length, with variable radius r is found, of course, from observations of light and substituting (3.9) in the above Equation (3.17-*i*).

Now let stationary observers ($v = 0$) be distributed along the entire radius from point to point. Each observer in their own space (point), measure their proper length and time. To measure the overall length of the radius in units of measurement given by observer, we follow the radial movement of the light signal to center field. By integration we find the length of the radius between two points:

$$\Delta r_0 = \int_{r_1}^{r_2} \frac{dr}{1 - \frac{\gamma m}{c^2 r}} = r \Big|_{r_1}^{r_2} + \frac{\gamma m}{c^2} \ln \left(r - \frac{\gamma m}{c^2} \right) \Big|_{r_1}^{r_2} \tag{3.18-i}$$

or

$$\Delta r_0 = \int_{r'_1}^{r'_2} \left(1 + \frac{\gamma m}{c^2 r'} \right) dr' = r' \Big|_{r'_1}^{r'_2} + \frac{\gamma m}{c^2} \ln r' \Big|_{r'_1}^{r'_2} \tag{3.18-ii}$$

Schwarzschild solution has the form:

$$Sh: \Delta r_0 = \int_r^{r_0} \frac{dr}{\sqrt{1 - \frac{2\gamma m}{c^2 r}}} = \sqrt{r \left(r - \frac{2\gamma m}{c^2} \right)} \Big|_r^{r_0} + \frac{2\gamma m}{c^2} \ln \left(\sqrt{r} + \sqrt{r - \frac{2\gamma m}{c^2}} \right) \Big|_r^{r_0} \tag{3.19}$$

It is shown that there is a difference between the length of Δr_0 (3.18-*i*) and Schwarzschild values Δr_0 (3.19). It is the largest when the radius r takes gravitational radius r_g ; length (3.19) has the final value, and quantities (3.18-*i*) and (3.18-*ii*) tend towards infinity. If $\Delta r_0 \rightarrow \infty$, means that between a given observer at a distance r and the gravitational radius r_g could accommodate another observer (point) and also in infinity, never reach the radius r_g , but infinitely gravitate to it. This result coincides with (3.9-*i*), if it was a finite number of observers deployed along the radius, time to reach r_g would finite and in contradiction with Equation (3.9-*i*).

3.3.4. Free Falling Particles

Velocity of a particle freely falling along the radius, measured from the absolute reference system is in the form:

$$\frac{dr'}{dt'} = \frac{dr}{dt} = \sqrt{h} \sqrt{\gamma^{11}} \sqrt{1 - h(1 - v_{pA}^2/c^2)}, \quad (3.20-i)$$

this is equivalent to:

$$u_0 = \frac{E_0}{mc^2} = \frac{1}{\sqrt{1 - v_{pA}^2/c^2}} = \frac{\sqrt{g_{00}}}{\sqrt{1 - v^2/c^2}}, \quad (3.20-ii)$$

where v_{pA} initial velocity in the absolute system. Time measured from the absolute system is found from the relation:

$$dt = \frac{dr}{\left(1 - \frac{\gamma m}{c^2 r}\right)^2 \sqrt{1 - \left(1 - \frac{\gamma m}{c^2 r}\right)^2}}; \quad (3.20-iii)$$

In their proper system of photon time as we know it stands $d\tau = 0$. As time runs in the proper system of particle that freely fall, we find from the equation:

$$cd\tau = \frac{\sqrt{h} \sqrt{\gamma^{11}}}{\sqrt{1 - v_{pA}^2/c^2}} \frac{dr'}{\sqrt{1 - h(1 - v_{pA}^2/c^2)}} \quad (3.21)$$

According to these equations for the metric (3.3), and (3.6), the particle falls from the absolute system with an initial velocity v_{pA} . In this example, the particle begins to fall from the proper system of observer. If the observer is at a distance R ; movement is described by metric that joins the reference system at a distance R :

$$\begin{aligned} ds^2 &= c^2 dt_R^2 \left(\frac{\sqrt{h}}{(\sqrt{h})_R} \right)^2 - \gamma_{11} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \\ &= c^2 dt_R^2 \left(\frac{1 - \frac{\gamma m}{c^2 r}}{1 - \frac{\gamma m}{c^2 R}} \right)^2 - \frac{dr^2}{\left(1 - \frac{\gamma m}{c^2 r}\right)^2} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \end{aligned} \quad (3.22-i)$$

$$\begin{aligned}
 ds^2 &= c^2 dt_R'^2 \left(\frac{\sqrt{h}}{(\sqrt{h})_R} \right)^2 - \left(\frac{\sqrt{\gamma_{11}}}{(\sqrt{\gamma_{11}})_R} \right)^2 \left(dr_R'^2 + r_R'^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right) \\
 &= c^2 dt_R'^2 \left(\frac{1 + \frac{\gamma m}{c^2 R'}}{1 + \frac{\gamma m}{c^2 r'}} \right)^2 - \left(\frac{1 + \frac{\gamma m}{c^2 R'}}{1 + \frac{\gamma m}{c^2 r'}} \right)^2 \left(dr_R'^2 + r_R'^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right)
 \end{aligned} \tag{3.22-ii}$$

The time interval for this metric depends on the place of the observer (R):

$$dt_R = (\sqrt{h})_R dt = \left(1 - \frac{\gamma m}{c^2 R} \right) dt = \frac{d\tau}{\frac{\sqrt{h}}{(\sqrt{h})_R} \sqrt{1 - v^2/c^2}} = \frac{1 - \frac{\gamma m}{c^2 R}}{1 - \frac{\gamma m}{c^2 r}} \frac{d\tau}{\sqrt{1 - v^2/c^2}}. \tag{3.23}$$

Again determine proper time of free falling body, but now from the perspective of the observer at a distance R . We follow a body that begins to fall from the absolute reference system, then its velocity which is measured by an observer at a distance R we denote by v_{pR} and that is the initial velocity for that observer, (R). Now according to (3.22) we can write:

$$\begin{aligned}
 \frac{dr'_R}{dt'_R} &= \sqrt{\frac{h}{(h)_R} \frac{\gamma^{11}}{(\gamma^{11})_R} \left(1 - \frac{h}{(h)_R} (1 - v_{pR}^2/c^2) \right)}; \\
 cd\tau &= \left(c^2 \frac{h}{(h)_R} \left(\frac{dt'_R}{dr'_R} \right)^2 - \frac{\gamma^{11}}{(\gamma^{11})_R} \right)^{\frac{1}{2}} dr'_R = \left(\frac{\frac{\gamma_{11}}{(\gamma_{11})_R} (1 - v_{pR}^2/c^2)}{1 - \frac{h}{(h)_R} (1 - v_{pR}^2/c^2)} \right)^{\frac{1}{2}} dr'_R
 \end{aligned} \tag{3.24}$$

For observer at a distance R we have:

$$\left(\frac{E_0}{mc^2} \right)_R = \frac{1}{\sqrt{1 - v_{pR}^2/c^2}} = \frac{\sqrt{h}/(\sqrt{h})_R}{\sqrt{1 - v_{pR}^2/c^2}}, \tag{3.25}$$

where v_{pR} is the velocity of a body at a distance r to the observer (R). After replacement we have:

$$cd\tau = \left(\frac{\frac{\gamma_{11}}{(\gamma_{11})_R} \frac{h}{(h)_R} (1 - v_{pR}^2/c^2)}{1 - \frac{h}{(h)_R} (1 - v_{pR}^2/c^2)} \right)^{\frac{1}{2}} dr'_R \tag{3.26}$$

Since the proper time remains unchanged with respect to (3.21), as we follow the movement of the body coming from the absolute system, $v_{pA} = 0$; (3.26) and (3.21) we find the equality:

$$\sqrt{1 - v_{pR}^2/c^2} = (\sqrt{h})_R \tag{3.27}$$

The whole situation is better seen if we look at relations that are related to the absolute observer ($R \rightarrow \infty$):

$$\frac{E_0}{mc^2} = \frac{(\sqrt{h})_R}{\sqrt{1 - v_{pR}^2/c^2}} = \frac{\sqrt{h}}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - v_{pA}^2/c^2}} = 1, \tag{3.28}$$

where v'_R denotes the velocity of particles at a distance R , which measures the absolute observer, and v' is the velocity at a distance r for the same observer. It follows the equality $v'_R = v_{pR}$ and $v_{Rr} = v'$ in (3.28), (3.27) and (3.25).

3.3.5. Maximum Speed of Falling Particle

We said that viewed from the outside observer, gravitational field in addition to accelerating particle (accelerated free fall) due to interaction with particle offers and resistance to their movement, which is expressed in strong fields. Before the start of deceleration particle have acceleration of free fall to the point where a maximum velocity; after that, as to dominate some resistant force field. Maximum velocity is found from the expression $dv'/dr = 0$. Looking at the radial velocity from the absolute system we can calculate the distance r at which the acceleration changes sign:

$$\begin{aligned} v' &= \frac{dr}{dt} = \left(1 - \frac{\gamma m}{c^2 r}\right)^2 \sqrt{1 - \left(1 - \frac{\gamma m}{c^2 r}\right)^2}; \quad \frac{dv'}{dr} = 0 \\ \Rightarrow r'_{v_{\max}} &= \frac{\gamma m}{c^2} \frac{1}{1 - \sqrt{2/3}} \approx 5.5 \frac{\gamma m}{c^2} \end{aligned} \quad (3.29)$$

The same way of solving gives the result for isotropic metric (3.6):

$$r'_{v_{\max}} = \frac{\gamma m}{c^2} \frac{1}{\sqrt{3/2} - 1} = r'_{v_{\max}} - \frac{\gamma m}{c^2} \approx 4.5 \frac{\gamma m}{c^2}. \quad (3.29-i)$$

3.3.6. Experiment with Interferometer Michelson Type

Here we used Michelson type interferometer, only now one arm is facing the center of the sun, along the radial direction, the other arm of the same dimensions is normal at first (both arm are mutually compared). The influence of the earth's gravity to interference less than influences of sun around thirty times and can be neglected. Interference shift of spectral lines depends on the difference path two light signals. This time we will follow time the return signal in place of interference and thus to determine the path difference.

Isotropic metric (3.7) gives the following result:

$$(I) \quad dt'_R = dr'_R \frac{\left(1 + \frac{\gamma m}{c^2 r'}\right)^2}{\left(1 + \frac{\gamma m}{c^2 R'}\right)^2}, \quad \Delta t'_R = \int_{r'_1}^{r'_2} dr' \frac{\left(1 + \frac{\gamma m}{c^2 r'}\right)^2}{1 + \frac{\gamma m}{c^2 R'}}, \quad (3.29-ii)$$

$$r'_2 = R', \quad r'_2 - r'_1 = \Delta l', \quad \Delta l' \ll r'_1$$

$$\Delta t_R = \frac{1}{1 + \frac{\gamma m}{c^2 R}} \left[\Delta l + \frac{2\gamma m}{c^2} \ln \left(1 + \frac{\Delta l}{r'_1}\right) \right], \quad \Delta t_R \approx \Delta l \left(1 + \frac{\gamma m}{c^2 R}\right) \quad (3.29-iii)$$

$$(II) \quad \frac{dl'_R}{dt'_R} = \frac{\left(1 + \frac{\gamma m}{c^2 R'}\right)^2}{\left(1 + \frac{\gamma m}{c^2 r'}\right)^2} \Rightarrow \Delta t'_R = \Delta l' \frac{\left(1 + \frac{\gamma m}{c^2 r'}\right)^2}{1 + \frac{\gamma m}{c^2 R'}} = \Delta l' \left(1 + \frac{\gamma m}{c^2 R'}\right) \quad (3.29-iv)$$

The item (I) refers to the radial direction, while (II) describes the movement

in the normal direction. The difference of the two time $\Delta t_{rad.} - \Delta t_{nor.} \approx 0$, indicates that it is not possible to observe the shift in the line using the most advanced devices⁶. Note the necessity of measuring the length of the two arms of the interferometer Δl in the same place in the system of reference.

If for a solution to this problem we look for the second approximation, according to the metric (3.7) we obtain the following solutions; radial direction applies $\Delta t_R \approx \Delta l [1 + \gamma M / c^2 R (1 + \Delta l / R)]$. While in the normal direction of correction due to the curvature of the sphere in relation to the arm of the interferometer which represents the measuring rod is $\Delta l_c \approx \Delta l (1 + \Delta l^2 / 24 R^2)$; then if we take correction due to change G-potential, along the movement of light found $\Delta t_R \approx \Delta l_s [1 + \gamma M / c^2 R + 2 \gamma M / c^2 R (\Delta l / R)^2] \approx \Delta l (1 + \gamma M / c^2 R + \Delta l^2 / 24 R^2)$. The difference of these two times is equal to $c(\Delta t_{Rrad.} - \Delta t_{Rnor.}) \approx \Delta l (\gamma M / c^2 R \Delta l / R - \Delta l^2 / 24 R^2)$. How is valid that is $\gamma M / c^2 \gg \Delta l$, we get the result $c(\Delta t_{Rrad.} - \Delta t_{Rnor.}) \approx \gamma M / c^2 (\Delta l / R)^2$, that cannot be tested experimentally.

3.4. Shifting Spectral Lines

Let the light source be at a distance R_s (sun surface) from the center of the sun, and the observer at a distance R . We find the energy balance for a photon by measuring its energy in both reference systems; using the isotropic metric (3.6) we obtain:

$$\frac{h\nu_R}{1 + \frac{\gamma m}{c^2 R}} = \frac{h\nu_{R_s}}{1 + \frac{\gamma m}{c^2 R_s}}. \quad (3.30)$$

Here, the ν_R frequency of a photon is measured in the system at a distance R , and the ν_{R_s} frequency of the same photon in the system at a distance R_s . This would mean that atoms on the sun's surface emit photons with energies corresponding to the difference in energy levels, so that at a great distance from the sun, the energies of the same photons would be slightly smaller. It should be noted that the influence of the gravitational field on the displacement of energy levels in the atom itself is negligible. Energy splitting is of the order of $m_e \phi a_0 / R$, so the difference in energy levels remains unchanged. The lack of photon energy at a remote observer has just been replaced by an observer near the sun from the gravitational field on the principle of equivalence of gravitational and EM energy, *i.e.* by transferring the gravitational potential energy of a particle to its kinetic energy.

By considering the motion of a particle in a field, we can arrive at the same result. If we linked the reference system to a free-falling particle, the observer would not register an increase in the particle energy (equivalence principle). In

⁶If the first approximation obtained: $c(\Delta t_{rad.} - \Delta t_{nor.}) \approx \Delta l \gamma M / c^2 R$; relation shift Δa and wide stripes a for a given light wavelength λ , would be equal to:

$\frac{\Delta a}{a} = \frac{2\Delta l' \gamma m}{\lambda c^2 R} \rightarrow \lambda = 5 \times 10^{-7} \text{ m}, R = 1.5 \times 10^{11} \text{ m}, \Delta l' = ? \Rightarrow \frac{\Delta a}{a} = 0.04 \cdot \Delta l'$. The calculated displacement can be noted in the experiment and conclude that interference exists, however this does not agree with the experimental results.

order to obtain a reference system that is stationary in the field, we must act with EM force and stop the accelerating particle, in which case the particle loses kinetic energy, which it obtains by constantly moving freely in the G-field, so that only the rest mass m_0 remains. This energy loss is equivalent to the EM energy invested to stop the particle. In the same way, a photon receives energy from a G-field; thus, its energy increases with respect to stationary observers with an increase in the absolute value of the gravitational potential, or decreases with a decrease in it. We find this amount of energy from the equation:

$$E_{R_S} = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} = m_0 c^2 \frac{1 + \frac{\gamma m}{c^2 R_S}}{1 + \frac{\gamma m}{c^2 R}}. \quad (3.31)$$

Here, E_{R_S} is the energy of the particle measured at the surface of the sun (R_S), and $m_0 c^2$ is the energy of the particle at location R , from where the particle begins to fall, so v denotes the velocity that the particle reaches at position R_S ; it may also fall from some other distance, so the rest mass need not be taken as reference energy. From the equivalence of mass and energy, as well as from the principle of equivalence, the equality of relations follows:

$$E_{R_S}/m_0 c^2 = h\nu_{R_S}/h\nu_R. \quad (3.32)$$

The confirmation that in the fixed reference systems the photon energy being monitored changes the measurements of the spectral shift towards blue on earth in a resonance absorption experiment where there is a difference of gravitational potential between the source and the absorber (atoms of the same element).

From the previous we get the following relation:

$$\frac{\nu_{R_S} - \nu_R}{\nu_R} = \frac{\frac{\gamma m}{c^2 R_S} - \frac{\gamma m}{c^2 R}}{1 + \frac{\gamma m}{c^2 R}}. \quad (3.33)$$

If instead of frequency we look at the wavelength and assume that it changes at the same length, we come to the following expression:

$$\frac{\lambda_R - \lambda_{R_S}}{\lambda_{R_S}} = \frac{\frac{\gamma m}{c^2 R_S} - \frac{\gamma m}{c^2 R}}{1 + \frac{\gamma m}{c^2 R}}. \quad (3.34)$$

3.5. The Movement of Light in a Centrally Symmetric Field

We now observe the path of the ray of light through the gravitational field. The equation of motion is determined by the equation of eiconals: $g^{ik} \frac{\partial \psi}{\partial x^i} \frac{\partial \psi}{\partial x^k} = 0$.

We will use the isotropic metric (3.6), and by putting $m_f = 0$ in the Hamilton-Jacobi Equation (3.46), we obtain the expression for the ray trajectory:

$$\left(\frac{d\sigma}{d\varphi}\right)^2 L^2 = \left[\frac{\omega_0^2}{c^2} \left(1 + \frac{A}{r}\right)^4 - \frac{L^2}{r^2}\right], \quad \sigma = \frac{1}{r}, \quad A = \frac{\gamma m}{c^2}, \quad \frac{cL}{\omega_0} = R. \quad (3.35)$$

In the next step, we differentiate by φ the above equation, then perform the appropriate approximation by neglecting higher order terms by small size. Finally, we get an equation that is easy to solve:

$$\frac{d^2\sigma}{d\varphi^2} + \sigma = \frac{2A}{R^2} \Rightarrow \sigma = \frac{2A}{R^2} + k \cos \varphi. \quad (3.36)$$

For the boundary condition $m = 0$, $A = 0$ follows, so that the value $k = 1/R$ is obtained from the above equation.

In $r = \infty$ ($\sigma = 0$) we look for the trajectory of the curve at small angles of turn $\delta\varphi$. We set the coordinate φ in the form $\varphi = \pm\pi/2 + 1/2\delta\varphi$ and finally we find the solution:

$$\delta\varphi = \frac{4A}{R} = \frac{4\gamma m}{c^2 R}. \quad (3.37)$$

3.6. The Movement of Light in an Isotropic and Inhomogeneous Medium

Suppose that a centrally symmetric gravitational field is precisely an isotropic and inhomogeneous environment. To determine the movement of light in the G-field, we use the eiconal equation:

$$g^{ik} \frac{\partial \psi}{\partial x^i} \frac{\partial \psi}{\partial x^k} = 0. \quad (3.38)$$

Putting the isotropic metric (3.6) in (3.38) gives the following equation:

$$[\text{grad}\psi]^2 = g^{00}\gamma_{11} \left(\frac{1}{c} \frac{\partial \psi}{\partial t} \right)^2. \quad (3.39)$$

An equation is obtained that has the same form as the geometric optics equation⁷:

$$[\text{grad}\psi(\mathbf{r})]^2 = \frac{\omega^2}{c^2} N^2(\mathbf{r}), \quad (3.40)$$

where $N(\mathbf{r})$ is the refractive index of a given medium at a given point. The refractive index for the central symmetric G-field is found by:

$$N(\mathbf{r}) = c/c' = \left(1 + \gamma m/c^2 r\right)^2 = \sqrt{g^{00}\gamma^{11}}.$$

For Equation (3.40) to be valid, *i.e.* for a weakly inhomogeneous and translucent medium through which wavelengths pass λ the condition must be fulfilled:

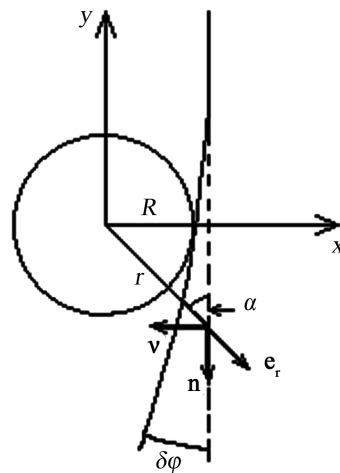
$$\lambda \left| \frac{\text{grad}N(\mathbf{r})}{N(\mathbf{r})^2} \right| \ll 1 \Rightarrow \lambda \frac{\gamma m}{c^2 R^2} \ll 1. \quad (3.41)$$

The visible light wavelength is about $10^{-8} \dots 10^{-9}$ m, so the upper inequality is certainly filled.

After some transformations, we get the following formula that describes the ray path in a given medium (see **Figure 1**):

$$\frac{1}{\Re} = \frac{1}{N(\mathbf{r})} \mathbf{v} \cdot \text{grad}N(\mathbf{r}), \quad (3.42)$$

⁷Look at the book, B. S. Milić, *Elektrodinamika* (Fizički fakultet u Beogradu).



Savijanje svjetlosnog zraica

Slika 1

Figure 1. Bending of a light ray.

where $(\mathbf{n}, \mathbf{v}, \boldsymbol{\beta})$ is a natural trihedron, and \mathfrak{R} is the radius of curvature of the ray.

When the refractive index $N(r)$ for the G-field is entered into Equation (2.35) we get:

$$\frac{1}{\mathfrak{R}} \approx -2 \frac{\gamma m}{c^2 r^2} \mathbf{e}_r \cdot \mathbf{v} = 2 \frac{\gamma m}{c^2 r^2} \frac{R}{r} \tag{3.43}$$

wherein:

$$\mathbf{e}_r \cdot \mathbf{v} = \cos\left(\frac{\pi}{2} + \alpha\right) = -\frac{R}{\sqrt{R^2 + y^2}} \tag{3.43-i}$$

We take the fact that the coordinate x changes very little along the ray path $x \approx R$, i.e. the change in ray deflection angle $\delta\varphi$ is very small. The formula for radius of curvature of the ray is obtained from differential geometry. If the curve is given in the parametric form $x = x(t)$, $y = y(t)$, where the parameter is time, then with the appropriate approximations for weak fields the radius of the curve \mathfrak{R} is calculated by the formula:

$$\frac{1}{\mathfrak{R}} = \frac{x' y'' - x'' y'}{(x'^2 + y'^2)^{3/2}} \approx \frac{-x'' y'}{y'^3} \approx \frac{-x''}{c^2} \tag{3.44}$$

From here we see that the acceleration x'' is negative, i.e. we set the problem by taking the path of ray from $y = 0$ to $y \rightarrow -\infty$ because of the symmetry of the system:

$$\frac{dy}{dt} \approx c, \quad -v_x = \frac{2\gamma m R}{c} \int \frac{dy}{(R^2 + y^2)^{3/2}} = \frac{2\gamma m R}{c} \frac{y}{R^2 \sqrt{R^2 + y^2}}; \tag{3.45-i}$$

$$-2 \int_R^{R-\Delta x/2} dx = \frac{2\gamma m}{c^2 R} 2 \cdot \int_0^{-\infty} \frac{y dy}{\sqrt{R^2 + y^2}} = \frac{4\gamma m}{c^2 R} \left(\sqrt{R^2 + y^2} \Big|_0^{-\infty} \right); \tag{3.45-ii}$$

$$\delta\varphi \approx \text{tg} \varphi = \lim_{\Delta y \rightarrow -\infty} \frac{\Delta x}{\Delta y} = \frac{4\gamma m}{c^2 R} \tag{3.45-iii}$$

3.7. Movement of Perihelion of Mercury

To determine the body trajectory, we use the Hamilton-Jacoby method [2]:

$$g^{ik} \frac{\partial S}{\partial x^i} \frac{\partial S}{\partial x^k} - m^2 c^2 = 0; \quad S = -E_0 t + M\varphi + S_r(r). \quad (3.46)$$

Instead of the Schwarzschild metric, we use the isotropic metric (3.6), which describes the field in a simpler way. From the equations of motion we find the constants in (3.46):

$$L = r^2 \frac{d\phi}{ds} \left(1 + \frac{A}{r}\right)^2$$

$$\frac{E_0}{mc^2} = \frac{dt}{ds} \left(1 + \frac{A}{r}\right)^2 = 1 / \left[\left(1 + \frac{A}{r}\right) \sqrt{1 - \frac{v^2}{c^2}} \right]; \quad A = \frac{\gamma m'}{c^2} \quad (3.46-i)$$

The above equations give the following differential equation:

$$\left(L \frac{d\sigma}{d\phi} \right)^2 = \frac{E_0^2}{c^2} \left(1 + \frac{A}{r}\right)^4 - \frac{L^2}{r^2} - m^2 c^2 \left(1 + \frac{A}{r}\right)^2. \quad (3.47)$$

Applying appropriate approximations and differentiating the whole equation by φ , we release the constants E_0 and arrive at the equation

$$\frac{d^2\sigma}{d\phi^2} + \sigma \left(1 - \frac{6A^2 E_0^2}{L^2 c^2} + \frac{m^2 c^2 A^2}{L^2} \right) = \frac{2AE_0^2}{L^2 c^2} - \frac{m^2 c^2 A}{L^2}. \quad (3.48)$$

The solution to this differential equation is an ellipse rotating in the plane:

$$\sigma = k_1 + k_2 \cos \left[\sqrt{1 - \frac{6E_0^2 A^2}{c^2 L^2} + \frac{m^2 c^2 A^2}{L^2}} \phi \right]. \quad (3.48-i)$$

From here we see that for one revolution of the planet the change in the angle of the perihelion is:

$$\delta\phi = \frac{5\pi\gamma m'}{c^2 a(1-e^2)}. \quad (3.49)$$

This value (about 37" per century for the planet Mercury) is slightly lower than the one that follows from the Schwarzschild metric and is available for measurement. The influence of other planets and the sun on the perihelion rotation must be calculated precisely (not an easy task), since it dominates with respect to (3.49). The measurement data collected should also be well analyzed. The permanent motion of Mercury in the weak G-field of the sun gives an effect that is measurable according to (3.49) and different from the solution that follows from the Schwarzschild metric. We obtained this result over metric coefficients using the second order of approximation. In the second order of approximation, a difference is observed from the solution of the general theory of relativity. In support of this, we can refer to the following papers [3] [4].

3.8. Electromagnetic Field and Gravity

Consider the case where in addition to the G-field there is an EM field, present

in a wide area of space. Since we tie mass to charges, the interaction between EM and G-fields is taken into account in the G-field equations, so the total action for this case of the field takes the form:

$$S_g + S_E + S_{EM} + S_I = \int (R + \Lambda^E + \Lambda^{EM} + \Lambda^I) \sqrt{-g} d\Omega \tag{3.50}$$

$$= \int \left[R - (F_{ik} \pm \sqrt{\gamma \epsilon_0} F_{ik}^{em}) (F^{ik} \pm \sqrt{\gamma \epsilon_0} F_{em}^{ik}) \right] \sqrt{-g} d\Omega$$

wherein the term describing the interaction between the EEM field and a given EM field has the form $\Lambda^I = \pm 2 F_{ik} F_{em}^{ik}$, where the above sign refers to a positive charge. From the principle of least action we obtain the equations of the field varying g_{ik} :

$$\delta (S_g + S_E + S_{EM} + S_I) = \int (-G_{ik} + T_{ik}^E + T_{ik}^{EM} + T_{ik}^I) \delta g^{ik} \sqrt{-g} d\Omega = 0 \tag{3.51}$$

From here, after the corresponding transformations, the expression follows:

$$\delta (S_g + S_E + S_{EM} + S_I) = \int (-G_i^k + T_i^{Ek} + T_i^{EMk} + T_i^{Ik})_{;k} \xi^i \sqrt{-g} d\Omega = 0, \tag{3.52}$$

so because of the arbitrariness ξ^i we can take

$$T_i^{Ek} + T_i^{EMk} + T_i^{Ik} = 0, \tag{3.53}$$

or when calculated (3.53) it becomes:

$$(F_{il} + \sqrt{\gamma \epsilon_0} F_{il}^{em}) (F^{lk}_{;k} + \sqrt{\gamma \epsilon_0} F_{em;k}^{lk}) = 0. \tag{3.54}$$

A system of 4 equations with 12 unknown scalar quantities is obtained: 6 independent g_{ik} , E_1 , E_2 , E_3 and B_1 , B_2 , B_3 ; added to (Equation (3.54)) 4 more non-source equations that follow from $(F_{ik;l} + F_{li;k} + F_{kl;i})_{EM} = 0$, then the equations of motion of the EM field source in the self-generated field should be taken, *i.e.* the field equations and the equations of motion of the field sources are solved in concert. However, in the approximation of the given densities ($j^i \approx \rho v^i$), we preset the vector j^i in advance and in such an approximation one should try to solve the set system of equations.

We now vary the potentials A_i (A_i)_{em} in action (3.50); as a result we get the equations of the field in the form⁸:

$$F^{ik}_{;k} + \sqrt{\gamma \epsilon_0} F_{em;k}^{ik} = \frac{4\pi}{c^2} j^i - \frac{4\pi}{c} j_{em}^i / \sqrt{\gamma \epsilon_0} \tag{3.55}$$

As it can be seen from Equation (3.55), the equations for the EM field in the presence of the G-field are not a generalization of Maxwell's equations, but contain an additional term as a consequence of considering the interactions between these fields.

Let's try using the static field example to find how the two fields interact. Consider a spherical body of uniform matter density and assume that the charge is uniformly distributed over the volume of the body. Determine the strength of

⁸In the presence of matter and charges in (3.50), terms appear describing the interaction of these fields with the medium present. Suppose in this case the interaction terms take the form:

$S_I = \int (A_i + \sqrt{\gamma \epsilon_0} A_i^{em}) (j^i - j_{em}^i / \sqrt{\gamma \epsilon_0}) \sqrt{-g} d\Omega$. When varying the action, the condition should be fulfilled $[j^i - (j^i)_{em}]_{;k} = 0$.

the EM field inside and outside the sphere. Since the field of the sphere in the system in which it is rest is centrally symmetric and for ease of calculation we will take the metric of the form:

$$ds^2 = c^2 e^\nu dt^2 - e^\lambda dr^2 - r^2 (d\theta^2 + \sin^2 \theta \cdot d\varphi^2). \quad (3.56)$$

Then we use Equations (3.51), which in the static field pass $G_{ik} = (T_{ik})_E + (T_{ik})_{EM} + (T_{ik})_B$ and in the area outside the sphere, from (3.56) we obtain the system of equations:

$$\left(\sqrt{\gamma \epsilon_0} E_1 e^{-\nu/2} - \frac{1}{2} \nu' \right)^2 e^{-\lambda} = -e^{-\lambda} \left(\frac{1}{r^2} - \frac{\lambda'}{r} \right) + \frac{1}{r^2} \quad (3.57-i)$$

$$\left(\sqrt{\gamma \epsilon_0} E_1 e^{-\nu/2} - \frac{1}{2} \nu' \right)^2 e^{-\lambda} = -e^{-\lambda} \left(\frac{1}{r^2} + \frac{\nu'}{r} \right) + \frac{1}{r^2} \quad (3.57-ii)$$

$$\left(\sqrt{\gamma \epsilon_0} E_1 e^{-\nu/2} - \frac{1}{2} \nu' \right)^2 e^{-\lambda} = \frac{1}{2} e^{-\lambda} \left(\nu'' + \frac{\nu'^2}{2} + \frac{\nu' - \lambda'}{r} - \frac{\nu' \lambda'}{2} \right). \quad (3.57-iii)$$

These equations are in agreement with Equation (3.55), from which in this case we find:

$$\left(\sqrt{\gamma \epsilon_0} E_1 - (\sqrt{h})_{,1} \right) e^{-\nu/2 - \lambda/2} = c_1 / r^2. \quad (3.58)$$

Solving these equations leads to the following result:

$$e^{-\lambda} = e^\nu = 1 + \frac{c_2}{r} + \frac{c_1^2}{r^2}; \quad \sqrt{\gamma \epsilon_0} E_1 = \frac{-\frac{c_2}{2r^2} - \frac{c_1^2}{r^3}}{\sqrt{1 + \frac{c_2}{r} + \frac{c_1^2}{r^2}}} + \frac{c_1}{r^2}, \quad (3.59)$$

where the constants c_1 and c_2 are determined from the conditions of potential continuity at the boundary of the given medium charge density ρ_e and mass density ρ .⁸ Suppose the densities ρ_{0e} and ρ_0 are constant in terms of Equation (4.4), i.e. $\rho = \rho_0 e^{-\lambda/2}$; and from (3.55) we obtain:

$$\left(\sqrt{\gamma \epsilon_0} E_1 - (\sqrt{h})_{,1} \right) e^{-\nu/2 - \lambda/2} = -\frac{4\pi}{3} (\rho_0 - \rho_{e0}) r. \quad (3.60)$$

We then use Equation (3.54), which now goes into

$$\sqrt{\gamma \epsilon_0} E_1 = (\sqrt{h})_{,1} \rho_{e0} / (\rho_{e0} - \rho_0 \sqrt{\gamma \epsilon_0}). \quad (3.61)$$

From (3.60) and (3.61) we find the solution:

$$e^{-\lambda/2} = -4\pi \left[-\frac{1}{3} (\rho_0 - \rho_{e0} / \sqrt{\gamma \epsilon_0})^2 / \rho_0 + \rho_0 \right] r^2 / 2 + const. \quad (3.62)$$

In the presence of matter, Equation (3.57-i) goes to

$$\left(\sqrt{\gamma \epsilon_0} E_1 e^{-\nu/2} - \frac{1}{2} \nu' \right)^2 e^{-\lambda} + \frac{8\pi\gamma}{c^2} \rho_0 e^{-\lambda/2} = -e^{-\lambda} \left(\frac{1}{r^2} - \frac{\lambda'}{r} \right) + \frac{1}{r^2} \quad (3.63)$$

The agreement of solutions (3.62) and (3.63) leads to the following equation $\rho_{e0} = 2\rho_0 \sqrt{\gamma \epsilon_0}$, so we find a solution in the case of a central field in space inside

and outside the spherical body⁹. Outside the sphere, Equation (3.59) goes into

$$e^{-\lambda/2} = e^{v/2} = 1 - \frac{\gamma M}{c^2 r}; \quad E_1 = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}; \quad Q = 2M\sqrt{\gamma\epsilon_0}. \quad (3.64)$$

Some other metric choices in a centrally symmetric field, for example:

$$ds^2 = c^2 e^v dt^2 - e^\lambda (dr^2 + r^2 (d\theta^2 + \sin^2 \theta \cdot d\varphi^2)), \quad (3.65)$$

according to (3.51), gives rise to free-field equations:

$$\left(\sqrt{\gamma\epsilon_0} E_1 e^{-v/2} - \frac{1}{2} v' \right)^2 e^{-\lambda} = \frac{8\pi\gamma}{c^4} T_0^0 = e^{-\lambda} \left(-\lambda'' - \frac{2\lambda'}{r} - \frac{1}{4} \lambda'^2 \right) \quad (3.66-i)$$

$$\begin{aligned} \left(\sqrt{\gamma\epsilon_0} E_1 e^{-v/2} - \frac{1}{2} v' \right)^2 e^{-\lambda} &= \frac{8\pi\gamma}{c^4} T_1^1 \\ &= -e^{-\lambda} \left(\frac{1}{r} (\lambda' + v') + \frac{1}{4} (\lambda' + v')^2 - \frac{1}{4} v'^2 \right) \end{aligned} \quad (3.66-ii)$$

$$\begin{aligned} -\left(\sqrt{\gamma\epsilon_0} E_1 e^{-v/2} - \frac{1}{2} v' \right)^2 e^{-\lambda} &= \frac{8\pi\gamma}{c^4} T_2^2 = \frac{8\pi\gamma}{c^4} T_3^3 \\ &= -e^{-\lambda} \left(\frac{1}{2} (\lambda'' + v'') + \frac{1}{2r} (\lambda' + v') + \frac{1}{4} v'^2 \right) \end{aligned} \quad (3.66-iii)$$

Now according to (3.65) Equation (3.55) takes the form:

$$\left(\sqrt{\gamma\epsilon_0} E_1 - (\sqrt{h})_{,1} \right) e^{-v/2+\lambda/2} = c_1/r^2. \quad (3.67)$$

From (3.67) and (3.55) the solution is:

$$e^{-\lambda/2} = e^{v/2} = \frac{1}{c_2 + \frac{c_1}{r}}; \quad \sqrt{\gamma\epsilon_0} E_1 = \frac{2c_1/r^2}{\left(c_2 + \frac{c_1}{r} \right)^2}, \quad (3.68)$$

We then search for a solution within a body characterized by constant densities ρ_0 and ρ_{e0} , to coincide with (3.68) at the boundary of matter. Thus according to (3.66-ii) and (3.66-iii) we have $e^v = e^{-\lambda}$, so from (3.55) and (3.61) we find:

$$e^{-v/2} = e^{\lambda/2} = -\frac{4\pi}{3} \frac{r^2 (\rho_0 - \rho_{e0}/\sqrt{\gamma\epsilon_0})^2}{2\rho_0} + const. \quad (3.69)$$

Since Equations (3.51), (3.61) and (3.66) are mutually consistent, they are coupled by the expression $\rho_{e0} = 2\rho_0\sqrt{\gamma\epsilon_0}$. From here we get a solution in the area outside the sphere:

$$e^{\lambda/2} = e^{-v/2} = 1 + \frac{\gamma M}{c^2 r}; \quad E_1 = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \left/ \left(1 + \frac{\gamma M}{c^2 r} \right)^2 \right.; \quad Q = 2M\sqrt{\gamma\epsilon_0}, \quad (3.70)$$

where the coordinate r is determined in units of the absolute observer.

Now consider the case of a charged particle in the outer G-field, for example,

⁹From the equation $\rho_{e0} = 2\rho_0\sqrt{\gamma\epsilon_0}$ we find that the elementary charge the restriction applies

$m = e/\sqrt{\gamma\epsilon_0} \sim 10^{-8}$ kg, i.e. we obtain the smallest mass for which the above equations are applicable in which we used the gravitational constant γ .

a charge q in the G-field of the sun at a distance R from the center. According to (3.55) and (3.65) we find the equation:

$$F_{em;k}^{0k} \approx 0 \Rightarrow E_1 = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \left/ \left(1 + \frac{\gamma m_s}{c^2 R} \right)^2 \right., \quad (3.71)$$

where r is the distance from the particle, where $r/R \rightarrow 0$.

3.9. Particle Motion Equations in the Gravitational Field

The geodesic line equation is derived from the principle of least action $\delta \int ds = 0$; the same is obtained when we move along the unit vector of its tangent in parallel $u^i = dx^i/ds$:

$$\frac{Du^i}{ds} = \frac{du^i}{ds} + \Gamma_{kl}^i u^k u^l = 0. \quad (3.72)$$

It should be emphasized that Equation (3.72) describes the motion of a free particle, which is that particle that moves only under the action of gravity. These equations are contained in the G-field maintenance Equations (2.9), since for the free particle we obtain (3.72).

In case that particle is affected by external forces of EM nature, Equation (3.72) goes into the form:

$$m \frac{du^i}{ds} + m \Gamma_{kl}^i u^k u^l = \frac{e}{c} F^{ik} u_k, \quad (3.73-i)$$

$$m \frac{du^i}{ds} + m \Gamma_{kl}^i u^k u^l = - \frac{m}{\sqrt{1-v^2/c^2}} F^{ik} u_k \quad (3.73-ii)$$

The term on the right expresses a “4-force” that acts on a body of mass m . In Equation (3.73-ii), this EM force is directed opposite to the G-force. Thus, in a static field, the body can remain at rest under the action of EM force, which is equivalent to gravitational force. From Equation (3.73-ii) for $v=0$ we obtain the scalar potential of the static field:

$$\Gamma_{00}^1 u^0 u^0 = -F^{10} u_0 \Rightarrow F_{10} = \frac{\partial \sqrt{h}}{\partial r} \Rightarrow A_0 = \sqrt{h} \quad (3.74)$$

In a stationary field the particle can be located at a point which is stationary with respect to the center of symmetry of the field, because the field is invariant in time; to keep the particle stationary, *i.e.* for $v=0$ it should be subjected to the same force as in the case of a static field, regardless of the existence of g_α . If we put the potentials (1.1) in the EM field tensor in (3.73-ii), then an external force acting on the body is represented, which is the force of the equivalent EM field, so the following expression is found:

$$\frac{du^\alpha}{ds} + \frac{\lambda_{\beta\gamma}^\alpha v^\beta v^\gamma}{c^2 (1-v^2/c^2)} = 0; \quad (3.75)$$

showing that in this case the resultant force acting on the body is equal to zero,

i.e. the force acting on the particle in a constant G-field is equal to the equivalent EM force with opposite sign¹⁰. For example, when Equation (3.75) is applied to a body in a static field moving along the radial direction, its velocity is shown to have a constant value.

From this, we can conclude that the expression for an equivalent EM force in a constant G-field has the form:

$$f^\alpha = mc^2 F^\alpha_k u^k = \frac{mc^2}{\sqrt{1-v^2/c^2}} \left[-\frac{(\sqrt{h})^{\alpha}}{\sqrt{h}} + \sqrt{h} \left(\frac{\mathbf{v}}{c} \times \text{rot} \mathbf{g} \right)^\alpha \right]. \quad (3.76)$$

We use the radial motion of a particle in a static field as an example:

$$f^1 = mc^2 F^1_k u^k = -mc^2 F_{10} u^0 \gamma^{11} = mc^2 \sqrt{1-v^2/c^2} \left(\frac{du^1}{ds} + \lambda_{11}^1 u^1 u^1 \right); \quad (3.77)$$

$$\lambda_{11}^1 = \frac{\gamma^{11}}{2} \left(\frac{\partial \gamma_{11}}{\partial r} \right), \quad v^1 = \sqrt{\gamma^{11}} \sqrt{1-h};$$

$$f^1 = -mc^2 F_{10} \frac{\gamma^{11}}{h} = mc^2 \sqrt{h} \left[\frac{1}{2} (\gamma^{11})' \frac{1-h}{h} - \frac{\gamma^{11} h'}{2h^2} - \frac{1}{2} (\gamma^{11})' \frac{1-h}{h} \right]; \quad (3.78)$$

$$f^1 = -mc^2 F_{10} \frac{\gamma^{11}}{h} = -mc^2 \frac{\partial \sqrt{h}}{\partial r} \frac{\gamma^{11}}{h}$$

4. Field in the Matter. Tensor T_{ik} for the Macroscopic Body

We study the central symmetric field in matter; the energy-momentum tensor of a perfect fluid has the following form:

$$T^i_k = (p + \varepsilon) u^i u_k - p \delta^i_k. \quad (4.1-i)$$

From here for $v=0$ we have:

$$T^0_0 = \varepsilon, \quad T^1_1 = T^2_2 = T^3_3 = -p \quad (4.1-ii)$$

A fluid particle at a distance r feels the pressure of the fluid above it as a result of the action of gravitational force on all particles of matter. Suppose that the element of volume dV is stationary, then the gravitational force of attraction is balanced by the pressure gradient, as can be seen from the divergence equation of the tensor T_{ik} :

$$T^k_{i;k} = 0 \Rightarrow (p + \varepsilon) \frac{Du_i}{ds} = \frac{\partial p}{\partial x^i}. \quad (4.2-i)$$

In the static field for $v=0$ we get:

¹⁰The expression for force in a constant gravitational field is determined by the formula:

$$f^\alpha = c \sqrt{1-\frac{v^2}{c^2}} \frac{D_{(3)} p^\alpha}{ds} = c \sqrt{1-\frac{v^2}{c^2}} \frac{d}{ds} \frac{m v^\alpha}{\sqrt{1-\frac{v^2}{c^2}}} + \lambda_{\beta\gamma}^\alpha \frac{m v^\beta v^\gamma}{\sqrt{1-\frac{v^2}{c^2}}}. \text{ After a short calculation we find in the}$$

vector three-dimensional form the equation:
$$\mathbf{f} = -\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \left[-\text{grad} \ln \sqrt{h} + \sqrt{h} \left(\frac{\mathbf{v}}{c} \times \text{rot} \mathbf{g} \right) \right].$$

$$-\frac{\partial p}{\partial r} = \frac{1}{2} \frac{\partial g_{00}}{\partial r} \frac{1}{g_{00}} (\varepsilon + p), \quad (\varepsilon = \text{const}) \Rightarrow p = \varepsilon \left[\frac{(\sqrt{g_{00}})_R}{\sqrt{g_{00}}} - 1 \right]. \quad (4.2-ii)$$

Substituting (4.3) into (4.2-ii), we find the pressure in the spherical body:

$$p = \varepsilon \frac{\sqrt{1-qr^2} - \sqrt{1-qa^2}}{3\sqrt{1-qa^2} - \sqrt{1-qr^2}}. \quad (4.2-iii)$$

In $r = 0$ we get:

$$p = \varepsilon \frac{1 - \sqrt{1-qa^2}}{3\sqrt{1-qa^2} - 1}, \quad \left(p \rightarrow \infty \Rightarrow R \geq \frac{9}{8} \frac{2\gamma m}{c^2} \right), \quad \left(\frac{p}{\varepsilon} \leq \frac{1}{3} \Rightarrow R \geq \frac{9}{5} r_g \right). \quad (4.2-iv)$$

Applying Einstein's equations and taking the tensor (4.1), for $\rho = \text{const}$ and $v = 0$, we obtain the following relations:

$$e^{-\lambda} = 1 - ar^2, \quad e^{\nu/2} = \frac{3}{2} \sqrt{1-aR^2} - \frac{1}{2} \sqrt{1-ar^2}, \quad a = \frac{2\gamma m}{c^2 R^3}. \quad (4.3)$$

Field energy is excluded in this consideration, so the mass of the substance is the only quantity for the field calculation. Suppose that the interior mass of a sphere is:

$$m(r) = M \frac{r^3}{R^3}. \quad (4.4)$$

Using the analogy with classical gravity, suppose that the coefficient of the metric is $e^{\nu} = r^2$, and that the field intensity on the surface of a sphere of mass $m(r)$ according to the Schwarzschild metric is:

$$\frac{\partial \sqrt{h}}{\partial r} = \frac{\frac{\gamma m(r)}{c^2 r^2}}{\sqrt{1 - \frac{2\gamma m(r)}{c^2 r}}}. \quad (4.5)$$

Substituting (4.4) into (4.5) gives us the known expression:

$$\sqrt{h} = \frac{3}{2} \sqrt{1 - 2 \frac{\gamma M}{c^2 R}} - \frac{1}{2} \sqrt{1 - 2 \frac{\gamma M}{c^2 R^3} r^2} \quad (4.5-i)$$

However, the choice of (4.4) does not mean that the density is a constant, since the element of volume $dV = 4\pi r^2 e^{\lambda/2} dr$, it turns out that the density is equal to $\rho(r) = dm(r)/dV = \rho_0 e^{-\lambda/2}$. We see that this is an element of proper volume, *i.e.* we observe from the system of reference that it is related to a given element of the volume of the body, and therefore, if the element of volume is stationary, it means that the belonging reference system also remains stationary in relation to the absolute system.

The second approach to this problem comes from the assumption that we can add a term that describes the energy of the field. Through the action of the gravitational field on particles and substance we find an equivalent EM field. If a given element of matter is stationary, it means that the force by which the gravitational field acts on this particle is balanced by the force of an equivalent EM

field that originates from the interaction of the given element with the surrounding matter. For example, suppose that a particle falls freely in matter, and this is done by conducting a narrow tunnel along the radial direction of the massive body. The energy of the EM field required to stop this particle is equivalent to the energy that the test body receives in the gravitational field. From here is the corresponding tensor of equivalent EM field $(T_{ik})_E$.

Suppose the tensor is T_{ik} for matter of the form: $T_{ik} = \rho u_i u_k$, so in the static field the only component other than zero is $T_0^0 = \rho$. If we describe the energy of the G-field by a tensor of equivalent EM field $(T_{ik})_E$, the divergence condition of the tensor goes into the form:

$$\left[(T_i^k)_m + (T_i^k)_E \right]_{;k} = 0. \quad (4.6)$$

In the static field, this expression takes the form:

$e^{-\lambda} \left(\frac{\nu''}{2} - \frac{\nu' \lambda'}{4} + \frac{\nu' \mu'}{2} \right) = \frac{4\pi\gamma}{c^2} \rho$; it agrees with equation $F^{0k}_{;k} = \frac{4\pi\gamma}{c^2} j^0$. In general, the divergence of the tensor (4.6) becomes:

$$\rho c^2 \frac{Du^i}{ds} = F^i_{;k} j^k. \quad (4.7)$$

The equation of motion of a particle of matter, or element of volume dV and density ρ , is obtained. In the general field j -na (4.7) is valid only if the motion of a particle of matter has a quasi-stationary character². In the free field, the right term of the above expression is zero. However, it would cause the massive body to collapse and shrink it to a point. The element of the continuous medium density ρ is not free, but is in the environment of the other matter with which it interacts.

No pressure appears in these equations; we use it to describe the motion of a given dV element in a continuous medium. In the static field, the pressure is obtained from Equations (4.2-ii) with the knowledge of the energy density ε of a given massive body and the coefficient g_{00} .

The equations of a centrally symmetric field in matter can be represented in the following form; but let's first choose an elementary interval in the form:

$$ds^2 = c^2 e^\nu dt^2 - e^\lambda dr^2 - e^\mu (d\theta^2 + \sin^2 \theta d\varphi^2). \quad (4.8)$$

The calculations give the following result¹¹:

$$\frac{8\pi\gamma}{c^4} T_0^0 = -e^{-\lambda} \left(\mu'' + \frac{3}{4} \mu'^2 - \frac{\mu' \lambda'}{2} \right) + e^{-\mu}, \quad (4.9-i)$$

$$-\frac{8\pi\gamma}{c^4} T_1^1 = \frac{1}{4} e^{-\lambda} \left((\mu' + \nu')^2 - \nu'^2 \right) - e^{-\mu}, \quad (4.9-ii)$$

$$-\frac{8\pi\gamma}{c^4} T_2^2 = \frac{1}{4} e^{-\lambda} \left(2(\nu'' + \mu'') + \nu'^2 + (\mu' + \nu')(\mu' - \lambda') \right) \quad (4.9-iii)$$

We set the tensor components of T_{ik} in the form $T_k^i = \rho u_k u^i$, but now assume that the density is:

¹¹Videti, L. D. Landau, *Teorija polja*.

$$\rho(r) = \rho_0 e^{-\lambda/2} e^{-\mu} r^2. \quad (4.10)$$

These equations agree with the divergence condition (4.6), so we have four equations with three unknowns at our disposal.

Using (4.9), (4.10) and (4.4), as well as choosing $e^\mu = r^2$, we obtain a solution in the form:

$$e^{-\lambda/2} = 1 - \frac{\gamma M}{c^2 R^3} r^2, \quad e^{\nu/2} = \left(1 - \frac{\gamma M}{c^2 R^3}\right)^{3/2} / \sqrt{1 - \frac{\gamma M}{c^2 R^3} r^2}. \quad (4.11)$$

In the next step, we look for the relationship between the coefficients e^μ i e^λ . From (4.9-*i*) and (4.9-*ii*) it follows:

$$e^{\mu-\lambda/2} \frac{\lambda' \mu'}{2} + e^{\mu/2} \mu' - \frac{A}{4\pi} 4\pi \rho e^\mu e^{\lambda/2} = e^{\mu-\lambda/2} (\mu'' + \mu'^2), \quad (4.12-i)$$

$$\int -\left(e^{-\lambda/2}\right)' (e^\mu)' dr + 2e^{\mu/2} + const - \frac{2\gamma}{c^2} m(r) = \int e^{\mu-\lambda/2} (d\mu' + \mu' d\mu), \quad (4.12-ii)$$

$$-e^{-\lambda/2} (e^\mu)' + 2e^{\mu/2} - \frac{2\gamma}{c^2} m(r) = const, \quad m(r) = \int 4\pi \rho e^\mu e^{\lambda/2} dr. \quad (4.12-iii)$$

The above equation allows us to obtain the desired metric form by a suitable choice of the coefficient e^μ .

Let us look at the reference systems in which the particle is stationary, in which the mass of the particle remains unchanged, this implies for the local proper frame, but it should also apply to a coordinate system whose center does not coincide with the coordinates of the particle. If a particle moves in a reference system, the variable mass m' is represented as the zero physical-coordinate of the momentum vector $p^\mu = mcu^\mu$; in the static field, $g_a = 0$, so we find:

$$m' = mu_0 \sqrt{g^{00}} = \frac{m}{\sqrt{1-v^2/c^2}}, \quad (4.13)$$

where za $v=0$ we have $m' = m$.

The fixed observers distributed along the radius of the massive body represented by the field source at a distance r measure the mass $m(r)$. The function $m(r)$ can be set in advance, so as the unit of proper volume depends on the place in the field, so does the density $\rho = dm/dV$ change, that is, we can write that the mass $m(r)$ has the form:

$$m(r) = \int 4\pi \rho e^\mu e^{\lambda/2} dr. \quad (4.13-i)$$

By choosing a metric form where $e^\mu = r^2 e^\lambda$ (ie referring to an absolute observer measuring its own standard of length), and specifying a mass in the form $m(r) = Mr^3/R^3$, since it is a different coordinate system. Equation (4.12-*iii*) gives the following equation:

$$ds^2 = \frac{c^2 dt^2}{\left(1 + \frac{3\gamma M}{2c^2 R^3} - \frac{1\gamma M}{2c^2 R^3} r'^2\right)^2} - \left(1 + \frac{3\gamma M}{2c^2 R^3} - \frac{1\gamma M}{2c^2 R^3} r'^2\right)^2 \left(dr'^2 + r'^2 (d\theta^2 + \sin^2 \theta d\phi^2)\right) \quad (4.14)$$

From here it can be seen that the speed of light and particles depends on the field potential. As the kinetic energy of a particle grows in the field, so the field provides greater resistance to its motion due to the interaction of the field and the particle, so it should be expected that in strong fields the particle can be slowed down by an external observer.

5. Conclusions

This approach to describing the G-field should make sense, since the physical assumptions are clear and the mathematical apparatus used is rounded. The derived field equations are in agreement with the particle motion equations and complement each other. For example, in a centrally symmetric field from the equation of motion of a particle, we find metric coefficients based on the Principle of Equivalence, which are added to the field equations to have a sufficient number of equations to solve the metric form.

The obtained equations and solutions were compared with the General Theory of Relativity and the differences were analyzed. The solutions of the equations where this difference is significantly observed are: (2.4), (3.3), (3.6), (3.18), (3.49), (3.64), (3.70), (4.11) and (4.14).

Thus, the example of the equation $R = 0$, where R is a scalar curve, is interesting, as a special case of the equation that follows from the General Theory of Relativity. As we have seen in the static field, this equation is also valid for the case when the EEM field is introduced. As from the condition $R = 0$ we obtain a solution of the metric coefficients with two constants, which by symmetry corresponds more to the equations for the EEM field, than the Schwarzschild solution with one constant in the metric coefficients.

Conflicts of Interest

The author declares no conflicts of interest.

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