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# Explore on the Wave Character of Thermodynamic Entropy and Its Cause

**Yicheng Chen**

College of Physical Science and Technology, Central China Normal University, Wuhan, China
Email: cyc4618@163.com

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## Abstract

Based on the existing basic theories of thermodynamics, this paper uses the physical thought contained in the entropy balance equation itself to spread it, and compare it with the laws of mass conservation and momentum conservation in mechanics, so as to obtain the wave equation satisfied by entropy. It is found that the thermodynamic entropy fluctuates in the system, and the solution of the wave equation is not a simple harmonic, but determined by the divergence of the term of the viscosity coefficient of the fluid in the system and the time change rate of the intensity of the entropy source, and explains the cause of this wave from the point of view of mechanics! This paper naturally denies the "heat death theory" in history in theory! It has recently been reported that the wave characteristic of entropy has been experimentally verified! At the same time, it is pointed out that there is still a lot of work to be done.

## Subject Areas

Thermodynamics

## Keywords

Thermodynamic Entropy, Non-Equilibrium Thermodynamics, Continuum Mechanics, Wave Character of Thermodynamic Entropy

## 1. Introduction

This paper is a basic theoretical study. More than a hundred years ago, this subject was a hot topic. Today, although the research is less hot, it is still a major theoretical subject! Pure basic theoretical research may not be seen to have any effect at present, so what about the future! We all know that When Hertz discovered electromagnetic waves, and he called them useless, but now what?

As you all know, the theory of heat death has puzzled people for a long time. Various theories have been put forward to get out of this dilemma, including "the universe is not an isolated system", Maxwell's demon device, Boltzmann's theory of fluctuation and Zellmero's reappearance theory, as opinions vary, no unanimous conclusion can be drawn!

In fact, the entropy of a system can be reduced by force, as illustrated by a common phenomenon in nature.

We often see the following situation: on a small lake, when the wind is calm, the wastes such as grass and moss on the lake almost spread to cover the whole lake. We take the floating wastes on the lake as the system, and then the entropy of the garbage system increases to the maximum.

When a gust of wind blows, the junk is blown sideways, reducing its entropy; the stronger the wind, the faster the entropy decreases! The wind blows the garbage, indicating the interaction of forces with the system. That means the force is going to decrease the entropy! And there are all kinds of forces in nature, so entropy can't just go up and down on its own!

More than 80 years ago, Landau developed the two-fluid theory, which successfully explained the superflow of helium-4 liquids (strongly interacting bose systems) and predicted that entropy or temperature would propagate in the form of waves in superflows.

In this paper, based on the inherent idea of thermodynamic entropy theory, the wave equation is derived theoretically.

The author finds that the equilibrium equation of entropy contains some conservation, which is very similar to the law of conservation of mass in mechanics. Starting from the existing basic theory and using the inherent physical thought of entropy equilibrium equation itself, this paper diverges it and compares it with the conservation law of mechanics, so as to obtain the theoretical conclusion of wave character of entropy. Early last month, the University of Science and Technology of China observed the critical divergence of entropy waves in Fermi superflow for the first time, which is probably the best experimental proof of entropy waves!

## 2. The Wave Equation Satisfied by Entropy Is Obtained by Using the Physical thought Contained in the Entropy Balance Equation Itself, Which Is Similar to the Law of Mass Conservation and Momentum Conservation in Mechanics

The authors found that the equilibrium equation of entropy contains some conservation property, which is very similar to the law of conservation of mass in mechanics. Based on the EXISTING basic theory, THIS paper uses the inherent physical thought of entropy balance equation itself to spread it out, and discusses it with the conservation law of mechanics, so as to get the theoretical conclusion of entropy fluctuation. In early February this year, the University of Science and Technology of China first observed the critical divergence of entropy waves in

Fermi superflow, which may be the best experimental proof of entropy waves [1]!

Consider a thermodynamic system consisting of $l$ components. For simplicity, assume that there is no chemical reaction between these components. Let the mass of the $k$-th component in the unit volume be $\rho_k$ (mass density), then the total density is $\rho = \sum_{k=1}^{l} \rho_k$ .

Now let's focus on the entropy equilibrium equation ([2], Chapter 3 (10)):

$$\frac{\partial(\rho s)}{\partial t} = -\mathrm{div}\vec{J}_{s,\mathrm{tot}} + \sigma \tag{1}$$

where $s$ is the entropy per unit mass; $\sigma$ is called entropy growth, that is, the entropy increased in unit volume per unit time, also known as entropy source strength; The other thing to notice here is that ( $\rho s$ ) matches $\vec{J}_{s,\mathrm{tot}}$, that these two quantities appear in the equation at the same time, which is useful in the following discussion. $\vec{J}_{s,\mathrm{tot}}$ is the total entropy flow per unit area in unit time ([2], Chapter 3 (13)):

$$\vec{J}_{s,\mathrm{tot}} = \vec{J}_s + \rho s \vec{\upsilon} \tag{2}$$

The second term on the right of this formula shows that the total entropy flow is related to the entropy contained in the unit volume mass moving at the center of mass velocity $\vec{\upsilon}$; The first term $\vec{J}_s$ is called entropy flux (some literature calls it entropy flow), that is, the entropy flow passing through the unit area in unit time, and its mathematical expression is ([2], Chapter 3 (20)):

$$\vec{J}_s = \frac{1}{T}\left(\vec{J}_q - \sum_{k=1}^{n} \mu_k \vec{J}_k\right) \tag{3}$$

The reason why it is not obvious to write the mathematical expression of $\sigma$ here is because the discussion here has nothing to do with the specific expression of $\sigma$.

The entropy equilibrium Equation (1) tells us that if the entropy at a certain point increases, there must be entropy flowing to that point, which is the physical meaning of the first term on the right of Equation (1). In (3) of the entropy flow expression, $\vec{J}_q$ is called heat flow, and the heat flow is determined by the following formula ([2], Chapter 2 (33)):

$$\vec{J}_e = \rho e \vec{\upsilon} + \vec{P} \cdot \vec{\upsilon} + \sum_{k=1}^{n} \psi_k \vec{J}_k + \vec{J}_q \tag{4}$$

where $e$ is the energy per unit mass, $\vec{J}_e$ is the energy flux per unit surface area per unit time, $\vec{P}$ is the pressure tensor, $\psi_k$ is the conservative force potential, and $\vec{J}_k$ is the "diffusion flow" of material $k$ moving relative to the centre of mass:

$$\vec{J}_k = \rho_k\left(\vec{\upsilon}_k - \vec{\upsilon}\right) \tag{5}$$

According to the definition Formula (4) of $\vec{J}_q$, the first term on the right of the equal sign indicates that this is the movement of energy per unit volume of mass at its center of mass velocity $\vec{\upsilon}$, so it is called heat flow.

The purpose of the above discussion is that entropy flow has a clear physical meaning, which is reflected in heat flow. It also reflects how these concepts are established and evolved step by step, which provides a theoretical basis for the following discussion.

The mathematical form of entropy equilibrium Equation (1) and the law of mass conservation in mechanics ([3], Chapter 5 (5.8)):

$$\frac{\partial \rho}{\partial t} = -\mathrm{div}\left(\rho \vec{\upsilon}\right) \tag{6}$$

Completely similar, compared with the two equations, it can be considered that the total entropy flow $\vec{J}_{\mathrm{s,tot}}$ corresponds to momentum $\rho\vec{\upsilon}$ and entropy $\rho s$ per unit volume mass corresponds to mass density $\rho$.

Obviously, the heat flow is not a constant vector, so the total entropy flow must have acceleration, that is: $\frac{\partial}{\partial t}\vec{J}_{\mathrm{s,tot}}$; It is undeniable that the entropy of different points in the system is also different, so the entropy should have a gradient. After the above analogy, there should also be an analogy of the time rate of change of momentum, that is, acceleration. Let's discuss this problem.

In a continuous medium, the motion of matter in a region V is governed by the law of conservation of momentum description ([3], Chapter 3 (3.26)):

$$\int_{\upsilon} \rho \vec{a}\mathrm{d}\upsilon = \int_{\upsilon} \rho \vec{f}\mathrm{d}\upsilon + \int_{\partial\upsilon} \vec{F} \cdot \vec{N}\mathrm{d}S \tag{7}$$

This formula is called Euler type equation, where $\vec{a} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{\upsilon}$ represents the acceleration of the material point, the scalar $\mathrm{d}\upsilon$ is the volume element, and $\vec{f}$ represents the physical force per unit mass, for simplicity, assume it's zero; $\vec{F}$ is the stress tensor on a surface element near the volume element. In order to avoid confusion with the entropy source strength $\sigma$, the Cauchy stress of the second-order tensor field there is rewritten as $\vec{F}$. The second term on the right of Formula (7) is the closed surface integral of the interface of region V, according to the Gauss theorem in Mathematics: the closed surface integral of tensor $\vec{F}$ is equal to the volume integral of the tensor divergence, *i.e.*: $\int_{\partial\upsilon} \vec{F} \cdot \vec{N}\mathrm{d}S = \int_{\upsilon} \mathrm{div}\vec{F}\mathrm{d}\upsilon$, substitute this result into Equation (7), since the integral of Equation (7) is valid for any volume, it can be obtained from Equation (7):

$$\rho\vec{a} = \mathrm{div}\vec{F} \tag{8}$$

This formula has considered $\vec{f} = 0$. For non-viscid non-compressible fluid, there is a constitutive equation ([3], Chapter 5 (5.2)):

$$\vec{F} = -p\vec{I} \tag{9}$$

where $p$ is the pressure and $\vec{I}$ is the second-order unit tensor. Substitute Equation (9) into the right side of Equation (8). Considering that the "dot product" of a vector and the unit tensor is equal to the vector, Equation (8) becomes:

$$\rho\vec{a} = -\text{grad}p \tag{10}$$

Since $\text{grad}p = \frac{\text{d}p(\rho)}{\text{d}\rho}\text{grad}\rho$, let $\frac{\text{d}p(\rho)}{\text{d}\rho} \equiv \kappa^2$, then Equation (10) becomes:

$$\rho\frac{\text{d}\vec{\upsilon}}{\text{d}t} = -\kappa^2\text{grad}\rho \tag{11}$$

In this formula, as defined above, $\kappa$ is an unknown scale coefficient, considering the $\frac{\text{d}}{\text{d}t} = \frac{\partial}{\partial t} + \vec{\upsilon}\cdot\text{grad}$, Equation (11) becomes:

$$\rho\left(\frac{\partial\vec{\upsilon}}{\partial t} + (\vec{\upsilon}\text{grad})\cdot\vec{\upsilon}\right) = -\kappa^2(\rho)\text{grad}\rho \tag{12}$$

For small perturbations, after $p$ takes the derivative of $\rho$, $\rho$ can be made constant, so $\rho$ can be regarded as a constant . Meanwhile, in Equation (12), $\left|(\vec{\upsilon}\text{grad})\cdot\vec{\upsilon}\right|$ is a small quantity of higher order than $\left|\frac{\partial\vec{\upsilon}}{\partial t}\right|$, so the former can be ignored. Therefore, the original form $\rho\frac{\partial\vec{\upsilon}}{\partial t}$ on the left-hand side of Equation (12) can be written as the present form $\frac{\partial(\rho\vec{\upsilon})}{\partial t}$. Then Equation (12) becomes:

$$\frac{\partial(\rho\vec{\upsilon})}{\partial t} = -\kappa^2(\rho)\text{grad}\rho \tag{13}$$

This formula applies only to non-compressible non-viscous fluids. This formula tells us that in continuum mechanics, the time change rate of momentum is directly proportional to the absolute value of the gradient of mass density, and its direction increases in the opposite direction of the gradient of mass density.

Multiply both sides of Equation (13) by the entropy $s$ contained in unit mass. Similar to the same reason above, entropy $s$ can be placed under the derivative symbol, then:

$$\frac{\partial(\rho s\vec{\upsilon})}{\partial t} = -\kappa^2(\rho)\text{grad}(\rho s) \tag{14}$$

It can be obtained from Equation (2):

$$\rho s\vec{\upsilon} = \vec{J}_{\text{s,tot}} - \vec{J}_{\text{s}} \tag{15}$$

Substitute Equation (15) into Equation (14), and get:

$$\frac{\partial}{\partial t}\vec{J}_{\text{s,tot}} = \frac{\partial\vec{J}_{\text{s}}}{\partial t} - \kappa^2\text{grad}(\rho s) \tag{16}$$

For non-compressible viscous fluid the viscosity term shall be added to the constitutive equation $\vec{F} = -p\vec{I}$ of non-viscous fluid, that is, $\vec{F}$ in Formula (9) shall be ([3], Chapter 5 (5.5)):

$$\vec{F} = -p\vec{I} + 2\eta\vec{D} \tag{17}$$

where $\eta$ is the viscosity coefficient and $\vec{D}$ meets the non-compressible condition $\text{tr}\vec{D} = \text{div}\vec{\upsilon} = 0$. After considering the viscosity of the fluid, the divergence of the second term on the right in Equation (17) is:

$$Y \equiv \operatorname{div}\left(2\eta \vec{D}\right) = 2\operatorname{grad}\eta \cdot \vec{D} \tag{18}$$

In this case, Equation (16) should be changed to:

$$\frac{\partial}{\partial t}\vec{J}_{s,\text{tot}} = \frac{\partial \vec{J}_s}{\partial t} - \kappa^2 \operatorname{grad}\left(\rho s\right) + Y \tag{19}$$

$Y$ on the right of this equation is a term related to the viscosity coefficient $\eta$ without entropy $s$ and total entropy flow $\vec{J}_{s,\text{tot}}$.

Calculate the divergence of Equation (19), and obtain:

$$\operatorname{div}\left(\frac{\partial}{\partial t}\vec{J}_{s,\text{tot}}\right) = \operatorname{div}\frac{\partial \vec{J}_s}{\partial t} - \kappa^2 \operatorname{divgrad}\left(\rho s\right) + \operatorname{div}Y \tag{20}$$

Newton's view of space-time tells us that spatial coordinates and time are independent variables. Therefore, when a function take the derivative with respect to spatial coordinates and time at the same time, the derivation order of spatial coordinates and time can be exchanged, the result remains the same. Therefore, the left side of Equation (20) can be written as: $\operatorname{div}\left(\frac{\partial}{\partial t}\vec{J}_{s,\text{tot}}\right) = \frac{\partial}{\partial t}\operatorname{div}\vec{J}_{s,\text{tot}}$, and $\operatorname{divgrad} = \nabla^2$, $\nabla^2$ is called Laplace operator, so Equation (20) becomes:

$$\frac{\partial}{\partial t}\operatorname{div}\vec{J}_{s,\text{tot}} = \frac{\partial}{\partial t}\operatorname{div}\vec{J}_s - \kappa^2 \nabla^2\left(\rho s\right) + \operatorname{div}Y \tag{21}$$

The Equation (1) derivative with respect to time, and obtain:

$$\frac{\partial^2\left(\rho s\right)}{\partial t^2} = -\frac{\partial}{\partial t}\operatorname{div}\vec{J}_{s,\text{tot}} + \frac{\partial}{\partial t}\sigma \tag{22}$$

Then, replace the first item on the right of (21) with Equation (21) and sort it out to obtain:

$$\kappa^2\nabla^2\left(\rho s\right) - \frac{\partial^2\left(\rho s\right)}{\partial t^2} = \operatorname{div}\left(Y + \frac{\partial}{\partial t}\vec{J}_s\right) - \frac{\partial}{\partial t}\sigma \tag{23}$$

Let $\kappa^2 = n^2$, then $n = \kappa$, and Equation (23) becomes:

$$n^2\nabla^2\left(\rho s\right) - \frac{\partial^2\left(\rho s\right)}{\partial t^2} = \operatorname{div}\left(Y + \frac{\partial}{\partial t}\vec{J}_s\right) - \frac{\partial}{\partial t}\sigma \tag{24}$$

For non-viscous fluids (superfluids), $Y = 2\operatorname{grad}\eta \cdot \vec{D} = 0$, Then Equation (24) can be written as:

$$n^2\nabla^2\left(\rho s\right) - \frac{\partial^2\left(\rho s\right)}{\partial t^2} = \frac{\partial}{\partial t}\operatorname{div}\vec{J}_s - \frac{\partial}{\partial t}\sigma \tag{25}$$

Equations (24) and (25) are the wave equations of entropy, which is a non-homogeneous linear wave equation, and $n$ is the wave velocity of this wave.

$$n = \sqrt{\frac{\mathrm{d}p\left(\rho\right)}{\mathrm{d}\rho}} \tag{26}$$

It is related to the rate of change of pressure to mass density.

## 3. The Cause of Such Entropy Waves Is Discussed

At a certain moment, at some point in the system, let's say $\frac{\partial(\rho s)}{\partial t} > 0$ in Equation (1) of entropy equilibrium for the sake of simplicity, we only discuss the $x$ component in the rectangular coordinate system, so Equations (1), (11) and (16) are respectively:

$$\frac{\partial(\rho s)}{\partial t} = -\frac{\partial}{\partial x} J_{s,\text{tot}} \tag{27}$$

$$\rho \frac{\mathrm{d}\upsilon_x}{\mathrm{d}t} = -\kappa^2 \frac{\partial \rho}{\partial x} = -\frac{\partial p}{\partial x} \tag{28}$$

$$\frac{\partial}{\partial t} J_{s,\text{tot}} = -\kappa^2 \frac{\partial(\rho s)}{\partial x} \tag{29}$$

where, the $x$ component of $\vec{J}_{s,\text{tot}}$ is represented by $J_{s,\text{tot}}$, and the selection of $\sigma = 0$ in Equation (27) does not affect our discussion.

According to Equation (10), when the stress gradient force $-\frac{\partial p}{\partial x} > 0$ in the positive direction of *x*, which is also $-\frac{\partial \rho}{\partial x} > 0$ from Equation (28), and the acceleration $\rho \frac{\mathrm{d}\upsilon_x}{\mathrm{d}t} > 0$ in the *x* direction can be obtained. Since Equation (29) corresponds to Equation (28), there is correspondingly $\frac{\partial}{\partial t} J_{s,\text{tot}} > 0$ and then $-\kappa^2 \frac{\partial(\rho s)}{\partial x} > 0$. In Equation (27), since $\frac{\partial(\rho s)}{\partial t} > 0$ has been assumed previously, there is $\frac{\partial}{\partial x} J_{s,\text{tot}} < 0$ near point *A*. As we know, $J_{s,\text{tot}}$ is the total entropy flow, while $\frac{\partial}{\partial x} J_{s,\text{tot}}$ is the gradient of the total entropy flow. If the gradient is less than zero, it indicates that the total entropy flow is flowing in a positive direction along the X-axis, that is, $\frac{\partial}{\partial x} J_{s,\text{tot}} < 0$ indicates that the total entropy flow will flow along the positive direction of *x* to the point *B* adjacent to point *A*, increasing the entropy density of point *B* to the right of point *A*. And so on, the entropy of point *A* in turn spreads to other places, forming an entropy wave.

The entropy waves in question are generated by stresses in the medium. In fact, there are other forces in nature, and there may be others at work; At least there is universal gravitation in the universe, the title of this paper contains "explore" two words, so we can further discuss!

## 4. Conclusion

From the above discussion, it can be seen that the thermodynamic entropy fluctuating in the system and the solution of the wave equation is not a simple harmonic. Its solution is determined by the divergence of *Y* of viscosity coefficient

$\eta$ containing fluid and the time rate of change of entropy flux $\vec{J}_s$ divergence and entropy source intensity $\sigma$!

As mentioned in reference [1], heat usually travels by diffusion, but in some cases it can also travel in waves like sound. Entropy waves are similar to conventional sound waves in that they decay as they travel. Landau named it second Sound. Therefore, Equation (26) is usually called "speed of sound".

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## Conflicts of Interest

The author declares no conflicts of interest.

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