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Entropy of State of Quantum System and Dynamic of This System

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Abstract

This work defined the concept of entropy for states of a quantum system and built a stochastic model of a dynamic closed quantum system. This paper also is considered new approaches to the concepts of statistics and spin of elementary particles.

Subject Areas

Modern Physics, Quantum Mechanics

Keywords

State, Quantum System, Entropy, Random Value, Random Process

1. Introduction

In the algebraic approach, a quantum system is represented by the pair $(U,\Im)$ where the U is some C^*-algebra observables, the $\Im$ is some set of states: positive linear functionals $f:U\to C$ with unit norm [1]. In work [2], for any state f we define probability measure on the spectrum P_U of any C^*-algebra U, which is the set of all pure states on this algebra. The set $P_U\subset\prod_u\sigma_u$ where $\prod_u\sigma_u$ is Tikhonov product of spectrums σ_u of all Hermite operators in U.

On any $\sigma_u\subset[\alpha_u,\beta_u]$ we have Lebesgue measure l_u. On $\prod_u\sigma_u$ we also have product measure $L=\otimes_u l_u$ [3]. This measure induces on P_U measure L_{P_U}. In work [4] we prove that there exists density distribution function $\phi_f:P_U\to R$ for measure μ_f, such that $0\le\phi_f(p)\le 1$, $f=\int_{P_U}p\phi_f(p)\mathrm{d}L$, $\int_{P_U}\phi_f(p)\mathrm{d}L=1$.

Consider the function $F_f(p,t)$ which the module squar $\varphi_f^2=\phi_f$.

Every elementary particle is dinamical system in the unperturbed state. They are stationary, it follows that function or functional which represents the particle must depend on the time. The function which depends on time and has module φ_f is function $F_f(p,t) = \varphi_f(p)(\cos\beta(p,t) + i\sin\beta(p,t)) = \varphi_f e^{i\beta(p,t)}$, this complex value function uniquely defines state *f*, the stationarity of F_f gives that $\frac{d\beta'(p,t)}{dt} = const$ and $\beta(p,t) = \omega(p)t + \theta(p)$. Hence $F_f(p,t) = \varphi_f(p)e^{i(\omega(p)t+\theta(p))}$. This function is like the solution of stationary Sch-rodinger equation. Let's call it the wave function. $\omega t = \frac{Et}{\hbar}$, $k(p) = -\frac{\theta(p)}{d(p_0,p)}$, where the *E* energy of particle, *d* any metric on the space P_U It follows $F_f(p,t) = \varphi_f e^{i\left(\frac{E}{\hbar}t - k(p)(d(p_0,p))\right)}$. Identical particles in the same state have equal energies, so their wave functions may differ only in phase α, $F_f(p,t) = \varphi_f e^{i\left(\frac{E}{\hbar}t - k(p)(d(p_0,p)+\alpha)\right)}$ We, known $\omega(p)$ is called frequency and $k(p) = -\frac{\theta(p)}{d(p_0,p)}$ is called wave number.

2. Concept of Entropy of State of Quantum State and It's Dynamic

Let *f* is the state of quantum system it has some support $Supp\mu_f$, $\Gamma = \{h\}$ is a family of homeomorphisms $h: P_U \to P_U$ which save minimal open pseudo-convex cover [5] $\{O_i\}_{i=1,2,\cdots,k}$ of support $Supp\mu_f$, and save distribution function $\varphi_f \circ h = \varphi_f$ of measure μ_f defined by state *f*. Let k_f is the number of orbit of action of homeomorphism $\{h\}$ on the cover $\{O_i\}_{i=1,2,\cdots,k}$ and n_f number of elements in family $\{O_i\}_{i=1,2,\cdots,k}$. Let's call the number $H_f = \frac{n_f}{k_f}$ the entropy of state *f*.

Let sequence of states $f_1, f_2, \cdots, f_n, \cdots$ represents dynamic $h: P_U \to P_U$ of closed quantum system. As known, the entropy at the evolution closed system decreases, so $H_{f_1} \le H_{f_2} \le \cdots \le H_{f_n} \le \cdots$

Let $t_1 < t_2 < \cdots < t_n < \cdots$ ascending sequence of points in time *i.e.* entropy value $\Im_{t_n}, n = 1,2,\cdots$ the set of states of a quantum system whose entropy of is less than t_n, proceeding from the consideration that the entropy of the state of a quantum system should be quantized that the sets $\Im_{t_n}, n = 1,2,\cdots$ will be finite Define on the sets $\Im_{t_n}$ probability measures [6] μ_{t_n} as the measure μ_{t_n} of one point subset $\{f_\beta\}$ be number:

$$\mu_{t_n}(\{f_\beta\}) = \frac{H_{f_\beta}}{\sum_{\beta=1}^{l_n} H_{f_\beta}},$$

where $f_\beta \in \Im_{t_n}$ and l_n number of elements in the set $\Im_{t_n}$. Measure for subset $\{f_i\}_{i=1,2,\cdots,k\le l} \subset \Im_{t_n}$ is number:

$$\mu_{t_n}\left(\{f_i\}_{i=1,2,\cdots,k\le l_n}\right)=\sum_{\beta=1}^{k}\mu_{t_n}\left(\{f_\beta\}\right).$$

This probability measure defines on the set $\Im_{t_n}$ random value Ψ_{t_n} with distribution law [3] [5]:

$$\Psi_{t_n}=\begin{matrix}\{f_1\}, & \{f_2\}, & \cdots, & \{f_{l_n}\}\\ \mu_{t_n}(\{f_1\}), & \mu_{t_n}(\{f_2\}), & \cdots, & \mu_{t_n}(\{f_{l_n}\})\end{matrix}$$

Let $E(\Psi_{t_n})=\sum_{\beta=1}^{l_n} f_\beta\mu_{t_n}(f_\beta)$ is the mathematical expectation of random value Ψ_{t_n} [6] [7]. The stochastic dynamics of a closed quantum dynamic system which is represented by states on any C^*-algebra observables A may described by a sequence of random values:

$$\Psi_{t_1},\Psi_{t_2},\cdots,\Psi_{t_n},\cdots;n=1,2,3,\cdots$$

The most probability realization of this random process [6] [7] will be a sequence of states:

$$E(\Psi_{t_1}),E(\Psi_{t_2}),\cdots,E(\Psi_{t_n}),\cdots$$

3. New Approaches to the Concepts of Statistics and Spin of Elementary Particles

The family $\{O_i\}_{i=1,2,\cdots,k}$ is renumbered whit natural numbers, Let homeomorphism $h_{1j_12j_2\cdots kj_k}:P_U\to P_U$ which save the cover $\{O_i\}_{i=1,2,\cdots,k}$ of $Supp\mu_f$ such that $h_{1j_12j_2\cdots kj_k}(O_i)=O_{j_i}$, $h_{1j_12j_2\cdots kj_k}(O_{j_i})=O_i$, $i=1,2,\cdots,k$. We call such a map mirror map.

Let the family $\Phi=\left\{F_f^\delta(p,t)=\varphi_f e^{i\left(\frac{E}{\hbar}t-k(p)(d(p_0,p))\right)}\right\}$ of all wave function of all particles which are in state *f*. They have same module, and they differ only in phase $k(p)(d(p_0,p))$.

Theorem 1. If $F_f^\delta\circ h_{1j_12j_2\cdots kj_k}=-F_f^\delta$, where $F_f^\delta\in\Phi=\{F_f^\delta(p,t)\}$ is the family of wave functions of identical particles which are in one state such, mirror map $h_{1j_12j_2\cdots kj_k}:P_U\to P_U$ is only one.

Proof. let $F_f^\delta\circ h_{1j_12j_2\cdots kj_k}=-F_f^\delta$, and $h'_{1j_12j_2\cdots kj_k}:P_U\to P_U$ other mirror map, for which also $F_f^\delta\circ h'_{1j_12j_2\cdots kj_k}=-F_f^\delta$. For composition $h_{1j_12j_2\cdots kj_k}\circ h'_{1j_12j_2\cdots kj_k}$ mast be

$$F_f^\delta\circ h_{1j_12j_2\cdots kj_k}\circ h'_{1j_12j_2\cdots kj_k}=F_f^\delta\circ h'_{1j_12j_2\cdots kj_k}\circ h_{1j_12j_2\cdots kj_k}=F_f^\delta.$$

It means that $h_{1j_12j_2\cdots kj_k}\circ h'_{1j_12j_2\cdots kj_k}=h'_{1j_12j_2\cdots kj_k}\circ h_{1j_12j_2\cdots kj_k}=Id_{Supp\mu_f}$. Thus $h'_{1j_12j_2\cdots kj_k}=h^{-1}_{1j_12j_2\cdots kj_k}$, for mirror maps $h_{1j_12j_2\cdots kj_k}=h^{-1}_{1j_12j_2\cdots kj_k}$, It follows that $h_{1j_12j_2\cdots kj_k}=h'_{1j_12j_2\cdots kj_k}$. The theorem is proved.

Definition 1. We say that elementary particle is subject to the Bose-Einstein statistics if it's wave function is symmetric with respect to mirror maps $h_{1j_12j_2\cdots kj_k}:P_U\to P_U$, ($F_f^\delta\circ h_{1j_12j_2\cdots kj_k}=F_f^\delta$) and we say that elementary particle Is subject to the Fermi-Dirac statistic if it's wave function is anti-symmetric with respect mirror maps:

$$h_{1j_1 2j_2\cdots kj_k} : P_U \to P_U ,\quad \left(F_f^{\delta} \circ h_{1j_1 2j_2\cdots kj_k} = -F_f^{\delta}\right).$$

Theorem 2. In given quantum state may be located in only one Fermi-Dirac elementary particle.

Proof: let $F_f^{\delta} \in \Phi$, $F_f^{\delta'} \in \Phi$, and $F_f^{\delta'}(O_i) = F_f^{\delta}(h(O_i))$ two wave function which located in one state, where $h: P_U \to P_U$ homeomorphism which saves the cover $O_i \in \{O_i\}_{i=1,2,\cdots,k}$ and the module of the wave function. Every such homeomorphism performs a rotation at some angle area of value of function F_f^{δ} on complex plain around zero point. Obviously in case of mirror map this angle is π radian. Let for $h: P_U \to P_U$ angle of rotation is α, and $F_f^{\delta} \neq F_f^{\delta'}$. If $h_{1j_1 2j_2\cdots kj_k} : P_U \to P_U$ corresponds to F_f^{δ} and $h'_{1j_1 2j_2\cdots kj_k} : P_U \to P_U$ corresponds to $F_f^{\delta'}$, we have:

$$h \circ F_f^{\delta} \circ h'_{1j_1 2j_2\cdots kj_k} = -h \circ F_f^{\delta},$$

it follows:

$$F_f^{\delta} \circ h'_{1j_1 2j_2\cdots kj_k} = -F_f^{\delta},$$

hence

$$h'_{1j_1 2j_2\cdots kj_k} = h_{1j_1 2j_2\cdots kj_k}.$$

From the theorem 1, follows

$$h'_{1j_1 2j_2\cdots kj_k} \neq h_{1j_1 2j_2\cdots kj_k}.$$

We took the opposite, our assumption $F_f^{\delta} \neq F_f^{\delta'}$ was not correct. Hence $F_f^{\delta} = F_f^{\delta'}$.

The theorem is proved.

From Theorem 2, follows that in case Fermi-Dirac particles the angle of rotation which corresponds to cover saving homeomorphism $h: P_U \to P_U$ may be $n\pi$, for mirror maps is $\pi(2k+1)$ and $\pi 2k$, $k = 0,1,2,\cdots$ another for rotation maps.

For Bose-Einstein particles the mirror map $h_{1j_1 2j_2\cdots kj_k}$ for which $h_{1j_1 2j_2\cdots kj_k} \circ F_f^{\delta} = F_f^{\delta}$.

For Bose-Einstein particles, we do not have a theorem analogous to theorem 2. Therefore in one state may be any number of elementary particles.

Let new we have representation $T: G \to \text{turns}\left(E^2, E^2\right)$ of group G of mirror maps in tarns of complex flatness, which is considered a two-dimensional Euclidian vector space. For only one wave function F_f^{δ} of Fermi-Dirac particle which is in given state *f*, we have only one mirror map which transfers F_f^{δ} in $-F_f^{\delta}$. Let this mirror map is $h_{1j_1 2j_2\cdots kj_k}$ and $T\left(h_{1j_1 2j_2\cdots kj_k}\right) = h$ where h is tarn at angle $(2n+1)\pi, n \in Z$. In state In state *f* is particle with wave function F_f^{δ} or particle with wave function $-F_f^{\delta}$, if assume that on each particle comes the half of the rotation angle $\frac{(2n+1)\pi}{2} = \left(n+\frac{1}{2}\right)\pi, n \in Z$. Call the number $\left(n+\frac{1}{2}\right)\hbar, n \in Z$ a spin of Fermi-Dirac particle.

For Bose-Einstein particles for each mirror map and for wave function F_f^δ of this particle we have $F_f^\delta \circ h_{1j_1 2j_2 \cdots kj_k} = F_f^\delta$ it means that for representation $T: G \to \text{turns}\left(E^2, E^2\right)$ $T\left(h_{1j_1 2j_2 \cdots kj_k}\right) = h = const = n\pi, n \in Z$ in this case on each particle comes the total rotation angle $n\pi, n \in Z$. Call the number $n\hbar, n \in Z$ a spin of Bose-Einstein particle.

4. Conclusions

1) In the paper is defined entropy of states of some quantum system.

2) In the paper is constructed discrete random process describing the evolution of the states of a closed dynamical system.

3) In the article, we are in a new way to prove the fact that a particle subject to Fermi-Dirac statistics can only be one in a given state, while particles that are subject to Bose-Einstein statistics can be of any number in a given state.

4) In the article, we are a new way to introduce the notion of spin for elementary particles.

Conflicts of Interest

The author declares no conflicts of interest.

References

[1] Bratteli, O. and Robinson, D.W. (1979) Operator Algebras and Quantum Statistical Mechanics, C^*- and W^*- Algebras, Symmetry Groups, Decomposition of States. Springer, Heidelberg.

[2] Mumladze, M. and Zerakidze, Z. (2015) The Criteria of Checking Hypotheses of Quantum States of Quantum Physical System. *Journal of Algebra*, *Number Theory*: *Advanced and Applications*, **14**, 57-67.

[3] Bogachev, V.I. (2007) Measure Theory. Springer Link, 575.

[4] Mumladze, M. (2019) Wave Functions, Creation and Annihilation Operators of Quantum Physical System. *Open Access Library Journal*, **6**, e5166. https://doi.org/10.4236/oalib.1105166

[5] Mumladze, M. (2020) Entropy of Topological Space and Evolution of Phase Space of Dynamical Systems. *International Journal of Management and Fuzzy Systems*, **6**, 8-13.

[6] Doob, J.L. (1953) Stochastic Processes. John Wiley & Sons, Hoboken.

[7] Mumladze, M. (2021) Stchochastic Model of Evolution of Homotopy Type of Phase. *East European Science Journal*, **4**, 64-68. https://doi.org/10.31618/ESSA.2782-1994.2021.1.68.17