Not Even Wrong—A Reexamination

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Abstract

Fundamental elements of physical theories of systems of objects are examined. Consider the Newtonian mechanics, Maxwellian electrodynamics, relativistic mechanics, general relativity, and the Dirac theory of the electron. Each of these theories has specific differential equations whose solutions accurately describe the state and the time evolution of systems of particles that belong to the theory’s domain of validity. Well-known textbooks prove that these theories can be derived from the least action principle that applies to an appropriate Lagrangian or Lagrangian density. The Noether theorem proves that the Euler-Lagrange differential equations of these theories conserve energy-momentum, angular momentum, and charge. The electroweak theory, which is a part of the Standard Model of particle physics, is a contradictory example. Although this theory is about 50 years old, textbooks still do not show the explicit form of its differential equations that describe the electromagnetic interactions of the $W^\pm$ particles. Hence, the electroweak theory fails to show that solutions of its missing differential equations fit experimental data. For this reason, the electroweak theory belongs to the set of physical ideas that are not even wrong.

Subject Areas

Particle Physics

Keywords


1. Introduction

This work examines the fundamental properties of physical theories that apply to a given system of objects. Here predictions of a theory should adequately describe measurements of the state and the time evolution of that system. Con-
cerning this issue, one does not deny a physical theory only if it fails to describe the physical properties of every system of physical objects. In this case, one should consider the domain of validity of any physical theory.

Two well-known examples illustrate the significance of this concept. Newtonian mechanics provides a good description of the motion of objects whose velocity is much smaller than the speed of light. This theory is the subject of a standard university course [1] [2], and this evidence indicates that Newtonian mechanics is not regarded as an erroneous theory. Therefore, its failure to describe systems where the particles' velocity is not much smaller than the speed of light does not mean that Newtonian mechanics is an erroneous theory. Here the domain of validity of Newtonian mechanics says that it applies to systems where the particles' velocity is small enough.

Classical electrodynamics says that accelerating charged particles emit radiation (see [3], section 63; [4], p. 657). Obviously, the stability of the ground state of atomic electrons means that it emits no radiation. This effect denies classical electrodynamics. However, classical electrodynamics is a standard university course (see, e.g., [3] [4] [5]). It means that it is not regarded as an erroneous theory. Here one realizes that quantum effects simply do not belong to the domain of validity of classical electrodynamics.

These examples illustrate the significance of the concept called the domain of validity of a physical theory. Rohrlich’s book contains a good discussion of this concept and its application to the hierarchical order of physical theories (see [6], pp. 1-6). As a rule, in cases that belong to a theory’s domain of validity, the accuracy of a quantitative value that is predicted by this theory takes several orders of magnitude.

This work examines the validity test of several physical theories. The examination shows that a well-established theory provides differential equations whose solutions adequately describe the state and the time evolution of a system of objects that belongs to the theory’s domain of validity. It is also proved that the electroweak theory of the Standard Model of particle physics (SM) is an exceptional case because it fails in this test. For example, the Dirac equation is a partial differential equation that excellently describes the state and the time evolution of the electromagnetic interaction of a massive electron. The case of the hydrogen atom is a well-known example of this point (see [7], pp. 52-60). However, electroweak textbooks still do not show the explicit form of an analogous equation for its electrically charged $W^\pm$ particles. The discussion of this problem demonstrates the novelty of this work.

Units where $\hbar = c = 1$ are used. Therefore, just one dimension is required and the dimension of length $[L]$ is used. The Minkowski metric $g_{\mu\nu}$ is diagonal and its entries are $(1, -1, -1, -1)$. All expressions are written in the standard notation and readers may consult the relevant references.

2. Differential Equations

This section briefly describes the properties of several fundamental physical
theories. Each of these theories has a definite domain of validity. These theories describe the state and the time evolution of an appropriate system of objects. As stated above, for a system of objects that belongs to a given domain of validity of a theory, the results of the corresponding theory are very good and their accuracy is of several orders of magnitude. Here an object can be an electron, which is a pointlike elementary particle, or a planet whose motion around the sun is analyzed by an appropriate gravitational theory. The theories’ description takes the historical order of their development.

2.1. Newtonian Mechanics

Newtonian mechanics is the first physical theory, and it is regarded as the beginning of physics as an accurate science. It is recognized that the friction-free motion of particles can be calculated with very high accuracy. Thus, Newtonian mechanics together with the Newtonian gravitation law enable the calculation of the planetary motion data, like the Kepler laws and eclipses. These calculations provide proof of the accuracy of Newtonian mechanics. The backbone of Newtonian mechanics takes the form of differential equations. In particular, the standard notation of Newton’s second law is

\[ m \frac{d^2 r}{dt^2} = F \]  

For example, well-known textbooks state:

“The relations between the accelerations, velocities and coordinates are called the equations of motion. They are second-order differential equations for the functions \( q(t) \), and their integration makes possible, in principle, the determination of these functions and so the path of the system (see [1], p. 2).”

Furthermore, another textbook says:

“The equation of motion is thus a differential equation of second order, assuming \( F \) does not depend on higher-order derivatives (see [2], p. 2).”

These quotations indicate that the validity of Newtonian mechanics is inferred from the fit between the solutions to its differential Equation (1) and experimental data.

For the simple case of a free particle, the Lagrangian of Newtonian mechanics is

\[ L = \frac{1}{2} m v^2 \]  

(see [1], p. 7; [2], p. 22). This Lagrangian yields the inertial motion of a force-free object.

2.2. Maxwellian Electrodynamics

In the 1860s, Maxwell used experimental formulas of the electromagnetic fields, together with the law of charge conservation, and formulated the equations of
motion of the electromagnetic fields, which are called Maxwell equations (see [3], chapter 4; [4], pp. 217-218). The relativistic covariant form of these equations is (see [3], pp. 71, 79; [4], p. 551)

\[
F^\mu\nu - F^{\ast\mu\nu} = 0,
\]

where \( F^{\mu\nu} \) is the tensor of the electromagnetic fields, \( F^{\ast\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \), and \( j^\mu \) is the 4-current of the electric charge.

The Lorentz force shows how electromagnetic fields affect the time evolution of a charged particle. In terms of densities, this force is (see [3], p. 89)

\[
\rho_m \frac{dv^\rho}{dt} = eF^{\mu\nu} j_\nu,
\]

where \( \rho_m \) denotes mass density. The far-reaching meaning of Maxwell equations and the Lorentz force is stated as follows:

“When combined with the Lorentz force equation and Newton’s second law of motion, these equations provide a complete description of the classical dynamics of interacting charged particles and electromagnetic fields (see [4], p. 218).”

The Lorentz force describes the motion of a charged particle in electromagnetic fields (see [3], p. 51). Maxwell equations introduce a new concept called field. The Lorentz force and Maxwell equations are differential equations. As stated above, these equations describe the state and the time evolution of a classical system of charged particles and electromagnetic fields.

### 2.3. Relativistic Mechanics

The theory of special relativity was published in year 1905, which is about 40 years after the formulation of Maxwell equations. It turns out that Maxwellian electrodynamics is compatible with special relativity. On the other hand, Newtonian mechanics is the low-velocity limit of relativistic mechanics. Section 2 of [3] presents the laws of relativistic mechanics. Here the Lagrangian of a free particle is

\[
L = -m \sqrt{1 - v^2}.
\]

The power series expansion of the square root of (5) shows that its low-velocity limit agrees with the corresponding expression (2) of Newtonian mechanics. The Lorentz force (4) is a relativistic expression, and it shows that also the relativistic mechanics theory of a charged particle takes the form of differential equations.

### 2.4. General Relativity

The next development is the theory called general relativity (GR). This is a theory that describes the time evolution of a system of objects that are affected by gravitational interactions. The Einstein equations of the gravitational fields are (see [3], p. 297)
Here the second rank curvature tensor $R_{\mu\nu}$ depends on the metric $g_{\mu\nu}$ and its derivatives (see [3], sections 91, 92), $k$ is the gravitational constant, and $T_{\mu\nu}$ is the energy-momentum tensor of particles and electromagnetic fields. The corresponding equation of a particle in a gravitational field is

$$a^\alpha = \Gamma^\alpha_{\alpha\beta} u^\beta,$$  

where $\Gamma^\alpha_{\alpha\beta}$ denotes the Christoffel symbol, $u^\alpha$ and $a^\alpha$ denote the 4-velocity and the 4-acceleration, respectively (see [3], p. 264).

This information shows that GR corresponds to classical electrodynamics: The Einstein Equation (6) of the gravitational field correspond to Maxwell equations of the electromagnetic fields (3), and (7) corresponds to the Lorentz force (4). Each of these equations is a differential equation. Their solutions determine the time evolution of the system. The fit of these solutions to measurements is impressive [8].

2.5. Quantum Mechanics

Quantum mechanics is the last theory that is examined in this list. This theory accounts for the wave properties of a massive particle. The Schroedinger equation is the quantum version of Newtonian mechanics

$$i \frac{\partial \psi}{\partial t} = -\frac{1}{2m} \Delta \psi + V(r,t) \psi$$  

(see [9], p. 21).

The Dirac equation is the quantum version of relativistic mechanics. This equation says that the electromagnetic interaction of the electron determines its state and time evolution (see [7], p. 11)

$$i \frac{\partial \psi}{\partial t} = \left[ -\mathbf{\alpha} \cdot (i \mathbf{\nabla} + e \mathbf{A}) + \beta m + e \Phi \right] \psi$$

(9)

The Dirac quantum function $\psi(r,t)$ is a 4-component spinor that depends on the space-time coordinates. Hence, the Schroedinger and the Dirac equations are partial differential equations. Solutions of the Dirac equation of the hydrogen atom remarkably fit the data:

It was validated by accounting for the fine details of the hydrogen spectrum in a completely rigorous way [10].

2.6. Common Properties of the Above Mentioned Physical Theories

The foregoing discussion shows examples of fundamental physical theories that describe the state and the time evolution of systems of objects. An examination of these theories indicates that each of them holds these properties:

P.1 Every theory takes the form of a set of differential equations.

P.2 The substantiation of every theory depends on the fit of relevant experimental data to solutions of its differential equations.
It is interesting to point out that the solutions to the differential equations of each of these theories excellently fit to quantitative measurements of variables. In every case, the fit is of several orders of magnitude. Therefore, these theories belong to the well-established part of physics.

3. The Least Action Principle

Physics is a mature science, and it recognizes that every acceptable theory should abide by several laws of Nature. For example, every physical theory should be consistent with the conservation laws of energy-momentum, angular momentum, and charge. Furthermore, relativistic quantum theories should take a relativistic covariant structure. These issues provide constraints on the acceptability of a set of differential equations that describe the state and the time evolution of a given system of objects.

It is recognized that the principle of least action is an important tool for the construction of a physical theory that is consistent with these requirements. For example, Landau and Lifshitz, and Goldstein used this principle for the derivation of Newtonian mechanics [1] [2]. Moreover, Landau and Lifshitz used it for the derivation of classical electrodynamics [3]. As a matter of fact, this principle is used by all modern textbooks on Quantum Field Theory (QFT) of elementary particles. For example, a well-known textbook states:

“All field theories used in current theories of elementary particles have Lagrangians of this form (see [11], p. 300).”

The Noether theorem applies to the Euler-Lagrange equations that are derived from the least action principle. It states that symmetries of the Lagrangian/Lagrangian density yield Euler-Lagrange equations that abide by corresponding conservation laws (see [11], p. 307). For example, assume that a QFT Lagrangian density \( \mathcal{L} \) does not explicitly depend on the four space-time coordinates \((r,t)\). It means that it takes the form (see e.g. [11], p. 300)

\[
\mathcal{L}(\psi(r,t), \psi'(r,t))
\tag{10}
\]

where \( \psi(r,t) \) denotes the quantum function. The Euler-Lagrange equation of such a Lagrangian density is (see e.g. [12], p. 14)

\[
\frac{\partial \mathcal{L}}{\partial \psi} - \partial_{\mu} \left( \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \psi)} \right) = 0.
\tag{11}
\]

The Noether theorem proves that the Euler-Lagrange equations of such a Lagrangian density conserve energy, momentum, and angular momentum.

An important property of a Lagrangian density refers to special relativity. If the Lagrangian density is a Lorentz scalar whose dimension is \([L^{-4}]\) then its Euler-Lagrange equations satisfy relativistic requirements (see [11], p. 300). Spatial rotations are a part of the Lorentz transformation. It means that such a Lagrangian density yields equations of motion that conserve angular momentum.

Another important property of the QFT Lagrangian density is its indepen-
dence of the transformation of the global phase factor \( \exp(i\alpha) \) of a given quantum function

\[ \psi(r,t) \rightarrow \exp(i\alpha)\psi(r,t). \]  

(12)

It means that if a term of the Lagrangian density of the quantum particle depends on the quantum function \( \psi \), then it comprises the product \( \psi^\dagger \psi \). This product removes the phase \( \exp(i\alpha) \) of (12). Here one finds that the corresponding Euler-Lagrange equations conserve charge (see [13], pp. 314-315). Thus, the 4-current \( j^\mu \) is

\[ j^\mu = (\rho, \rho v), \]  

(13)

where \( \rho \) is the charge density and \( v \) is the 3-velocity. The Noether expression for the 4-current is

\[ j^\mu = a \frac{\partial L}{\partial \psi^\dagger \psi^\mu}, \]  

(14)

where \( a \) is an appropriate numerical coefficient, and \( j^0 \) is the particle’s density. Hence, \( a \) is fixed so that \( \int j^0 d^3 x = 1 \). This 4-current satisfies

\[ j^\mu_\mu = 0. \]  

(15)

This differential equation is called the continuity equation, which is the mathematical form of charge conservation (see [3], pp. 76, 77). Charge conservation and the continuity equation are crucial elements of Maxwellian electrodynamics (see [3], section 29; [4], p. 169).

The Lagrangian density of quantum electrodynamics (QED) is (see e.g. [12], p. 84; [14], p. 78)

\[ \mathcal{L}_{\text{QED}} = \bar{\psi} \left[ \gamma^\mu \left( i \partial_\mu - e A_\mu \right) - m \right] \psi - \frac{1}{16\pi} F^\mu_\nu F_{\mu\nu}. \]  

(16)

This expression demonstrates the virtues of the foregoing arguments. Thus the Euler-Lagrange equation of the quantum function \( \psi \) is the covariant form of the Dirac Equation (9)

\[ \left[ \gamma^\mu \left( i \partial_\mu - e A_\mu \right) - m \right] \psi = 0 \]  

(17)

(see [12], p. 84). Relation (14) yields the 4-current of a Dirac particle

\[ j^\mu = \bar{\psi} \gamma^\mu \psi \]  

(18)

(see [7], p. 23). It is important to note that the Dirac 4-current (18) is used in the electromagnetic interaction term of the QED Lagrangian term, (16) where it is coupled to the electromagnetic 4-potential \( A_\mu \).

Furthermore, Landau and Lifshitz prove that an application of the least action principle to the electromagnetic variables of (16) \( -\frac{1}{16\pi} F^\mu_\nu F_{\mu\nu} \) yields the Maxwell equations (see [3], chapter 4). Here is a quotation that demonstrates the far-reaching virtues of the QED Lagrangian density (16):

“That such a simple Lagrangian can account for nearly all observed pheno-
mena from macroscopic scales down to $10^{-13}$ cm is rather astonishing (see [14], p. 78).”

The following section uses the elements that are discussed above and examines the structure of the electroweak theory, which is the SM sector that aims to describe weak interactions. The structure of this theory is compared to the previously discussed theories in general, and the electromagnetic theory of a Dirac particle, in particular.

An important attribute of the application of the least action principle is that the quantum function acquires a dimension. The action is $S = \int \mathcal{L} d^4x$, and the unit system where $\hbar = 1$ means that the action is dimensionless. It follows that the dimension of the Lagrangian density is $[L^{-4}]$. The QED Lagrangian density has the term $\bar{\psi} m \psi$. Thus, the $[L^{-1}]$ dimension of mass means that the dimension of the Dirac function $\psi$ is $[L^{-3/2}]$. This outcome indicates the dimensional coherence of the Dirac 4-current (18). Thus, $j^0$ denotes density; the $\gamma$ matrices are dimensionless numerical quantities; the dimension of the product $\bar{\psi} \psi$ is $[L^{-1}]$ which is the dimension of density.

An analogous argument is utilized below in the analysis of the electroweak theory of the $W^\pm$ particles.

4. The Electroweak Theory

The electroweak theory is more than 50 years old, and it is discussed in many textbooks. However, this section points out crucial theoretical elements that textbooks of this theory fail to prove. The pair of electrically charged particle-antiparticle $W^\pm$ are an important part of the electroweak theory (see [15], section 21.3), and this evidence indicates that these particles belong the theory’s validity domain. The $W^\pm$ are the electroweak analogs to the electrically charged electron-positron pair of the Dirac theory. As stated above, the QED Lagrangian density provides the differential equations of the system that properly describe electromagnetic interactions: the Dirac equation of the electron (17) and a coherent expression for the Dirac 4-current that satisfies the continuity Equation (18). It is shown above that these equations are derived from the QED Lagrangian density (16).

Let us examine the electroweak description of the $W^\pm$ particles. The mass term of the Lagrangian density of these particles is

$$\mathcal{L}_{WM} = -M^2 W_{\mu} W^\mu$$

(see [15], p. 309). This expression and the $[L^{-4}]$ dimension of the Lagrangian density prove that the dimension of the electroweak $W^\pm$ quantum functions is $[L^{-1}]$. This result can also be derived from the relation between the electroweak $W^\pm$ quantum functions and the ordinary electromagnetic 4-potential $A_\mu$ (see [15], section 21.3). This dimension of the $W^\pm$ is a key element of the analysis of problematic attributes of the electroweak description of the $W^\pm$ particles.

It turns out that, unlike the case of the Dirac electron, electroweak textbooks
show neither an explicit form of the corresponding differential equations of the electromagnetic interaction of the $W^\pm$ particle nor a coherent 4-current of these particles that satisfies the continuity equation. The following lines explain the origin of this discrepancy. It is now recognized that the required expressions should be derived from an appropriate Lagrangian density.

Here are details of two textbooks that present an electromagnetic interaction term of the $W^\pm$ particles (many other electroweak textbooks just ignore this crucial issue). This term takes the form (see [16], p. 518),

$$W_{EM} \text{ term: } e F_{\mu\nu} \left( W^\pm W^{\mu\nu} - W^{\pm}W^{\mu\nu} \right)$$

(20)

where $F_{\mu\nu}$ is the electromagnetic field. An analogous term is shown in [17], p. 113. An examination of expression (20) proves that it is unacceptable. Indeed, the electric charge $e$ of a Lagrangian density should be multiplied by an expression that describes density. The electromagnetic field $F_{\mu\nu}$ is charge-independent. Hence the $W$ functions of (20) should account for density. However, the dimension of the product of two $W$ functions is $[L^{-2}]$ whereas the dimension of density is $[L^{-3}]$. This is certainly a gross error.

The following points explain why the electroweak theory cannot properly define a coherent 4-current that is required for the electromagnetic interaction of the $W^\pm$ particles.

W.1 The 4-current is a mathematically real expression. Therefore, the mathematically complex form of the $W$ function means that the 4-current of the $W$ particles should depend on the product of $W^\pm W$.

W.2 The dimension of the product $W^\pm W$ is $[L^{-2}]$ while the dimension of the 4-current is $[L^{-3}]$. Therefore, a quantity that has an odd dimension is required. The space-time coordinates cannot be used because of the requirement of energy-momentum conservation (see section 3). Hence, only the derivative operator of the coordinates $\partial_\mu$ that applies to the quantum function of the $W$ particle is available for this purpose.

W.3 The Noether expression for the 4-current (14) proves that a term of the Lagrangian density that depends on a derivative $\partial_\mu$ of the quantum function contributes to the overall expression of the 4-current. Hence, a derivative-dependent 4-current cannot be coherently used in an electromagnetic interaction term because it modifies the system’s 4-current.

This discussion explains why the electroweak theory of the $W^\pm$ particles has intrinsic unsettled problems and why electroweak textbooks ignore the electromagnetic interaction of these particles.

The Not Even Wrong Concept

As shown in item P.1 of subsection 2.6, the correctness of a given physical theory of a system of objects is tested by the comparison of solutions of the theory’s differential equations with experiments that measure an appropriate system of objects. The expression not even wrong is ascribed to W. Pauli, who has used it for describing his negative opinion about a certain physical idea. A definition of
this concept says:

The phrase “not even wrong” describes any argument that purports to be scientific but fails at some fundamental level, usually in that it contains a terminal logical fallacy or it cannot be falsified by experiment, or cannot be used to make predictions about the natural world [18].

About 15 years ago P. Woit used this concept as the title of his book that denies the string idea as a basis for an elementary particle theory [19]. P. Woit criticizes string theory as an example of ideas that are

“so incomplete that they could not even be used to make predictions to compare with observations to see whether they were wrong or not.”

The previous discussion examines several physical theories: Newtonian mechanics, Maxwellian electrodynamics, relativistic mechanics, GR, and QED of the electron. Each of these well-established theories has a specific set of differential equations whose solutions rigorously predict the state and the time evolution of physical systems that belong to its domain of validity. The principle of least action is a cornerstone of these theories. Unlike these theories, an analysis of the electroweak theory proves that this theory fails to satisfy fundamental elements of physics. In particular:

No textbook shows the explicit form of the differential equations of the electromagnetic interactions of the electroweak theory of the $W^{\pm}$ particles and no textbook shows the corresponding 4-current that satisfies the continuity equation!

It means that, unlike the standard QFT structure in general and the QED Dirac theory of the electron in particular, the electroweak theory of the $W^{\pm}$ particles lacks differential equations whose solutions fit experimental data of the electromagnetic interaction of these particles. As such, this theory is an example of a physical idea that is not even wrong.

5. Conclusions

This work examines physical theories of systems of objects. It shows that appropriate differential equations are a fundamental element of theories like classical mechanics, Maxwellian electrodynamics, relativistic mechanics, GR, and QED of the electron. Solutions of the differential equations of any of these theories accurately describe the states and time evolution of appropriate systems of objects that belong to the theory’s domain of validity. This fit is an acceptability criterion of a physical theory.

The principle of least action is an important element of these theories. Here the Euler-Lagrange equations are the system’s equations of motion. In this case, the Noether theorem proves that appropriate symmetries of the Lagrangian/Lagrangian density yield Euler-Lagrange equations that conserve quantities like energy-momentum, angular momentum and charge. Textbooks like [1] [2] [3] [12] derive the above mentioned theories from the principle of least action.
It is proved above that the electroweak theory is an exceptional case because it lacks the fundamental elements of these theories:

L.1 Most textbooks do not show the entire Lagrangian density of the electroweak theory.

L.2 No textbook shows the explicit form of the electroweak differential equations of the electromagnetic interactions of the $W^\pm$ particles.

L.3 Obviously, no textbook shows that solutions of the unknown differential equations of the electroweak theory fit the electromagnetic interaction of the $W^\pm$ particles experimental data.

The absence of items L.2 and L.3 proves that the electroweak theory belongs to the category of physical ideas that are not even wrong.

Conflicts of Interest

The author declares no conflicts of interest.

References


