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Elementary Particles' Electrodynamics

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Abstract

We suggest a field theory based on a generalization of Maxwell's equations of electromagnetism, which allows one to describe the various elementary particles' structures. It seems to us, that such a theory must exist because under suitable reactions, particles can turn into particles of other forms and that the photon which partakes in many reactions, being a quantum of electromagnetic field, is described by the Maxwell equations. The fact that particles are supposed to have an internal structure and not be singular points, as is generally accepted in modern physics, supports Plank's hypothesis that every free particle is associated with a harmonic oscillation related to its internal energy. The photon can serve as an example, whose internal motion is described by the interaction of electric and magnetic fields. In order for a particle to exist on its own, that is without external influence, the equations that describe its field, in our opinion, must be compatible with each other (source being determined by the field, and the field by the source) and nonlinear. Let us note that the photon is an exception to this rule, since it is defined by linear equations. This is related, apparently, to the fact that it travels with the maximum speed—the speed of light *c*. Unlike in Maxwell's theory where sources are independent of the field that they generate, we propose a model with nonlinear self-compatible equations in which the sources happen to be quadratic functions of the electric and magnetic fields. We give an approximate solution to the nonlinear equation for the electron and we show that the structure of its electric field is asymptotically determined by Coulomb's law away from the particle's center, while near the center the field changes its nature.

Subject Areas

Dynamical System, Fundamental Physics, Spectroscopy

Keywords

Elementary Particles, Generalized Maxwell's Equations of Electrodynamics, Nonlinear Self-Compatible Field Equations, Internal Particle Structure

1. Introduction

In modern physics, elementary particles are viewed as points (*i.e.* not having an internal structure) or consisting of several such point particles (e.g. quarks). The behavior of complex systems (atoms, molecules, atomic nuclei, etc.) is successfully described by quantum mechanics using the motion of external fields of point particles. The point explanation, regardless of the fact that it contradicts in a large way the development of modern science, is a necessary but useful idealization. Observing that according to Plank's hypothesis, in reality particles are not simply points and have internal structures, in our opinion that to every free particle is associated an oscillation frequency ω, related to its internal energy E by the expression $E = \hbar\omega$, where $\hbar$ is Plank's constant. That is the particle is associated to some internal motion that determines a periodic oscillation. The photon is an example, a quantum of electromagnetic wave, whose internal motion is described by Maxwell's field equations. In the photon, this motion is determined by the interaction of two components: the intensities of the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{H}$. This agrees well with the various philosophical concepts (ancient and modern), that the development and self-development happens as a result of the interaction of two beginnings, as in the living and non-living worlds (male and female beginnings). The fact that the Maxwell field equations describe an electromagnetic phenomenon in the whole range of frequencies without limits from above or below is worthy of awe, and hints at particular directions of thought.

The characteristic property of elementary particles is their capacity for mutual transformation while satisfying certain conservational laws. Photon for instance partakes in many such reactions. Both of these factors indicate at the fact the nature of elementary particles is one. Due to the fact that the photon is described electrodynamically by Maxwell's equations, the other elementary particles can also have an electrodynamic basis albeit a little bit modified.

Since a systematic theory of elementary particles does not exist at the moment, we attempt here to create a field theory of particles free of external influence on the basis of a generalized theory of Maxwell's equations of electrodynamics. The concept of a free particle remains relatively abstract, since the presence of external influence is always present. Such an ideal model is accepted by physics and for the most part has justified itself.

In order for a particle to exist on its own, that is without external sources, the equations describing its field, in our opinion, must be self-compatible (*i.e.* source is determined by the field, and the field by the source) and nonlinear. Einstein's general theory of relativity can serve as an example, in which the theory of gravity is described by self-compatible nonlinear tensor differential field equations. But this theory, based on the equality of inertial and gravitational mass, describes the behavior of macrobody and in our opinion is not entirely suitable for a construction of a theory of elementary particles. Most likely because of this the construction of a single field theory explaining all forms of interaction (including the theory of gravity), despite of numerous attempts and effort, has not been created as of yet.

For the creation of a field theory giving the internal structure of elementary particles, we choose Maxwell's model of electrodynamics, operating on only two vector quantities $\boldsymbol{E}$ and $\boldsymbol{H}$. Maxwell's equations are linear and the electromagnetic field is created by external independent sources (*i.e.* current and charges).

We propose a model that works with nonlinear self-compatible equations in which the sources of the field are quadratic forms of $\boldsymbol{E}$ and $\boldsymbol{H}$. The purpose of this investigation is the creation of a field theory describing the structure of various elementary particles on the basis of generalized Maxwell's equations of electrodynamics.

2. Basic Relations

First of all, let us note that the CGS absolute system of units will be used, since it forms a more convenient configuration in theoretical physics.

Maxwell's equations in empty space are as shown [1]:

$$\begin{aligned} &\operatorname{rot} \boldsymbol{H} = \frac{1}{c} \cdot \frac{\partial \boldsymbol{E}}{\partial t}, \\ &\operatorname{rot} \boldsymbol{E} = -\frac{1}{c} \cdot \frac{\partial \boldsymbol{H}}{\partial t}, \\ &\operatorname{div} \boldsymbol{E} = 0, \\ &\operatorname{div} \boldsymbol{H} = 0, \end{aligned} \tag{1}$$

where $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic fields respectively. For our analysis below we will need the fact that the present components $\boldsymbol{E}$ and $\boldsymbol{H}$ have different natures, even though both are vectors. $\boldsymbol{E}$ is a polar (usual) vector while $\boldsymbol{H}$ is an axial vector [1]. They differ by the fact that under a reflection of system of coordinates that is by a change of sign of all coordinates, the components of all vectors change sign. The components of the axial vector (that can be described as the vector product of two polar vectors) don't change sign under such a reflection. The scalar product of two vectors of the same type is a scalar, but of different types is a pseudoscalar, that is under a reflection of coordinates changes sign. We note that the "rot" operator changes one type of vector into the other. The fact that $\boldsymbol{E}$ is polar while $\boldsymbol{H}$ is axial is seen, for example, from the famous relation:

$$\boldsymbol{H} = \operatorname{rot} \boldsymbol{A}, \quad \boldsymbol{E} = -\frac{1}{c} \cdot \frac{\partial \boldsymbol{A}}{\partial t} - \operatorname{grad} \varphi$$

(e.g. see [1]), where $\boldsymbol{A}$ is a vector potential (a polar vector), φ is a potential, and c is the speed of light.

It's interesting to note that Maxwell's equations allow a beautiful formulation in complex notation if one introduces a new complex vector quantity $\boldsymbol{\Psi}$:

$$\boldsymbol{\Psi} = E + i\boldsymbol{H}. \tag{2}$$

Then the four equations in (1) become the two equations:

$$\operatorname{rot}\boldsymbol{\Psi} = \frac{i}{c}\cdot\frac{\partial\boldsymbol{\Psi}}{\partial t},\qquad \operatorname{div}\boldsymbol{\Psi} = 0. \tag{3}$$

Maxwell's equations have to relative invariants [1]:

$$\mathcal{J}_1 = \boldsymbol{E}^2 - \boldsymbol{H}^2,\qquad \mathcal{J}_2 = \boldsymbol{EH}. \tag{4}$$

Or alternatively:

$$\mathcal{J} = \boldsymbol{\Psi}^2,\qquad \mathcal{J} = \mathcal{J}_1 + 2i\mathcal{J}_2, \tag{4'}$$

where $\mathcal{J}_1, \mathcal{J}_2$ are a real scalar and real pseudoscalar respectively. The two invariants $\mathcal{J}_1$ and $\mathcal{J}_2$ are independent [1].

For an electromagnetic monochromatic planar wave in empty space (photon), the following is true:

$$\mathcal{J}_1 = \mathcal{J}_2 = 0,\qquad \boldsymbol{\Psi}^2 = 0. \tag{5}$$

We note that

$$\boldsymbol{\Psi\Psi}^* = E^2 + H^2 = 8\pi W$$

where W is the energy density of the electromagnetic field.

$$\boldsymbol{\Psi}\times\boldsymbol{\Psi}^* = -2i\boldsymbol{E}\times\boldsymbol{H} = -\frac{8\pi}{c}i\boldsymbol{S}$$

where $\boldsymbol{S}$ is Poynting's vector, the power flow density of the electromagnetic field [1].

3. Generalization of Maxwell's Equations

From the quadratic forms in (4) we construct two vectors: $\operatorname{grad}\mathcal{J}_1$ and $\operatorname{grad}\mathcal{J}_2$, and taking into account that the first is a polar vector while the second is an axial vector, and that quantities in the first equation in (1) are polar vectors, in the second equation are axial, in the third are scalars, and in the fourth are pseudoscalars, we generalize the Maxwell equations (1) in the following way:

$$\begin{aligned}
\operatorname{rot}\boldsymbol{H} &= \frac{1}{c}\cdot\frac{\partial\boldsymbol{E}}{\partial t} + \beta_1\operatorname{grad}\mathcal{J}_1,\\
\operatorname{rot}\boldsymbol{E} &= -\frac{1}{c}\cdot\frac{\partial\boldsymbol{H}}{\partial t} + \beta_2\operatorname{grad}\mathcal{J}_2,\\
\operatorname{div}\boldsymbol{E} &= \alpha_1\mathcal{J}_1,\\
\operatorname{div}\boldsymbol{H} &= \alpha_2\mathcal{J}_2,
\end{aligned} \tag{6}$$

where $\alpha_1, \alpha_2, \beta_1, \beta_2$ are real constants.

We now require that the system of Equation in (6) is reducible into the complex form (3) using the complex vector Ψ (2). Then setting:

$$\beta_1 = \beta;\ \ \beta_2 - 2\beta;\ \ \alpha = \alpha;\ \ \alpha_2 = 2\alpha,$$

the system (6) takes the form:

$$\begin{aligned} &\operatorname{rot}\boldsymbol{\Psi} = \frac{i}{c}\cdot\frac{\partial\boldsymbol{\Psi}}{\partial t} + i\beta\,\operatorname{grad}\boldsymbol{\Psi}^2, \\ &\operatorname{div}\boldsymbol{\Psi} = \alpha\boldsymbol{\Psi}^2, \end{aligned} \tag{6'}$$

where instead of four constants, only two are involved: α and β. Or in explicit form:

$$\begin{aligned} &\operatorname{rot}\boldsymbol{H} = \frac{1}{c}\cdot\frac{\partial\boldsymbol{E}}{\partial t} + \beta\,\operatorname{grad}\mathcal{J}_1, \\ &\operatorname{rot}\boldsymbol{E} = -\frac{1}{c}\cdot\frac{\partial\boldsymbol{H}}{\partial t} - 2\beta\,\operatorname{grad}\mathcal{J}_2, \\ &\operatorname{div}\boldsymbol{E} = \alpha\mathcal{J}_1, \\ &\operatorname{div}\boldsymbol{H} = 2\alpha\mathcal{J}_2. \end{aligned} \tag{6''}$$

We point out that (6") is analogous to Maxwell's equations with external sources: with electric current density $\boldsymbol{j}_e$ and the charge density ρ_e [1]:

$$\begin{aligned} &\operatorname{rot}\boldsymbol{H} = \frac{1}{c}\cdot\frac{\partial\boldsymbol{E}}{\partial t} + \frac{4\pi}{c}j_e, \\ &\operatorname{rot}\boldsymbol{E} = -\frac{1}{c}\cdot\frac{\partial\boldsymbol{H}}{\partial t}, \\ &\operatorname{div}\boldsymbol{E} = 4\pi\rho_e, \\ &\operatorname{div}\boldsymbol{H} = 0. \end{aligned} \tag{7}$$

By Comparing (7) and (6"), it can be seen that $\beta\operatorname{grad}\mathcal{J}_1$ is analogous to the electric current density $\frac{4\pi}{c}\boldsymbol{j}_e$, $\alpha\mathcal{J}_1$ is analogous to the electric charge density $4\pi\rho_e$. However, unlike the Maxwell's equations written in (7), the magnetic current density $\boldsymbol{j}_m$ and the magnetic charge density ρ_m are present if we define:

$$\begin{aligned} &\frac{4\pi}{c}\boldsymbol{j}_m = -2\beta\,\operatorname{grad}\mathcal{J}_2, \\ &4\pi\rho_m = 2\alpha\mathcal{I}_2. \end{aligned} \tag{8}$$

We note that the "electric" current and charge densities and the "magnetic" current and charge densities in (6") are quadratically dependent on both the fields $\boldsymbol{E}$ and $\boldsymbol{H}$, but in different forms, and if the former are a scalar and vector respectively, then the latter is a pseudoscalar and pseudovector. Comparing the obtained system of Equation (6") with the Maxwell Equation (7), we observe two additional differences between them. The main and substantial difference is that if the electromagnetic field in Equation (7) is excited by external sources, then (6") is excited by the field itself and are nonlinear equations.

1) The Maxwell Equation (7) is invariant relative to a change in sign in the time variable and the magnetic field [1] that is:

$$t \to -t,\ \boldsymbol{E} \to \boldsymbol{E},\ \boldsymbol{H} \to -\boldsymbol{H}.$$

This way, processes in electromagnetic fields can run in one direction as in the other. However, solutions to (6") don't allow for such a phenomenon because of the second equation, that is the processes inside particles exhibit a one way nature in time.

2) It's possible to show that the Equation in (6"), unlike the Maxwell Equation (7), is not relativistically invariant with respect to a Lorentzian change of coordinates. This seems to imply that the physics of processes inside particles, that is for $r < 10^{-12}$ cm (the radius of heavy nuclei) is somewhat different, comparable to that physical process inside atoms ($r < 10^{-8}$ cm) is described not by classical, but quantum mechanics.

By applying the divergence operator to the first Equation in (6'), considering the second Equation in (6'), it is easy to see that:

$$\frac{\mathrm{d}\boldsymbol{\Psi}^2}{\mathrm{d}t} + a\,\mathrm{div}\,\mathrm{grad}\,\boldsymbol{\Psi}^2 = 0 \quad \text{or} \quad \frac{\mathrm{d}\boldsymbol{\Psi}^2}{\mathrm{d}t} + a\Delta\boldsymbol{\Psi}^2 = 0, \tag{9}$$

where $a = \beta c/\alpha$ and Δ is the Laplacian. The first Equation in (9) is from a certain point of view an analog of the charge continuity equation of electromagnetic theory [1]:

$$\frac{\partial \rho_e}{\partial t} + \mathrm{div}\,\boldsymbol{j}_e = 0,$$

and of that in quantum mechanics [2]:

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\,\boldsymbol{j} = 0,$$

where we have that

$$\rho = \Psi\Psi^* \ \text{and} \ \boldsymbol{j} = \frac{\hbar}{2mi}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right)$$

where m is the mass of the particle and Ψ, unlike the complex vector [2], is the complex scalar wave function satisfying the Schrödinger equation conveying a physical explanation the space time wave [3].

4. Static Solutions

The system of Equations (6') and (6"), as well as Maxwell's Equation (7), allow us to obtain static solutions $\boldsymbol{\Psi}_0$, *i.e.* time independent solutions:

$$\begin{aligned} \mathrm{rot}\,\boldsymbol{\Psi}_0 &= i\beta\,\mathrm{grad}\,\boldsymbol{\Psi}_0^2, \\ \mathrm{div}\,\boldsymbol{\Psi}_0 &= \alpha\boldsymbol{\Psi}_0^2, \end{aligned} \tag{10}$$

where

$$\boldsymbol{\Psi}_0 = \boldsymbol{E}_0 + i\boldsymbol{H}_0,$$

$$\boldsymbol{\Psi}_0^2 = \boldsymbol{E}_0^2 - \boldsymbol{H}_0^2 + 2i\boldsymbol{E}_0\boldsymbol{H}_0.$$

By applying the rotor operator to the first Equation in (10), considering the second Equation in (10) and the fact that that for any vector $\boldsymbol{A}$

$$\mathrm{rot}\,A = \mathrm{grad}\,\mathrm{div}\,\boldsymbol{A} - \Delta\boldsymbol{A},$$

we get

$$\Delta\boldsymbol{\Psi}_0 = \alpha\,\mathrm{grad}\,\boldsymbol{\Psi}_0^2. \tag{10'}$$

Equation (10) can be written explicitly in the following form:

$$\begin{aligned} \operatorname{rot} \boldsymbol{H}_0 &= \beta \operatorname{grad}\left(\boldsymbol{E}_0^2 - \boldsymbol{H}_0^2\right), \\ \operatorname{rot} \boldsymbol{E}_0 &= -2\beta \operatorname{grad}\left(\boldsymbol{E}_0 \boldsymbol{H}_0\right), \\ \operatorname{div} \boldsymbol{E}_0 &= \alpha\left(\boldsymbol{E}_0^2 - \boldsymbol{H}_0^2\right), \\ \operatorname{div} \boldsymbol{H}_0 &= 2\alpha \boldsymbol{E}_0 \boldsymbol{H}_0. \end{aligned} \tag{11}$$

From (11) it follows that separately, unlike in the Maxwell Equation (7), the fields $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$ don't exist. In the case $\boldsymbol{E}_0 \perp \boldsymbol{H}_0$, $\operatorname{rot} \boldsymbol{E}_0 = 0$ the electric field is simply a potential, while the magnetic is a rotational field ($\operatorname{div} \boldsymbol{H}_0 = 0$) and the Equation (11) simplify as

$$\begin{aligned} \operatorname{rot} \boldsymbol{H}_0 &= \beta \operatorname{grad}\left(\boldsymbol{E}_0^2 - \boldsymbol{H}_0^2\right), \\ \operatorname{rot} \boldsymbol{E}_0 &= 0, \\ \operatorname{div} \boldsymbol{E}_0 &= \alpha\left(\boldsymbol{E}_0^2 - \boldsymbol{H}_0^2\right), \\ \operatorname{div} \boldsymbol{H}_0 &= 0. \end{aligned} \tag{12}$$

An approximated solution for (12) is shown on **Appendix**.

5. Dynamic Solutions

The functions $\boldsymbol{E}(\boldsymbol{r},t)$, $\boldsymbol{H}(\boldsymbol{r},t)$, and $\boldsymbol{\Psi}(\boldsymbol{r},t)$ are all dependent on the coordinate $\boldsymbol{r}$ and time *t*.

We will later analyze the dynamically stationary solutions that are dependent on time through a harmonic rule $\mathrm{e}^{-i\omega t}$, that is analogous to the quantum states in Schrödinger equation with constant energy $E = \hbar\omega$ [2].

Let us note that while Schrödinger equation is linear, Equation (6') aren't, as a result of the existence of $\boldsymbol{\Psi}^2$. So stationary solutions are allowed in particular if the following relations are met:

$$\begin{aligned} \boldsymbol{\Psi}(\boldsymbol{r},t) &= \boldsymbol{\Psi}_0(\boldsymbol{r}) + \tilde{\boldsymbol{\Psi}}(\boldsymbol{r})\mathrm{e}^{-i\omega t}, \\ \boldsymbol{\Psi}^2(\boldsymbol{r},t) &= \boldsymbol{\Psi}_0^2(\boldsymbol{r}) + 2\boldsymbol{\Psi}_0\tilde{\boldsymbol{\Psi}}(\boldsymbol{r})\mathrm{e}^{-i\omega t}, \end{aligned} \tag{13}$$

under the following condition:

$$\tilde{\boldsymbol{\Psi}}^2 = 0, \tag{14}$$

which doesn't imply that $\tilde{\boldsymbol{\Psi}} = 0$. Yet, as in the electromagnetic field Condition (5), the following conditions are met:

$$\begin{aligned} \tilde{\boldsymbol{\Psi}}_1^2 - \tilde{\boldsymbol{\Psi}}_2^2 &= 0, \\ \tilde{\boldsymbol{\Psi}}_1\tilde{\boldsymbol{\Psi}}_2 &= 0, \end{aligned} \tag{14'}$$

where

$$\tilde{\boldsymbol{\Psi}} = \tilde{\boldsymbol{\Psi}}_1 + i\tilde{\boldsymbol{\Psi}}_2,$$

where $\tilde{\boldsymbol{\Psi}}_1$, $\tilde{\boldsymbol{\Psi}}_2$ are real vectors.

Considering Equations (13) and (14), Equation (6') can be divided into a static part $\boldsymbol{\Psi}_0(\boldsymbol{r})$ defined in Equations (10) and (10') and a dynamic part $\tilde{\boldsymbol{\Psi}}(\boldsymbol{r})$

$$\begin{aligned} \operatorname{rot}\tilde{\boldsymbol{\Psi}}(\boldsymbol{r}) &= k\tilde{\boldsymbol{\Psi}}(\boldsymbol{r}) + i\beta \operatorname{grad}\tilde{\mathcal{J}}(\boldsymbol{r}), \\ \operatorname{div}\tilde{\boldsymbol{\Psi}}(\boldsymbol{r}) &= \alpha\tilde{\mathcal{J}}(\boldsymbol{r}), \end{aligned} \tag{15}$$

where

$$\tilde{\mathcal{J}}(\boldsymbol{r}) = 2\Psi_0(r)\tilde{\Psi}(\boldsymbol{r}),$$

$$k = \frac{\omega}{c}.$$

The relationship between $\boldsymbol{E}(\boldsymbol{r},t)$, $\boldsymbol{H}(\boldsymbol{r},t)$, and $\tilde{\Psi}(\boldsymbol{r})$ can be obtained in the following relations:

$$\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}_0(\boldsymbol{r}) + \tilde{\boldsymbol{E}}(\boldsymbol{r},t), \quad \boldsymbol{H}(\boldsymbol{r},t) = \boldsymbol{H}_0(\boldsymbol{r}) + \tilde{\boldsymbol{H}}(\boldsymbol{r},t),$$

$$\tilde{\boldsymbol{E}}(\boldsymbol{r},t) + i\tilde{\boldsymbol{H}}(\boldsymbol{r},t) = \tilde{\Psi}(\boldsymbol{r})\mathrm{e}^{-i\omega t}.$$

Note that $\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{H}}$ are real vectors, but $\tilde{\Psi}$ is a complex one.

Considering Equations (13) and (14), Equation (9) for Ψ^2 can also be divided into constant and varying parts:

$$-i\omega\Psi_0(\boldsymbol{r})\tilde{\Psi}(\boldsymbol{r}) + a\Delta\left[\Psi_0(\boldsymbol{r})\tilde{\Psi}(\boldsymbol{r})\right] = 0, \tag{16}$$

$$\Delta\Psi_0^2(\boldsymbol{r}) = 0. \tag{17}$$

It is seen that for stationary (*i.e.* stable in time) particles, the dynamic part of the field $\tilde{\Psi}$ is always connected with the static part Ψ_0.

Another form of dynamic equations can be obtained by applying the rotor operator to Equation (6'):

$$\Delta\Psi(\boldsymbol{r},t) - \frac{1}{c^2}\frac{\partial^2\Psi(\boldsymbol{r},t)}{\partial t^2} = \alpha\,\mathrm{grad}\,\Psi^2(\boldsymbol{r},t) + \frac{\beta}{c}\mathrm{grad}\frac{\partial\Psi^2(\boldsymbol{r},t)}{\partial t}. \tag{18}$$

For an electromagnetic field, Equation (18) becomes the known wave equation [1]:

$$\Delta\Psi(\boldsymbol{r},t) - \frac{1}{c^2}\frac{\partial^2\Psi(\boldsymbol{r},t)}{\partial t^2} = 0. \tag{19}$$

To find the stationary solutions of Equation (18) let us assume, as was done above, that Conditions (13), (14) are satisfied; then Equation (18) can be shown as follows:

$$\Delta\tilde{\Psi}(\boldsymbol{r}) + k^2\tilde{\Psi}(\boldsymbol{r}) = 2(\alpha - ik\beta)\mathrm{grad}\left[\Psi_0(r)\tilde{\Psi}(\boldsymbol{r})\right], \tag{20}$$

$$\Delta\Psi_0(r) = \alpha\,\mathrm{grad}\,\Psi_0^2(r). \tag{21}$$

Equation (21) corresponds, of course, to Equation (10').

Equation (20) holds as a generalized wave equation for a stationary particle (perhaps it is a neutrino). It can be seen that this particle is coupled to a charged particle Ψ_0; let us say that it is an electron.

6. Some Commentary

Let us note that for a particle to be stable, strong enough conditions have to be satisfied (13), (14), that is why probably most free particles are not stable, and decay at one time or another. Hence, stable particles are, in a way, an unusual thing.

Note

It is interesting to consider a few other questions, especially those regarding ball lightning whose nature remains unknown to this day. It can be assumed that a ball lightning is an electrostatic field, which is held by itself. The intensity of the electric field is defined through (A.3).

Conflicts of Interest

The author declares no conflicts of interest.

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Appendix

Despite its seeming simplicity, the nonlinearity of Equation (12) makes them difficult to solve. For this reason, we simplify the problem by assuming that $\boldsymbol{H}_0^2 \ll \boldsymbol{E}_0^2$. This way, we are led to the following system of differential equations:

$$\begin{aligned} &\operatorname{rot} \boldsymbol{E}_0(\boldsymbol{r}) = 0, \\ &\operatorname{div} \boldsymbol{E}_0(\boldsymbol{r}) = \alpha \boldsymbol{E}_0^2(\boldsymbol{r}), \end{aligned} \tag{A.1}$$

For a spherically symmetric particle, assuming that the vector $\boldsymbol{E}_0(\boldsymbol{r})$ has only one component, E_{0_r}, which depends only on the radius r (with this the first equation in (A.1) is automatically satisfied) and Equation (A.1) becomes

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left[r^2 E_{0_r}(r)\right] = \alpha E_{0_r}^2(r), \tag{A.2}$$

or

$$\frac{\mathrm{d}E_{0_r}(r)}{\mathrm{d}r} + \frac{2}{r}E_{0_r}(r) = \alpha E_{0_r}^2(r).$$

This is a Bernoulli equation [4]:

$$\frac{\mathrm{d}y}{\mathrm{d}x} + P(x)y = Q(x)y^n,\ \ n = 2,$$

which is analytically solvable:

$$E_{0_r} = \left(\alpha r + \bar{c} r^2\right)^{-1}, \tag{A.3}$$

where $\bar{c}$ is an arbitrary constant.

For $r \gg 1$, away from the particle's center,

$$E_{0_r} \sim \frac{1}{\bar{c} r^2},$$

which corresponds to Coulomb's law:

$$E(r) = \frac{q}{r^2},$$

where q is the particle's charge.

We get:

$$\bar{c} = \frac{1}{q} \tag{A.4}$$

Close to the particle's center, the field does not match Coulomb's law:

$$E_{0_r} \sim \frac{1}{\alpha r}.$$

The value of the constant $\bar{c}$ can be found directly by equating the third Equation in (7) and (A.1), so that the following is obtained:

$$4\pi\rho \to \alpha E_0^2(r).$$

Hence, the charge q equals to:

$$q = \int \rho \mathrm{d}V = \alpha \int_0^\infty \frac{r^2 \mathrm{d}r}{\left(\alpha r + \overline{c} r^2\right)^2} = \frac{1}{\overline{c}},$$

which satisfies (A.4) as well.

Now let us estimate the value of the constant α. For this we introduce the particle's radius r_0. We will assume that inside the volume, defined by r_0, most of the charge q exists. We will also assume that inside there is $0.9q$.

Hence,

$$0.9q \cong \alpha \int_0^{r_0} \frac{r^2 \mathrm{d}r}{\left(\alpha r + q^{-1} r^2\right)^2},$$

we get:

$$\alpha \cong \frac{0.1 r_0}{q}.$$

For an electron, $q = e \cong 10^{-19}\ \mathrm{C} \cong 3 \times 10^{-10}\ \mathrm{esu}$. For the electron's radius r_0, let us assume that it is equal to $r_0 \sim 10^{-14} - 10^{-15}\ \mathrm{cm}$ (a radius of a heavy nucleus is about 10^{-12} cm).

Therefore,

$$\alpha \sim 10^{-6} - 10^{-7}\ \frac{\mathrm{cm}}{\mathrm{esu}}.$$