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New Models of the Physical Microcosm and Their Optimality

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Abstract

A solution to the Schrödinger equation completely mapped to the microlevel of the matter is obtained. The solution allows mathematically to ground the phenomenon of formation of the elementary antiparticles, emergence of the elementary particles from the vacuum at a high electric field intensity, to create a model of wave-particle duality and obtain a model of a gravitational wave. To obtain a model of antiparticles, a model of the emergence of elementary particles from a physical vacuum at a high electric field strength, a model of wave-particle duality, as well as a model of a gravitational wave, and a solution of the non-stationary Schrödinger equation is given. In order to obtain a solution to the non-stationary Schrödinger equation, the values that are in the real area are transferred to the imaginary area. This is achieved by jointly solving the equations of stochastic mechanics and the ARG function introduced by the author. A steady-state solution of the Rikkati-type equation written for imaginary dispersion is obtained. The steady-state solution of this equation at a constant value of the potential field makes it possible to obtain the models listed above.

Subject Areas

Particle Physics

Keywords

Schrödinger Equation, Mapping, Microlevel, Particle, Antiparticle, Corpuscle, Gravity Wave

1. Introduction

At the intersection of two sciences, there are always some unexplored areas. The priority of the present work is to identify the areas and prove their optimality. For us, such an area lies between the theory of optimization and the physical theory of elementary particles.

E. Schrödinger was the first to express the idea about the existence of the optimal properties of the elementary particles, when he was writing his equation for the particles of the physical microcosm using Hamiltonian function. After establishing the adequacy of modeling by means of the equation of stationary processes of the microcosm it became clear that the microcosm is an organisation on the basis of optimal principles. However, existed solution of the Schrödinger equation did not allow for a model of the wave function of the elementary particles on the microlevel. The present work solved that problem: the results obtained allow to model the wave function of the elementary particles existing at the microlevel.

Let us now define the essence of the optimization principle prevailing in the physical microcosm. According to that principle, under the action of conservative forces, any dynamic system moves in such a way as to minimize the time average value of the difference between kinetic and potential energies, *i.e.*

$$\delta\int_{t_1}^{t_2}(T-V)\,\mathrm{d}t=0 \tag{1*}$$

or taking into account the Equation (1*), we can write

$$\int_{t_1}^{t_2}\delta L\,\mathrm{d}t=0\,, \tag{2*}$$

where $T(q,p)$—kinetic energy,

$V(q)$—potential energy,

$L(q,p)$—Lagrange function,

q = generalized coordinate,

$p=\dot{q}$ generalized impulse.

The variation of the Lagrange function in the integrand (2*) is

$$\begin{aligned}\int_{t_1}^{t_2}\delta L\,\mathrm{d}t&=\int_{t_1}^{t_2}\frac{\partial L}{\partial p}\delta p\,\mathrm{d}t+\int_{t_1}^{t_2}\frac{\partial L}{\partial q}\delta q\,\mathrm{d}t\\&=\int_{t_1}^{t_2}\frac{\partial L}{\partial q}\delta q\,\mathrm{d}t+\frac{\partial L}{\partial p}\delta q\Big|_{t_2}^{t_1}-\int_{t_1}^{t_2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial p}\right)\delta q\,\mathrm{d}t\\&=\int_{t_1}^{t_2}\left[\frac{\partial L}{\partial q}-\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial p}\right)\right]\delta q\,\mathrm{d}t=0.\end{aligned}$$

In the last expression, it is assumed that $\delta q=0$ for $t_1=t$, and $t_2=t$.

Since the number of generalized coordinates q is equal to the number of degrees of freedom and as δq does not depend on time, the latter equality is satisfied if the expression in square brackets is equal to zero, *i.e.*

$$0=\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial\dot{q}}-\frac{\partial L}{\partial q}\equiv\frac{\mathrm{d}p}{\mathrm{d}t}+\frac{\partial H}{\partial q}=0\Rightarrow\dot{p}=-\frac{\partial H}{\partial q}, \tag{1}$$

$$0=\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial\dot{p}}-\frac{\partial L}{\partial p}\equiv 0-\dot{q}+\frac{\partial H}{\partial p}=0\Rightarrow\dot{q}=\frac{\partial H}{\partial p}, \tag{2}$$

where $H = T + V$ is the Hamiltonian function (Hamiltonian). Expressions (1*) and (2) show that the Euler-Lagrange equations are equivalent to the Hamilton equations, representing the right-hand side (with respect to the equivalence signs $<<\equiv>>$) of expressions (1) and (2). Schrödinger used the Hamilton function H as a basis for the synthesis of his equation.

2. The Methods of Estimation of State of Systems

From the point of view of optimality, the concept of the integrity of a dynamic system [1], *i.e.* its indivisibility into separate subsystems is very important. It is convenient to interpret the integrity property in terms of observations (measurements).

Let the observation system be given by scalar equations:

$$\dot{x} = -\alpha x + \xi(t), \tag{3}$$

$$y = x + \varsigma(t) \tag{4}$$

of the object (3) and observation channel (4). In expressions (3) and (4) $\xi(t)$ and $\varsigma(t)$ are scalar random processes of the white noise type with the following stochastic characteristics:

$$E[\xi(t)] = 0,\ E[\xi(t)\xi(t')] = \rho\delta(t-t'),$$

$$E[\varsigma(t)\varsigma(t')] = r\delta(t-t'),\ \ E[\varsigma(t)] = 0,$$

where E is the mathematical expectation operator, δ -Dirac function, parameters α, ρ, r are constant.

And the following designations: $E[x(0)] = 0$, $E[x^2(0)] = v_0$, $v = E\left[(\hat{x} - x)^2\right]$, where $\hat{x}$ denotes the conditional variable evaluation *x*, obtained by the least squares method, and v is the dispersion of the variable *x*. In such a case, the equation for dispersion v will be given by [1]:

$$\dot{v} = -2\alpha v - (1/r)v^2 + \rho,\ \ v(0) = v_0.^{1} \tag{5}$$

Expression (5) is the Riccati equation. The right-hand side of Equation (5) can be written as a soliton [2]

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -A\,\mathrm{sech}^2(\beta^* t - \phi). \tag{6}$$

Soliton solutions of the integrity dynamical systems have an important property. The property lies in the optimality of the soliton solution of the Riccati Equation (5). The general solution of Equation (5) has the following form [3]:

$$v = v_1 + \frac{v_1 + v_2}{\left[(v_0 + v_2)/(v_0 - v_1)\right]\mathrm{e}^{2\beta^* t} - 1}, \tag{5a}$$

where

[1]Equation (5), where the constant term is equal to zero, *i.e.* $\rho = 0$, we refer to as the Riccati equation.

$$\beta^* = \sqrt{\alpha^2 + \rho/r}\,, \tag{7}$$

$$v_1 = r\left(\beta^* - \alpha\right), \tag{8}$$

$$v_2 = r\left(\beta^* + \alpha\right), \tag{9}$$

$\phi = \ln\left(\sqrt{c}\right)^{-1}$, $c = \dfrac{v_2 + v_0}{v_1 - v_0}$, $v_1 > v_0$, $A = D\beta^*$, $D = \dfrac{v_1 + v_2}{2\sqrt{c}}$ and v_0 is the dispersion value v at the initial moment of time $t_0 = 0$, *i.e.* $v_0 = v(0)$.

Finally, solution of Equation (6) allows us to determine the dispersion

$$v = -A\int_{t_0}^{t} \operatorname{sech}^2\left[\beta^*\left(t' - t_0\right) - \phi\right]\mathrm{d}t'\,. \tag{10}$$

Representation of the observation (measurement) system in the form of object (3) and observation channel (4) is formal. In natural conditions the observation system is an integrity formation; it cannot be divided into an object (3) and an observation channel (4). The observation channel (4) is an integral part of the observation object (3). Representation of a real observation system in the form of expressions (3) and (4) is appropriate for mathematical processing of the results of indirect observations. The class of integrity dynamic systems includes the systems modelled simulated by the following Riccati equations:

$$\dot{z} = mz\left(n - z\right),\quad z\left(t_0\right) = z_0, \tag{11a}$$

$$\dot{z} = -mz\left(n - z\right),\quad z\left(t_0\right) = z_0. \tag{11b}$$

The solution of Equations (11a) and (11b) is given by:

$$z = \frac{1}{4}n^2 m\int_{t_0}^{t} \operatorname{sech}^2\left[\frac{1}{2}mn\left(t' - t_0\right)\right]\mathrm{d}t'\,, \tag{12a}$$

$$z = -\frac{1}{4}n^2 m\int_{t_0}^{t} \operatorname{sech}^2\left[\frac{1}{2}mn\left(t' - t_0\right)\right]\mathrm{d}t'\,. \tag{12b}$$

From the parity property of the soliton it follows that the parameter n can have both positive and negative signs in solutions (12a) and (12b). It should be noted that Equations (11a) and (11b) are the particular forms of Equation (5).

Solutions of integrity dynamical systems (10), (12*a*), (12*b*) *have the dissipative property. Dissipative functions are not invertible with respect to the corresponding argument. Conservative functions are invertible, their second derivative with respect to the argument does not reverse the sign.*

The t *time derivative of both sides of solution* (5*a*) *is the soliton differential Equation* (6), *whose solution* (10) *satisfies the Euler—Lagrange optimization equations*. It is easy to verify that the functionals $L = L(v)$ and $L = L(z)$ (see (10), (12a), (12b)) satisfy the Euler—Lagrange Equations (1) and (2).

The fact that the functional L satisfies the Euler-Lagrange equation means that the variance is zero $v = E\left[\left(\hat{x} - x\right)^2\right] = 0$ or $x/y = x$, *i.e.* the object (3) and the observation channel (4) represent one whole: the system (3), (4) is integrity, *Thus, soliton solutions of integrity dynamical systems have the following important* *properties*:

1) *They satisfy the Euler-Lagrange optimization equations*.

2) *They are dissipative in time functions*, *i.e. these functions are time irreversible*.

3) *They do not allow to represent the equations of the object and the observation channel separately*.

Further, *the solution to the problem of mathematical modeling of the dispersion of the elementary particle is given at the microlevel of the matter having those properties*.

*Denotations p and. r given above are independent from those given below.

3. The Analysis of Schrödinger's and the Stochastic Mechanic's Equations

In the middle of the twenties of the last century, Austrian physicist Erwin Schrödinger using de Broglie's hypothesis of optico-mechanical analogy for the behavior of the micro particles and based on the Hamilton optimization principle, synthesized the key equation of quantum mechanics named after him:

$$j\varepsilon \frac{\partial \Psi}{\partial t} = -\frac{1}{2}\varepsilon^2 \frac{\partial^2 \Psi}{\partial x^2} + \left(\frac{U(x)}{m}\right)\Psi, \tag{13}$$

where $j = \sqrt{-1}$, $\varepsilon = \hbar/m$, $\hbar = 1.05459 \times 10^{-34}\ \mathrm{J \cdot s}$ is the Planck constant divided by 2π, Ψ, the wave function of the particle to be found, $U(x)$ the potential energy of the particle with mass m and coordinate x.

Schrödinger's equation is extraordinary. The extraordinary nature of the equation lies in the fact that it simultaneously belongs to two levels of the matter, partly to the microlevel (the left-hand side of the equal sign "=") and to the mesolevel (the right-hand side); the mesolevel of the matter is between the microlevel and the macrolevel.

The solution to the Schrödinger equation can be found in three ways.

The first way is used to solve the nonstationary Equation (13). The second one is used to solve the stationary equation, *i.e.* for $\dot{\Psi} = 0$. This method was used by Schrödinger himself. Finally, the third way of solution uses the function close to a generalized function.[2] The latter method, applied by the author for solution of Equation (13), allows obtaining the wave function of an elementary particle at the microlevel. In such a case, the Schrödinger equation entirely belongs to the left-hand side of the plane with respect to the equal sign "=".

Consider the solutions to the Schrödinger equation at three levels of the matter separately.

1) Introduce denotations $\Psi(x,t) = \psi(x)\varphi(t)$[3]. In such a case, Equation (13) can be written as follows

$$-j\varepsilon \frac{1}{\varphi}\frac{\partial \varphi}{\partial t} = \frac{1}{2}\varepsilon^2 \frac{1}{\psi}\frac{\partial^2 \psi}{\partial x^2} - \frac{U(x)}{m}. \tag{14}$$

[2]The function close to a generalized function will hereafter be referred to as a normalized algorithmically realizable generalized function (ARGF).

[3]Such an approach is valid if the potential energy of the particle does not depend on time.

Since the left-hand side of Equation (14) is the function of time, and the right-hand side is the function of coordinates, Equation (14) is satisfied if and only if both parts are equal to a constant value [4]. We denote this constant value by $-W/m$, where W is the total energy of the particle. When the above condition is satisfied, Equation (14) is divided into two equations

$$\begin{gathered} j\varepsilon \frac{1}{\varphi}\frac{\partial \varphi}{\partial t} = \frac{W}{m}, \\ \frac{\partial^2 \psi}{\partial x^2} + 2\frac{1}{\varepsilon^2 m}(W-U)\psi = 0. \end{gathered} \tag{15}$$

Thus, the solution of the non-stationary Equation (13) has no practical value.

2) Schrödinger solved the stationary Equation (15) ($\dot{\Psi} = 0$) applied to the hydrogen atom (using a spherical coordinate system) and obtained a spectrum for the energy eigenvalues that coincides with the well-known experimental data. That showed that the stationary Equation (15) correctly describes the motion of the electron in the potential electric field. Therefore, Equation (15) was taken as the basic equation of stationary states of quantum mechanics;

3) The entire Schrödinger equation transferred to the microlevel was obtained neither by Schrödinger nor other scientists. In this work, the solution of the Schrödinger equation is transferred to the microlevel of matter.

Such an approach to the solution of the Schrödinger equation will make it possible to solve number of problems up to now known from heuristic considerations.

To pass from the solution of the continuous Equation (13) to the solution of the equation transferred to the microlevel, it is necessary to use the system of equations of stochastic mechanics transferred to the microlevel with the help of ARGF. ARGF is considered to translate dispersions and diffusions from real areas into emajine areas.

The system of equations of stochastic mechanics has the following form:

$$\frac{\partial P}{\partial t} + \Delta \cdot (Pv) = 0, \tag{16}$$

$$\frac{\partial v}{\partial t} + (v \cdot \Delta)v = \frac{F}{m} + (u \cdot \Delta)u - \frac{1}{2}\varepsilon \Delta^2 u, \tag{17}$$

$$Pu = -\frac{1}{2}\varepsilon \Delta P, \tag{18}$$

where v and u are unidimensional vectors of real dispersion and diffusion of the elementary particle; $\Delta = \frac{\partial}{\partial x}$, unidimensional vector-operator; $F = \frac{\mathrm{d}U}{\mathrm{d}x}$, the gradient of the field U, *i.e.*, $F = \operatorname{grad} U$; of the point "." denotes scalar product.

Since in such a case the angle between the vectors equals to 0 degrees, the point in Equations (16) and (17) can be omitted, *i.e.* the scalar product can be replaced by the ordinary product.

Equation (18) can be written as follows:

$$u = -\frac{1}{2}\varepsilon\left(\ln P\right)'_x = -\frac{1}{2}\varepsilon\frac{\Delta P}{P}, \tag{18a}$$

where $\left(\ln P\right)'_x$, is the rate of change of continuous probability density P of the real random process of the diffusion with coefficient $-\frac{1}{2}\varepsilon$.

Further, Equations (16)-(18) already transferred to the microlevel of the matter are used. And formula (18a) is used to transfer diffusion u to the microlevel of the matter.

4. General Considerations for Transfering the Solution to the Schrödinger Equation Mapped to the Microlevel of the Matter

It is well known that the wave function of an elementary particle satisfying the Schrödinger Equation (13) can be given by:

$$\Psi = \sqrt{P}\exp\left\{\frac{j}{\varepsilon}\int v\mathrm{d}x\right\}. \tag{19}$$

Substituting the wave function (19) in Equation (13), taking into consideration Equation (18), we will have the differential relation

$$-j\varepsilon\Delta\Psi = \left(v + ju\right)\Psi .$$

From that relation we can pass to the wave function which is solution of nonstationary Schrödinger equation

$$\Psi\left(x,t\right) = \mathrm{e}^{\frac{j}{\varepsilon}\int_{\tau_0}^{\tau} v(x,t)\mathrm{d}x - \frac{1}{\varepsilon}\int_{\tau_0}^{\tau} u(x,t)\mathrm{d}x}, \tag{20}$$

where τ_0 is the very little time but other than zero, *i.e.* $\tau_0 \neq 0$.

The role of ARGF is, together with the equations of stochastic mechanics, to transfer the real dispersion function v and diffusion function u into the class of imaginary functions. Such a transfer allows to transform the real wave function Ψ into an imaginary wave function and, consequently, to obtain a solution to the Schrödinger equation at the microlevel of the matter [5].

The Laplace transform from ARGF is given by $\frac{1}{s}th\left(\frac{\tau_0 s}{2}\right)$, where $s = \sigma + j\omega$, $\tau_0 = const$.

If we use the symbol of correspondence between the Laplace transform and its original, then it will be possible to determine ARGF in the time domain:

$$\frac{1}{s}th\left(\frac{\tau_0 s}{2}\right) \quad \left(-1\right)^{n-1}, \quad n-1 < \frac{t}{\tau_0} < n, \tag{21}$$

where t is the current time, $n = 1, 2, \cdots$

Apart from formula (21) ARGF can also be defined by the use of the inverse Laplace transform operator L^{-1}:

$$L^{-1}\left\{\frac{1}{s}th\left(\frac{\tau_0 s}{2}\right)\right\} = \left(-1\right)^{n-1}, \quad n-1 < \frac{t}{\tau_0} < n. \tag{22}$$

If in formula (22) we take into consideration the equality $-1 = \mathrm{e}^{\pi j(2k+1)}$, $k = 0,1,2,\cdots$ then the last expression will be written as follows:

$$L^{-1}\left\{\frac{1}{s}th\left(\frac{\tau_0 s}{2}\right)\right\} = \mathrm{e}^{\pi j(2k+1)(n-1)\tau_0},\ (n-1)\tau_0 \le t < n\tau_0 \ \text{ for } \ n \gg 1. \tag{23}$$

Introduce designation:

$$\tau = n\tau_0 = x. \tag{24}$$

Clearly, $t = 0$ if $n = 1$. If $(n-1)\tau_0 = t$, according to (23) for $n = 2,3,\cdots$ we will have

$$L^{-1}\left\{\frac{1}{s}th\left(\frac{\tau_0 s}{2}\right)\right\} = \mathrm{e}^{\pi j(2k+1)t}. \tag{25}$$

Let us introduce the distribution function of an imaginary random diffusion process defining the function by the right-hand side of expression (25). Then the density function $P(j,x,t,k)$ of *the probability distribution of an imaginary random diffusion process will be found according to the expression*

$$\left(L^{-1}\left\{\frac{1}{s}th\left(\frac{\tau_0 s}{2}\right)\right\}\right)'_t = \pi j(2k+1)\mathrm{e}^{\pi j(2k+1)t} \equiv P(j,x,t,k). \tag{26}$$

Further, we use the wave function of just an imaginary random diffusion process, *i.e.*, solution $\Psi(j,x,t,k)$, of non-stationary Schrödinger equation, which is the mapping of solution (20) to the microlevel of the matter:

$$\Psi(j,x,t,k) = \mathrm{e}^{\frac{j}{\varepsilon}\int_{\tau_0}^{n\tau_0} v(j,x,t,k)\mathrm{d}x - \frac{1}{\varepsilon}\int_{\tau_0}^{n\tau_0} u(j,k)\mathrm{d}x}, \tag{20a}$$

where $v(j,x,t,k)$, dispersion of an imaginary random diffusion process mapped to the microlevel. According to formula (20a), it is necessary to substitute diffusion $u(j,k)$ in it. The imaginary diffusion of an elementary particle is determined by the formula (18a) transferred to the microlevel of the matter. To that end, we use the density function $P(j,x,t,k)$ of distribution of the probability of an imaginary random diffusion process (26) taking into consideration the notations (24), (25):

$$u(j,k) = -\frac{\varepsilon}{2}\pi j(2k+1). \tag{27}$$

It can be seen from (27) that the diffusion of an imaginary random process for a concrete k is constant.

5. Obtaining of the Dispersion Equation for the Elementary Particles Mapped onto the Microlevel of the Matter

In what follows, it is assumed everywhere that the system of equations of stochastic mechanics (16)-(18) consists of the functions $P(j,x,t,k), u(j,k)$ and $v(j,x,t,k)$ mapped[4] to the microlevel. It should be noted that $P(j,x,t,k)$ and $u(j,k)$ are already known, they are defined by the formulae (26) and (27). The mapped diffusion function $u(j,k)$ and the dispersion $v(j,x,t,k)$ function

[4]Further, instead of the words "transfer to", their synonym "mapping to" will be used.

are used to substitute them in the mapped wave function formula (20a).

Below given sequence of mathematical operations allows determining the equation satisfying the mapped dispersion $v(j,x,t,k)$.

To find the equation of the dispersion v mapped to the microlevel, we substitute the density $P(j,x,t,k)$ determined according to (26) into Equation (16)

$$-\pi^2(2k+1)^2 e^{\pi j(2k+1)t} - v\pi^2(2k+1)^2 e^{\pi j(2k+1)t}$$
$$= -\pi j(2k+1)e^{\pi j(2k+1)t}\frac{\partial v}{\partial x} \Big| : \pi j(2k+1)e^{\pi j(2k+1)t}.$$

The latter gives the differential equation

$$\frac{\partial v}{\partial x} = -\pi j(2k+1)(v+1). \tag{28}$$

If we put the diffusion value (27) into the Nelson Equation (17) mapped to the microlevel, we will have

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \frac{F(x)}{m}. \tag{29}$$

The joint solution of equations (28) and (29) leads to $2k+1(k=0,1,2,\cdots)$ number of Riccati-type equations mapped to the microlevel

$$\frac{\partial v}{\partial t} = \pi j(2k+1)v + \pi j(2k+1)v^2 + F(x)/m. \tag{30}$$

For a certain k the expression* (30) *is the mapped scalar Riccati equation with constant coefficients.

Consider Equation (30) as Equation (5) mapped to the microlevel of the matter. In this reflection, the parameters $-2\alpha, -\frac{1}{r}$ and ρ of equations (5) are mapped to the parameter of Equation (30), respectively. The mapping process can be schematically represented as follows:

$$-2\alpha \quad \text{is mapped to} \quad \pi j(2k+1), \tag{31}$$

$$-\frac{1}{r} \quad \text{is mapped to} \quad \pi j(2k+1), \tag{32}$$

$$\rho \quad \text{is mapped to} \quad F(x)/m. \tag{33}$$

Thus, the joint solution of the equations of stochastic mechanics mapped to the microlevel (16)-(18) allows to obtain the Riccati Equation (30) mapped to the microlevel, satisfying the dispersion $v(j,x,t,k)$ of elementary particle at the microlevel of the matter.

6. Solution of the Equation Determining the Dispersion of Elementary Particle at the Microlevel in a Steady State

If instead of parameters $-2\alpha, -\frac{1}{r}$ and ρ we take into consideration their mapped values (31)-(33), then the structure of the solution to Equation (30) will be the same (see (5a)) as it was in solution of Equation (5). In such a case, the parameters β, v_1, v_2 are determined by taking into consideration the mappings (31)-(33), in accordance to formulae (7)-(9):

$$\beta(x) \equiv \beta(j,x,k) = \sqrt{-\frac{\pi^2}{4}(2k+1)^2 - j\pi(2k+1)\frac{F(x)}{m}}, \tag{34}$$

$$v_1(x) \equiv v_1(j,x,k) = -\left[j\pi(2k+1)\right]^{-1}\left[\beta(x) + j\frac{\pi}{2}(2k+1)\right], \tag{35}$$

$$v_2(x) \equiv v_2(j,x,k) = -\left[j\pi(2k+1)\right]^{-1}\left[\beta(x) - j\frac{\pi}{2}(2k+1)\right]. \tag{36}$$

Taking the above parameters into consideration, the solution of the mapped dispersion Equation (30) for a certain k will be written as follows:

$$v(j,x,t) = v_1(j,x) + \frac{v_1(x)+v_2(x)}{\dfrac{v_0(x)+v_2(x)}{v_0(x)-v_1(x)}\mathrm{e}^{2\beta(x)t} - 1}, \tag{37}$$

where $v_0(x) \equiv v(j,x,0)$ is the imaginary dispersion for $t=0$.

Without losing generality, in solution of Equation (30), we can assume that $v_0 = 0$. In such a case, the soliton solution of Equation (30) will be given by

$$v(x,t) = -D(x)\beta(x)\int_0^\infty \operatorname{sech}^2\left[\beta(x)t - \phi(x)\right]\mathrm{d}t \tag{38}$$

where $D(x) = \dfrac{v_1(x)+v_2(x)}{2\sqrt{c}}$, $c = \dfrac{v_2(x)}{v_1(x)}$, $\phi(x) = \ln\left(\sqrt{c}\right)^{-1}$.

The functional determining the dispersion $v(x,t)$ in the time interval $t \in (0,\infty)$ depends on the coordinates of the particle in a complex way; therefore, the substitution of the mapped dispersion $v(x,t)$ in formula of wave function (20a) greatly complicates the calculation of the function, making calculation practically impossible. However, to determine the dispersion $v(x,t)$ in the stationary case, *i.e.* when $t=\infty$ and at the initial moment when $t=0$, the calculation of the dispersion is possible.

Indeed, for $t=\infty$, according to formula (37), we have $v(x) = v_1(x)$, and for $t=0$, then from (37) we receive $v_0(x)=0$. Consequently, calculation of integral (38) in the stationary state and at the initial moment will be given by

$$v\left(x, t =_0^\infty\right) \equiv -v(x) = v_1(x) - 0 = v_1(x). \tag{39}$$

According to formula (35), expression (39) will be written as

$$v(x) = v_1(x) = -\left[j\pi(2k+1)\right]^{-1}\beta(x) - \frac{1}{2}. \tag{40}$$

If in formula (34) we take out the term $-\dfrac{\pi^2}{4}(2k+1)^2$ for the radical sign, then we will have

$$\beta(x) = j\frac{\pi}{2}(2k+1)\sqrt{1 + j\frac{4}{\pi m(2k+1)}F(x)}. \tag{41}$$

If in expression (40) we take into consideration (41), then we get[5]

$$v(x)=\frac{1}{2}\sqrt{1+j\aleph F(x)}+\frac{1}{2}=\frac{1}{2}\sqrt{1+j\chi(x)}-\frac{1}{2}, \tag{42}$$

where, $\aleph=\frac{4}{\pi m(2k+1)}$, $\chi(x)=\aleph F(x)$.

With account of the formula

$$\sqrt{1+j\chi}=\pm\left[\sqrt{\frac{r+1}{2}}+j\sqrt{\frac{r-1}{2}}\right],\quad r=\sqrt{1+\chi^2}, \tag{43}$$

expression (41) will be written as

$$\beta(x)=\pm\left[j\eta(x)-\gamma(x)\right],$$

where, $\eta(x)=\frac{\pi}{2}(2k+1)\sqrt{\frac{r+1}{2}}$, $\gamma(x)=\frac{\pi}{2}(2k+1)\sqrt{\frac{r-1}{2}}$.

7. Obtaining a Mathematical Model of an Antiparticle

In 1930, guided by physical considerations, P. Dirac predicted the existence of antiparticles: each elementary particle corresponds to its antiparticle; the positron is the antiparticle for the electron. All predictions on the existence of antiparticles were confirmed experimentally and the antiprotons, antineutrons, etc. were discovered.

The mathematical substantiation of this phenomenon is of interest. For mathematical confirmation of this phenomenon, it is reasonable to consider the wave function of the particle and antiparticle at the microlevel of the matter. According to formula (20a), mathematical operations are performed in exponential order; the exponential order will be obtained, if we take into consideration the expressions (27) and (42) in formula (20a):

$$\frac{j}{2\varepsilon}\int_{\tau_0}^{n\tau_0}\sqrt{1+j\chi(x)}\,\mathrm{d}x+\frac{j}{2\varepsilon}\int_{\tau_0}^{n\tau_0}\mathrm{d}x+\frac{1}{2}j\pi(2k+1)\int_{\tau_0}^{n\tau_0}\mathrm{d}x. \tag{44}$$

Represent the expression $\sqrt{1+j\chi(x)}$ separately according to formula (43): the terms with the positive sign and with the negative sign will be considered separately.

First, we take into consideration the plus sign (+) and then the minus sign (−). In such a case, expression (44) will be written in two variants:

$$\frac{jt}{2\varepsilon}\left\{+\left[\sqrt{\frac{r+1}{2}}+j\sqrt{\frac{r-1}{2}}\right]+1\right\}+\frac{jt}{2}\pi(2k+1), \tag{45}$$

[5]If both sides of the formula (42) are multiplied by the expressions conjugate to that formula, respectively, *i.e.* to the $v^*=\frac{1}{2}\left[\sqrt{1+j\chi(x)}-1\right]$,we get the square of the dispersion of an elementary particle at the microlevel $v^2=\frac{1}{4}\left[1+j\chi(x)-1\right]=\frac{j}{\pi m(2k+1)}F(x)$. Hence, $\chi(x)=4v^2$ and, consequently, $r=\sqrt{1+\chi^2}=\sqrt{1+16v^4}$.

$$\frac{jt}{2\varepsilon}\left\{-\left[\sqrt{\frac{r+1}{2}}+j\sqrt{\frac{r-1}{2}}\right]+1\right\}+\frac{jt}{2}\pi(2k+1). \tag{46}$$

Further, it is assumed that the particle and the antiparticle are in a constant potential field, *i.e.* $U = const$ and, consequently, $r = 1$.

According to expressions (45) and (46), the wave function can be written as follows:

$$\Psi_{1,2}=\left\{e^{j\left[\frac{1}{\varepsilon}+\frac{\pi}{2}(2k+1)\right]}\right\}^t, \tag{47}$$

$$\Psi_0=\left[e^{j\frac{\pi}{2}(2k+1)}\right]^t. \tag{48}$$

It should be noted that the wave function (47) does not contain the mass of the elementary particle, therefore, further, it will not be taken into consideration. The wave function (48) contains both the wave function of the particle and the wave function of the antiparticle. If we take into consideration Euler's formula $\left(e^{jg}=\cos g+j\sin g\right)$, then for obtaining the wave function of the antiparticle it is necessary to open the brackets of expression $\frac{\pi}{2}(2k+1)$. In such a case, the arguments of the sinusoidal and cosinusoidal functions will consist of three summands $\varepsilon^{-1},\pi k,\frac{\pi}{2}$:

$$\begin{aligned}\Psi_1&=\left[e^{j\left(\frac{1}{\varepsilon}+k\pi+\frac{\pi}{2}\right)}\right]^t\\&=\left[\cos(A+B+C)+j\sin(A+B+C)\right]^t\end{aligned}. \tag{49}$$

To obtain the wave function of the particle it is not necessary to open the brackets of expression $\frac{\pi}{2}(2k+1)$; in such a case, the arguments of the sinusoidal and cosinusoidal functions will consist of two summands $\varepsilon^{-1},\frac{\pi}{2}(2k+1)$:

$$\Psi_2=\left\{e^{j\left[\frac{1}{\varepsilon}+\frac{\pi}{2}(2k+1)\right]}\right\}^t=\left[\cos(a+b)+j\sin(a+b)\right]^t, \tag{50}$$

where $A=a=\frac{1}{\varepsilon}$, $B=\pi k$, $C=\frac{\pi}{2}$, $b=\frac{\pi}{2}(2k+1)$, $k=1,3,5,\cdots$, for particle and $k=0,2,4,\cdots$, for antiparticle.

For the antiparticle we will have

$$\begin{aligned}\cos(A+B+C)&=\cos A\cos B\cos C-\sin A\sin B\cos C\\&\quad-\sin A\cos B\sin C-\cos A\sin B\sin C\\&=-\sin\left(\frac{1}{\varepsilon}\right),\end{aligned} \tag{51}$$

$$\begin{aligned}\sin(A+B+C) &= \sin A\cos B\cos C+\cos A\sin B\cos C\\ &\quad+\cos A\cos B\sin C-\sin A\sin B\sin C\\ &=\cos\left(\frac{1}{\varepsilon}\right).\end{aligned} \tag{52}$$

For the particle we will have

$$\cos(a+b)=\cos a\cos b-\sin a\sin b=\sin\left(\frac{1}{\varepsilon}\right), \tag{53}$$

$$\sin(a+b)=\sin a\cos b+\cos a\sin b=-\cos\left(\frac{1}{\varepsilon}\right). \tag{54}$$

Consequently, the difference between the particle and the antiparticle is reduced to the group or the absence of group of summands πk and $\frac{\pi}{2}$.

If we take the results (53) and (54) into consideration in formula (50), taking de Moivre formula $(\cos g+j\sin g)^t=\cos tg+j\sin tg$ into consideration, then we get the wave function of the particle

$$\Psi_2=\sin\left(\frac{t}{\varepsilon}\right)-j\cos\left(\frac{t}{\varepsilon}\right).$$

If we take the results (51) and (52) into consideration in formula (49) there we get the wave function of the antiparticle

$$\Psi_1=-\sin\left(\frac{t}{\varepsilon}\right)+j\cos\left(\frac{t}{\varepsilon}\right).$$

Only the real parts of a complex function have physical meaning; therefore the wave functions of the particle and the antiparticle will be written as follows:

$$\Psi_2=\sin\left(\frac{t}{\varepsilon}\right),$$

$$\Psi_1=-\sin\left(\frac{t}{\varepsilon}\right).$$

Consequently, we have the annihilation

$$\Psi_1+\Psi_2=0.$$

8. Model of Emergence of Elementary Particles from the Vacuum

The physical vacuum is teeming with virtual particles; it contains various kinds of virtual elementary particles. Physical vacuum exerts comprehensive pressure on any elementary particle both on the scale of the Universe and in laboratory conditions. It is considered that for any type of vacuum, there is a resonant wavelength. The data presented in [6] show how sharply the concentration of elementary particles decreases with the change of the vacuum type. The present work shows how significantly the free path of neutrinos changes depending on the resonance wave of a certain type of vacuum. Below, a model is constructed based on the results of this work, which shows how elementary particles emerge from "nothing" in a strong electric field [1].

To determine the gradient of the electric field, where the real particles emerge from virtual particles, we use formula (43) instead of the first term of the expression (44); and without taking into consideration the coefficient $\frac{1}{2}$, we obtain

$$j\int_{\tau_0}^{n\tau_0}\left\{\pm\frac{1}{\varepsilon}\left[\sqrt{\frac{r+1}{2}}+j\sqrt{\frac{r-1}{2}}\right]+\frac{1}{\varepsilon}-\pi(2k+1)\right\}\mathrm{d}x. \tag{55}$$

After multiplying (55) by $-j$, in formula (55), we will have imaginary and non-imaginary parts separately

$$\int_{\tau_0}^{n\tau_0}\left\{-\frac{1}{\varepsilon}\left[\pm\sqrt{\frac{r-1}{2}}\right]+\left[\pm\frac{j}{\varepsilon}\sqrt{\frac{r+1}{2}}\right]+\frac{j}{\varepsilon}-j\pi(2k+1)\right\}\mathrm{d}x. \tag{56}$$

If virtual particles are absent, in expression (56) the sum of the coefficients for imaginary terms will be equal to zero,

$$\frac{1}{\varepsilon}\left[\pm\sqrt{\frac{r+1}{2}}\right]+\frac{1}{\varepsilon}-\pi(2k+1)=0. \tag{57}$$

When the virtual particles are absent, we have only real particles. As $r=\sqrt{1+\aleph^2F^2}$, from equality (57) it follows that

$$\sqrt{1+\aleph^2F^2}=G(k), \tag{58}$$

where $G(k)=1-4\pi\varepsilon(2k+1)+2\pi^2\varepsilon^2(2k+1)^2$.

From equality (58) we find the gradient of the electric field, where the real particles emerge

$$F(k)=\left|\pm\frac{1}{\aleph(k)}\sqrt{G(k)^2-1}\right|. \tag{59}$$

The sign of the absolute value in formula (59) follows from the fact that the expression $\chi=\frac{4F}{\pi m(2k+1)}$ is positive and, consequently, $F>0$.

If we take into consideration (57) in expression (56) and also consider the formulae (20a), (27), (42) and the denotation $\tau_0(n-1)=t$ we obtain the final result of emergence of real particles from "nothing"

$$\Psi(k,t)=\mathrm{e}^{\frac{t}{2\varepsilon}\sqrt{\frac{r(k)-1}{2}}}, t=const.$$

The parameter *k*, where the elementary particles begin to emerge from the vacuum, is determined from the following inequality

$$\frac{\tau_0\sqrt{\frac{r(k)-1}{2}}}{2\varepsilon}>1.$$

9. Model of Corpuscular-Wave Dualism

After Planck's postulate about the discrete nature of energy radiation by atomsoscillators (1900), the idea of quantization was developed by A. Einstein (1905). He suggested that quantum properties are inherent in light in general. It follows from Einstein's hypothesis that light must be considered not as a wave, but as a stream of quanta (photons) with the energy $E_0 = h\nu_0$ [6] and impulse $p = \hbar\omega_0/c$ [7] each. In terms of cognition, this hypothesis did not accept the position in classical physics about the essential difference between the matter and radiation; it affirmed the fundamental principle of the physics of the microcosm of wave—a model of corpuscular-wave dualism is given. To obtain a mathematical model of corpuscular-wave dualism. The hypothesis of the great thinker was of considerable theoretical value.

In this section, one of the main problems of this work is solved—a model of corpuscular-wave dualism is given. To obtain a mathematical model of corpuscular-wave dualism, consider the wave function of an elementary particle taking into consideration expresion (55) and coefficient $\frac{1}{2}$ writing it as

$$\Psi(k,t) = \mathrm{e}^{\frac{1}{2}\int_{\tau_0}^{n\tau_0}\left\{\pm\left[-\frac{1}{\varepsilon}\sqrt{\frac{r-1}{2}}\right]\pm\left[\frac{j}{\varepsilon}\sqrt{\frac{r+1}{2}}\right]+\frac{j}{\varepsilon}-j\pi(2k+1)\right\}\mathrm{d}x}, \tag{60}$$

where $k = 1,3,5,\cdots$

The potential field in which the elementary particle is located is constant, *i.e.* $U = const$. This means that $r = 1$ and the content in the first square brackets goes to zero.

To avoid annihilation of number ε^{-1} consider the plus sign before the second square brackets; as a result, the wave function (60) takes the form

$$\Psi(k,t) = \mathrm{e}^{j\left[\frac{t}{\varepsilon}+\frac{\pi}{2}t(2k+1)\right]}. \tag{61}$$

We introduce the notation

$$a = \frac{1}{\varepsilon},\ b = \frac{\pi}{2}(2k+1). \tag{62}$$

Taking these designations into consideration, the wave function (61) will be written as

$$\Psi(k,t) = \left[\mathrm{e}^{j(a+b)}\right]^t. \tag{63}$$

Using the Euler formula in the square brackets, the wave function (63) will be given by

$$\Psi(k,t) = \left[\cos(a+b) - j\sin(a+b)\right]^t. \tag{64}$$

Transformation of trigonometric functions and taking into consideration notation (62) gives

$$\cos(a+b) = \cos a\cos b - \sin a\sin b = \sin a, \tag{65}$$

$$\sin(a+b) = \sin a\cos b + \cos a\sin b = -\cos a. \tag{66}$$

[6] h is the Planck constant, ν_0 —the frequency of electromagnetic radiation.

[7] c—the speed of light in emptiness, ω_0 —the angular frequency of electromagnetic radiation.

Taking into consideration expressions (65) and (66) in formula (64), we obtain

$$\Psi(m,t) = [\sin a + j\cos a]^t. \quad (67)$$

Since time is discrete $t = (n-1)\tau_0$, the Moivre formula can be used; as a result, formula (67) will be written as

$$\Psi(m,t) = \sin(at) + j\cos(at). \quad (68)$$

Taking into consideration the fact that only the real part of the complex function has physical meaning, with account of designation (62) formula (68) will be given by

$$\Psi(m,t) = \sin\left(\frac{1}{\varepsilon}t\right). \quad (69)$$

To pass from formula (69) to the wave-particle duality, it is necessary to use the de Broglie formula

$$\lambda = \frac{h}{p} = \frac{h}{mV}.[8] \quad (70)$$

Using formulas (70) and (69), the mass of an elementary particle can be eliminated. Replacing it with the speed of that particle and determining the mass $m = \frac{h}{\lambda V}$ from (70) and substituting it in formula (69) we will have

$$\Psi(V,t) = \sin\left(\frac{2\pi}{\lambda V}t\right) = \sin\left(\frac{\omega_0}{V}t\right), \quad (71)$$

where $\omega_0 = \frac{2\pi}{\lambda}$ is the angular frequency.

Since for small values ℓ we have $\sin\ell \approx \ell$, formula (71) can be given in two expressions

$$\Psi(V,t) = \sin\left(\frac{\omega_0}{V}t\right), \quad (71a)$$

$$\Psi(V,t) = \frac{\omega_0}{V}t. \quad (72)$$

Formula (71a) corresponds to the wave motion of the particle, and formula (72) to corpuscular motion. So, the motion of an elementary particle can be viewed from two positions: corpuscles and waves.

10. Gravitational Wave Model

Although the existence of gravitational waves was predicted by the general theory of relativity, their detection was possible only after a hundred years.

In the mid-seventies of the last century, the problem of indirect detection of gravitational waves was solved in [7]. In the article, the probabilistic (correlation) relationship between the earthquakes and flares passing through the solar

[8] λ —is the wavelength of the elementary particle; V—the velocity of an elementary particle; p—the impulse of an elementary particle.

chromosphere was proved. The studies presented in this work allows:

1) To ascertain that gravitational waves as such exist;

2) To ascertain that the gravitational waves consist of neutrinos as neutrinos freely pass through the Earth;

3) To ascertain that the flares in the solar chromosphere show that gravitational waves are composed of the matter, *i.e.* from neutrinos that have rest mass;

4) To determine the velosity of gravitational wave υ; since the distance from the earth to the sunis well known ($l = 15 \times 10^7$ km), the time t of flight of the neutrino cluster from the moment of the earthquake to the moment of the flares in the chromosphere, consequently, $\upsilon = \frac{l}{t}$.

Thus, it becomes clear that the gravitational wave is associated with the transfer of the matter in the form of a large aggregate of elementary particles of neutrino of the same type.

The necessity for introduction of neutrinos was determined by the law of conservation of energy in the process of the $\beta^{\div}$-decay of the atomic nuclei. W. Pauli suggested a hypothesis (in 1931-1932) about the existence of neutrinos. E. Fermi gave the name "neutrino" to the particle due to lack of charge and its very small dimensions. He also expressed the idea that the neutrino is not in a "ready-made form" in the nucleus of an atom, but in some way, it is instantly formed from the energy of the nucleus.

Researchers of the composition of the cosmic rays reached the conclusion that all the ordinary matters in the Universe consist of two lightest leptons, an electron e and an electron neutrino ν_e [9] [3].

For 30 years after the discovery of neutrino, it was believed that this particle had zero rest mass. The papers published at that time considered that gravitational waves carried energy and impulse, but they had nothing to do with the transfer of the matter [8] [9].

It became known to cosmology that the total mass of neutrinos in the Cosmos many times prevails the total mass of luminous objects and therefore the neutrino makes the main contribution to cosmic gravity [3].

Such an abundance of neutrinos in the Universe has created the prerequisites for creation of a new science—neutrino astronomy. Consequently, the detection of gravitational waves belongs to that science.

Finally, the time came for direct detection of the gravitational wave: it occurred on February 11, 2016, when two highly sensitive detectors of the LiGO gravitational observatory, located in Washington and state of Louisiana, simultaneously recorded the signal GW150914, lasting about 0.2 seconds.

The above model of the wave function of an elementary particle located at the microlevel, can be used (formula (69)) for modeling the gravitational wave, since neutrino belongs to the class of elementary particles. Therefore, the gravitational

[9]Further, the index e by ν_e will be omitted. In addition to the electron neutrino ν, there exist τ-neutrinos and μ-neutrinos, but they are rare. We consider all neutrinos as three states of one particle. This is possible in the case when the laws of conservation of the lepton charges are violated.

wave model is given by the formula:

$$\Psi(M,t)=\sin\left(\frac{M}{\hbar}t\right), \tag{73}$$

where *M*, is the relativistic mass of the neutrino beam. The neutrino beam consists of *N*-number of neutrinos of the same type; therefore, the relativistic mass of the neutrino beam is determined as follows

$$M=\sum_{i=1}^{N}m_i=Nm, \tag{74}$$

where m_i, is the relativistic mass of one neutrino particle. The relativistic mass of one neutrino particle is determined according to the Lorentz transformation

$$m=\frac{m_0}{\sqrt{1-\left(\frac{\upsilon}{c}\right)^2}}=\mu m_0, \tag{75}$$

where $\frac{1}{\sqrt{1-\left(\frac{\upsilon}{c}\right)^2}}=\mu$ is a constant parameter depending on the neutrino velocity, υ is the velocity of neutrinos; m_0, the rest mass of the neutrino.

Thus, the relativistic mass *M* of the neutrino beam is determined according to (74), taking into consideration the relativistic mass of the *i*-th particle (75):

$$M=N\mu m_0.$$

Since for a small value ℓ we have $\sin\ell\approx\ell$, then for the detected signal, according to formula (73), we obtain

$$\frac{M}{\hbar}=0.2.$$

From this relation, only the relativistic mass of the neutrino beam can be determined

$$M=0.2\hbar.$$

11. Result and Conclusions

There is no lacky of the estimates about the difficulty of solving the Schrödinger equation. One of the estimates is that: "In most cases, solution of an equation is a difficult mathematical problem that cannot be solved using functions studied in mathematics [10]." However, it is never mentioned what the difficult problem of the solution consists in, and how it can be solved. Therefore, the author considers it priority to clarify the problem of solving the Schrödinger equation and eliminate it.

Thus, the Schrödinger equation, mapped onto the microlevel of the matter, allows solving the problems known from the heuristic considerations or experimental results. And in regard to gravitational waves as L. Brillouin's expression is prophetic: "Nothing confirms that gravitational waves cannot represent

Ψ-waves of quantum mechanics. Each particle has its own Ψ-wave and, thanks to its mass, is a source of gravitational waves; then why we cannot suggest that Ψ-waves transmit gravitational interactions?"

In present work following results were obtained:

1) Was solved non-stationary Schrödinger equation.

2) On the base of solving of non-statyonary Schrödinger equation obtained model of antiparticle.

3) On the basis of solving of Schrödinger equation is onstructed mathematical model electric field at which appear real elementary particle.

4) On the basis of solving of non-stationary Sredinger equation mathematical model of corpuscular-wave dualism was obtained.

5) On the base of solving of non-stationary Schrödinger equation mathematical model of gravitation wave was received.

In addition to obtaining new models, it is shown that these models are integral systems, since the implementation of the constancy of the potential field (U = const) entails the condition r = 1 (see page 10) which takes place (see footnote 5 on page 9), if the variance is zero, *i.e.*

This means that these models are integrity formations and are in a medium without dispersion.

In the case of four-dimensional space, the constancy of the potential field is equivalent to its homogeneity and isotropy in all directions.

This position takes place by determining the constancy of the potential field by creating a model of a gravitational wave.

Conflicts of Interest

The author declares no conflicts of interest.

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