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The Role of Density of Physical Variable

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Abstract

The concept of density in quantum theories of an elementary particle is discussed. Density is, at least implicitly, recognized in contemporary textbooks on quantum field theories, where the Noether theorem is utilized for a derivation of a conserved 4-current $j^{\mu}_{,\mu} = 0$ and a conserved energy-momentum tensor $T^{\mu\nu}_{,\nu} = 0$. Here the component j^0 is the particle's density and the components $T^{\mu 0}$ are the energy-momentum density. The novelty of this work is the analysis of the particle's density and the energy-momentum density of these expressions and their application to several specific quantum theories. As of today, these tasks have not been adequately accomplished in contemporary textbooks. The results show that the first-order Dirac theory of an elementary massive spin-1/2 particle yields consistent results. In contrast, second-order quantum theories, such as the Klein-Gordon theory, the electroweak theory of the $W^{\pm}, Z$ particles, and the Higgs boson theory are inherently wrong.

Subject Areas

Particle Physics

Keywords

Physical Principles, Locality Attributes, Quantum Theories, Coherence Tests

1. Introduction

Classical physics fails to explain the behavior of the microscopic world. For example, an elementary classical particle is pointlike (see [1], pp. 46, 47). Hence, due to its acceleration, a classical electron of the hydrogen atom should radiate until it falls into the proton's center. However, the radius of the hydrogen atom is much larger than the proton's radius.

Quantum theories are based on concepts that are very different from those of

Classical Physics (CPH). In quantum theories, the wave function $\psi(\boldsymbol{x},t)$ describes the electronic state and the quantum equation of motion describes the time-evolution of ψ. Furthermore, appropriate operators yield the expectation value of physical quantities. Thus, the expectation value of the electron's energy is:

$$\langle E(t)\rangle = \int \psi(\boldsymbol{x},t)^* H\psi(\boldsymbol{x},t)\mathrm{d}^3r, \tag{1}$$

where $\langle E(t)\rangle$ denotes the energy expectation value, and the Hamiltonian H is the energy operator. This is an example of a quantum description of the expectation value of physical quantities (see [2], p. 145).

The plain meaning of (1) is that at the specific time of the calculation, the electron exists at all points of a given region and for an infinitesimal region, ΔV, the probability of its existence inside it is $\psi^*\psi\Delta V$. This outcome manifests the well-known uncertainty relation of quantum theories, where the particle's position is not well defined (see [2], p. 20):

$$\Delta x\Delta p_x \geq \hbar. \tag{2}$$

The first experiment of muon decay illustrates the local attributes of an elementary quantum particle. The emulsion tracks of the muon decay $\mu^- \to e + \nu_\mu + \bar{\nu}_e$ and of the outgoing electron are described in **Figure 1** (the original figures are shown in [3], p. 4 and in [4], p. 26). The charged particles ionize atoms along with their motion, bubbles settle on the ionized atoms, and the figures of [3], p. 4 and [4], p. 26 show the lines of bubbles. These lines are depicted in **Figure 1** of this work. The two neutrinos are unseen in the figures of [3] [4] because they do not induce ionization. The lines of **Figure 1** illustrate the local existence of a quantum particle—namely, the charged particles exist at a quite narrow region along the lines and do not exist elsewhere.

The foregoing discussion points out the significance of the notion of density in quantum theories. This paper is dedicated to the theoretical properties of this topic. It examines the relativistic form of the Lagrangian density of Quantum Field Theories (QFT) and shows several aspects of density that many textbooks do not discuss. This is the primary purpose of this work and it illustrates its novelty.

Units where $\hbar = c = 1$ are used. Therefore, just one dimension is required and the dimension of length [*L*] is used. The Minkowski metric $g_{\mu\nu}$ is diagonal and its entries are (1, −1, −1, −1). Relativistic expressions are written in the standard notation. The Dirac matrices $\boldsymbol{\alpha}, \beta$, and γ^μ take the form that is used in [5]. Section 2 describes several principles that are the basis of this work. Section 3 proves that the Dirac theory of a massive spin-1/2 quantum particle is consistent with the physical principles used herein. Section 4 analyses density

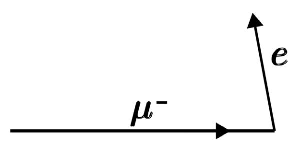

Figure 1. An illustration of muon decay (see text).

and proves that inconsistencies exist in theories where the quantum differential equations of motion are of the second order. Section 5 shows further arguments that corroborate the results of Section 4. The last section summarizes the main points of this paper.

2. General Principles

This work examines the properties of density in physical theories. A description of the physical concepts that are used below helps readers see the general structure of the analysis.

2.1. The Correspondence Relationships

An important element of a theory is an adequate definition of its domain of validity. For example, Non-Relativistic CPH (NRCPH) is a good theory for a description of processes that take place in the macroscopic world, and the velocity of the particles is much smaller than the speed of light. Quantum Mechanics (QM) is restricted to cases where relativistic effects can be ignored. Hence, the validity domain of NRCPH is a subset of the validity domain of QM. For this reason, NRCPH is regarded as a lower rank theory with respect to QM.

The correspondence principle says that an appropriate limit of quantities of a higher rank theory should agree with corresponding quantities of a lower rank theory. It means that QM must define particle density, energy density, and momentum density of the quantum particle, and the classical limit of these quantities should agree with the corresponding quantities of NRCPH. The need to prove the correspondence between QM and NRCPH was recognized in the QM early days when the Ehrenfest theorem was published (see [6], pp. 25-27, 137, 138).

QM does not explain everything. For example, experiments show that the proton comprises quark-antiquark pairs of the u, d, s flavor [7] [8]. In principle, this kind of evidence should be explained by QFT. Hence, the domain of validity of QM is a subset of the domain of validity of QFT. It means that analogous constraints apply to QFT: QFT must define particle density, energy density, and momentum density of the quantum particle, and the QM limit of these quantities should agree with the corresponding quantities of QM. This correspondence is clearly stated in Weinberg QFT textbook (see [9], p. 49):

"*First, some good news: quantum field theory is based on the same quantum mechanics that was invented by Schroedinger, Heisenberg, Pauli, Born, and others in* 1925-26, *and has been used ever since in atomic, molecular, nuclear, and condensed matter physics.*"

2.2. The Lagrangian Density

It is now recognized that a QFT of an elementary particle is based on a Lagrangian density whose general form is:

$$\mathcal{L}\left(\psi, \psi_{,\mu}\right), \tag{3}$$

(see e.g. [9], p. 300). The equations of motion of the particle are the Euler-Lagrange equations that are derived from a variation of the action S of (3) with respect to ψ:

$$\begin{aligned} 0 &= \delta S \\ &= \delta \int \mathcal{L} \mathrm{d}^4 x \\ &= \int \left[\frac{\partial \mathcal{L}}{\partial \psi} \delta \psi + \frac{\partial \mathcal{L}}{\partial \psi_{,\mu}} \delta \left(\psi_{,\mu} \right) \right] \mathrm{d}^4 x. \\ &= \int \left[\frac{\partial \mathcal{L}}{\partial \psi} \delta \psi - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \psi_{,\mu}} \right) \delta \psi + \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \psi_{,\mu}} \delta \psi \right) \right] \mathrm{d}^4 x. \end{aligned} \tag{4}$$

The integral of the last term of (4) yields the values of $\frac{\partial \mathcal{L}}{\partial \psi_{,\mu}} \delta \psi$ at spatial infinity. It is assumed that at spatial infinity, $\psi = \psi_{,\mu} = \delta \psi = 0$. Therefore, the form of (3) proves that the last term of (4) can be removed. Equating the variation δS to zero and remembering that $\delta \psi$ is an arbitrary variation, one finds that the first and second terms on the right-hand side of the last line of (4) yield the Euler-Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial \psi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \psi_{,\mu}} \right) = 0 \tag{5}$$

(see e.g., [9], p. 300).

In the units used herein, the action S is dimensionless. Therefore, the dimension of the Lagrangian density is $[L^{-4}]$. Moreover, if the Lagrangian density is a Lorentz scalar then the Euler-Lagrange equations take the required Lorentz invariant form (see e.g., [9], p. 300). This is an important feature of the application of the Lagrangian density as the basis of the theory: If it is a Lorentz scalar whose dimension is $[L^{-4}]$ then the theory abides by Special Relativity (SR).

Another important property of the Lagrangian density is its mathematically real form. Since the integration factor $\mathrm{d}^4 x$ is a mathematically real Lorentz scalar then the action S of such a Lagrangian density is a mathematically real Lorentz scalar. An action that is a mathematically real Lorentz scalar is used for the undulating factor of the particle's function of QM:

$$\Phi = \mathrm{e}^{iS} \tag{6}$$

(see e.g., [10], pp. 127, 128; [11], pp. 19, 20). Evidently, the power series expansion of the exponent of (6) proves that a coherent action S should be a mathematically real Lorentz scalar. Furthermore, this form of the action is used for proving the correspondence between QM and NRCPH (see e.g., [10], pp. 127, 128; [11], pp. 19, 20). Hence, the correspondence between QFT and QM shows that the action S of QFT should be a mathematically real Lorentz scalar. This outcome is consistent with [9], p. 300.

The Lagrangian density (3) is still an incomplete description of the state of a given elementary quantum particle. Indeed, the existence of a physical particle is recognized due to a measurement process where the particle affects the time-evolution of a measurement device. Hence, *the Lagrangian density requires an interaction term that depends on the particle's quantum function* ψ *and on an interaction carrying external field*. For example, the electromagnetic interaction of a Dirac electron is:

$$\mathcal{L}_{int} = -e\bar{\psi}\gamma^{\mu}A_{\mu}\psi, \tag{7}$$

where e is the electronic charge and the components of A_{μ} are the electromagnetic potentials $\phi, \boldsymbol{A}$ (see [12], p. 78). Here (7) is compatible with the classical expression for the electromagnetic interaction:

$$\mathcal{L}_{int} = -ej^{\mu}A_{\mu}, \tag{8}$$

where j^{μ} is the 4-current (see [1], p. 75; [13], p. 596). It is shown later that the Quantum Electrodynamics (QED) Lagrangian density comprises the term (7), which uses the Dirac 4-current (15).

2.3. The Noether Theorem

The Noether theorem is an important element of theories that are derived from a Lagrangian density. It shows that if the Lagrangian density is invariant under a given transformation then the Euler-Lagrange equations of this theory conserve an appropriate quantity (see e.g., [9], p. 307; [14], pp. 17-22).

This work examines the Noether expressions for the particle's density and its energy-momentum densities. The particle's density is the 0-component of a 4-vector (see [1], pp. 73-75). If the Lagrangian density is independent of the phase of the quantum function ψ, then the required 4-vector is (see [15], pp. 314-315):

$$j^{\mu} = a\frac{\partial\mathcal{L}}{\partial\psi_{,\mu}}\psi. \tag{9}$$

where a is an appropriate coefficient. j^0 is the particle's density, and a is fixed so that $\int j^0 \mathrm{d}^3x = 1$. This expression means that the dimension of the 4-current is $[L^{-3}]$, which is compatible with the concept of density. Furthermore, the 4-current (9) is conserved:

$$j^{\mu}_{,\mu} = 0. \tag{10}$$

An important feature of the conserved 4-current (9) holds for a charged particle. Here the charge Q is used for the definition of the electric 4-current Qj^{μ}. Charge conservation is a crucial property of Maxwellian electrodynamics; its mathematical expression is called the continuity equation; and (10) is the covariant form of this equation (see [1], pp. 76-78; [13], p. 549).

Energy is the 0-component of the energy-momentum 4-vector, and it is mentioned above that density is the 0-component of a 4-vector. Therefore, energy density is the 00-component of a 4-tensor $T^{\mu\nu}$, which is called the energy-momentum tensor. If the Lagrangian density does not explicitly depend on the space-time coordinates $(t, \boldsymbol{x})$, then the Noether theorem provides an expression for this tensor that proves energy-momentum conservation. This energy-momentum tensor is:

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \psi_{,\nu}} g^{\alpha\mu} \psi_{,\alpha} - g^{\mu\nu} \mathcal{L} \tag{11}$$

(see [12], p. 310). The components $T^{\mu 0}$ are energy-momentum density and they are conserved:

$$T^{\mu\nu}_{\ ,\nu} = 0. \tag{12}$$

The above-mentioned textbook references to (9) and (11) indicate that the foregoing expressions for conserved density are already known. Moreover, mainstream textbooks recognize density as an element of QFT. The novelty of this work is the analysis of particle density and energy-momentum density of these expressions and their application to several quantum theories. As of today, these tasks have not been adequately accomplished in contemporary textbooks.

3. The Dirac Particles

Consider the Lagrangian density of a free Dirac particle (see [12], p. 52; [14], p. 54):

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi, \tag{13}$$

where:

$$\bar{\psi} \equiv \psi^{\dagger}\gamma^{0}. \tag{14}$$

The Noether expression for the 4-current (9) and the Dirac Lagrangian density (13) yield the Dirac 4-current:

$$j^{\mu} = \bar{\psi}\gamma^{\mu}\psi. \tag{15}$$

(see [14], p. 56; [15], p. 315). The Dirac 4-current (15) also holds for the Dirac Lagrangian density that includes the electromagnetic interaction term (see e.g., [12], p. 78; [14], p. 84):

$$\mathcal{L}_{QED} = \bar{\psi}\left[\gamma^{\mu}\left(i\partial_{\mu} - eA_{\mu}\right) - m\right]\psi. \tag{16}$$

The interaction term $-e\bar{\psi}\gamma^{\mu}A_{\mu}\psi$ of (16) is derivative-free and the application of formula (9) to this term adds a null quantity.

Using the Noether expression for the energy-momentum tensor (11), one finds that the Dirac theory yields:

$$T^{\mu\nu} = \bar{\psi}i\gamma^{\nu}g^{\mu\lambda}\partial_{\lambda}\psi - g^{\mu\nu}\mathcal{L}. \tag{17}$$

As stated above, the T^{00} component of this tensor is the energy density. In the case of a free Dirac particle, one finds that this component of (17) boils down to:

$$T^{00} = \psi^{\dagger}\left(-i\boldsymbol{\alpha}\cdot\nabla + \beta m\right)\psi. \tag{18}$$

(The calculation depends on these points: (14) is used; in (17), $T^{00} \to \mu = \nu = 0$; for the running index λ of (17), $\lambda = 0$ is the time derivative that is canceled, due to the corresponding term in $-\mathcal{L}$; the $\lambda = 1,2,3$ terms yield $-i\boldsymbol{\alpha}\cdot\nabla$; $\gamma^{0}\boldsymbol{\gamma} = \boldsymbol{\alpha}$; the sign of the mass term of $-\mathcal{L}$ is changed.) The expression inside the parentheses of (18) is the Hamiltonian of a free Dirac particle (see [5], pp. 6, 7), and the 0-component of the Dirac 4-current (15) is $\psi^\dagger\psi$. Hence, the right-hand side of (18) is a coherent expression of the energy density of a free Dirac particle. An explicit calculation shows that the previous relativistic calculation also yields a coherent expression for the momentum density (see [16], chapters 5, 6).

This analysis proves that an application of the Noether theorem to the Dirac Lagrangian density yields coherent expressions for the particle density and its energy-momentum density. The $[L^{-3/2}]$ dimension of the Dirac function ψ is an important property of this theory because the dimension of the product $\psi^\dagger\psi$ is $[L^{-3}]$, which is the dimension of density.

4. Second-Order Theories

There are several QFT of second-order equations, such as the Klein-Gordon (KG) equation, the electroweak theory of the $W^\pm, Z$ particles, and the Higgs particle theory. A typical form of the Lagrangian density of these theories is:

$$\mathcal{L} = Am^2\phi^?\phi + F\left(\phi^\dagger_{,\mu}, \phi_{,\nu}\right) + OT, \tag{19}$$

where A is a numerical coefficient, F is a Lorentz scalar function of products of derivatives of the quantum functions $\phi^\dagger_{,\mu}\phi_{,\nu}$, and OT denotes other terms (see e.g., [12], pp. 16, 721; [17], pp. 515, 518). The Noether expression (11) for the T^{00} component is the energy density of the system. This issue proves that the first term of the Lagrangian density of second-order theories (19) entails a quadratic mass term of T^{00}. In contrast, energy density depends linearly on the particle's mass. This outcome indicates an inherent contradiction of any second-order quantum theory.

It should be pointed out that the $[L^{-4}]$ dimension of the Lagrangian density entails the dimension $[L^{-1}]$ of the quantum function ϕ of a second-order theory. Hence, the quadratic mass term is used in (19) because the dimension of mass is $[L^{-1}]$.

5. Discussion

Any specific mathematical analysis implicitly relies on the assumption that its basis comprises coherent expressions. This self-evident issue also holds for the Noether theorem, and the previous results of the Dirac Lagrangian density provide an example of this issue. In contrast, the incompatible result of the application of the Noether theorem to second-order quantum theories of an elementary massive quantum particle proves that these theories have an erroneous basis. Further arguments indicate that erroneous elements of second-order quantum theories substantiate this assertion.

As stated above, the Lagrangian density is a mathematically real quantity whose dimension is $[L^{-4}]$. In order to be mathematically real, it must be a product of the quantum functions $\phi^\dagger\phi$. The same is true for density. Considering density, one should note that in the case of a second-order quantum theory, the dimension of the quantum function ϕ is $[L^{-1}]$, whereas the dimension of density is $[L^{-3}]$. Therefore, dimensional considerations entail that each term of any coherent expression of the 4-current of a second-order quantum theory must contain a derivative of the quantum functions $\phi^{\dagger}_{,\mu}$ or $\phi_{,\mu}$.

This conclusion is unacceptable because the electromagnetic interaction term (8) depends on the particle's 4-current, and the Noether expression for the 4-current (9) says that a term that depends on a derivative of the quantum function $\phi_{,\mu}$ yields a new term of the 4-current. It means that the electromagnetic interaction term of a second-order quantum theory destroys the compatibility of the 4-current upon which it depends.

Conclusion: A second-order quantum theory of a charged particle cannot have a coherent expression for its electromagnetic interaction.

The following evidence provides strong support for this conclusion. Thus, about one month after the publication of the first-order Dirac theory of the electron [18], Darwin found an expression for the conserved 4-current of this particle [19]. This information is compatible with the coherence of the Dirac theory that is derived in Section 3. In contrast, the electroweak theory is about 50 years old, and large research centers, like Fermilab and CERN, still use a derivative-dependent "effective" and incoherent expression for the electromagnetic interaction of the $W^{\pm}$ particles [20] [21]. The derivative dependence of this expression certainly demonstrates an inherent gross error of the electroweak theory.

Here is another argument that emphasizes the vital role of density in QFT. Consider the experimental detection of the μ^{+}, μ^{-} decay channel of the Z particle [22] (see **Figure 2**). In this case, each of the two charged μ^{+}, μ^{-} particles hits a detector that measures its energy-momentum, together with the time and the position of the measurement. These data enable experimenters to draw the path of each particle. If the μ^{+}, μ^{-} emerged from a small space-time region and if their invariant energy is included inside the mass region of the Z particle then the event is recorded as a μ^{+}, μ^{-} decay of this particle. It means that an expression for the density is required from a coherent theory of the Z particle. This experiment shows a two-particle creation process. Hence, it belongs to the QFT validity domain. However, no Standard Model (SM) textbook shows an expression for the density of its mathematically real electroweak function of the Z particle. As a matter of fact, it is impossible to create a coherent expression for the density of a mathematically real quantum function of an elementary massive particle (see [16], pp. 44, 45).

A particular problem arises from the Lagrangian density of the Higgs particle (see e.g., [12], p. 721; [17], p. 515). Here the coefficient of the mass term is positive. Hence, formula (11) for the energy-momentum tensor proves that the mass term of the T^{00} component of this tensor is negative. It means that the Higgs

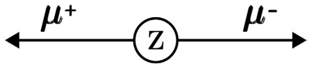

Figure 2. An illustration of the μ^{+}, μ^{-} decay channel of the Z particle (see text).

idea depends on a quantum theory of a particle where its self-mass makes a negative contribution to its energy! This grave contradiction is inconsistent with fundamental principles of physics where energy and mass of a given system are non-negative quantities. This outcome refutes the Higgs theory and supports the general arguments of the previous section.

Remark: *A second-order quantum theory proves that the dimension of its quantum function* ϕ *is* $[L^{-1}]$. *It is shown above that this property is the root of the problematic points of these theories—the quadratic dependence on mass and the need for a derivative* $\phi_{,\mu}$ *for the density. The inability to change this dimension means that the erroneous elements of second-order quantum theories are uncorrectable*.

6. Concluding Remarks

This work discusses the significance of density in quantum theories. It is proved that this quite neglected issue is required for the coherence of any specific quantum theory as well as for its ability to explain experimental data. Particle density is the 0-component of the 4-current j^μ, and energy-momentum densities are the $T^{\mu 0}$ components of the energy-momentum tensor. Contemporary QFT textbooks show general expressions for these quantities, and their derivation utilizes the Noether theorem.

The novelty of this work is the test of the coherence of specific quantum theories with respect to the Noether expressions for density. Standard QFT textbooks ignore this issue. Section 2 describes well-known principles that are used as the basis of the analysis: the correspondence principle, the variational principle, and the Noether theorem. These issues are discussed in mainstream textbooks. The analysis applies the dimension $[L^n]$ of physical quantities, which is a well-defined quantity. Section 3 proves that the first-order Dirac theory of a massive spin-1/2 particle yields coherent results. In contrast, Sections 4 and 5 prove that erroneous elements exist in second-order quantum theories, such as the KG theory, the electroweak theory, and the Higgs theory. The errors stem from the $[L^{-1}]$ dimension of the second-order quantum function ϕ of these theories. The inability to change the dimension of ϕ means that second-order quantum theories are uncorrectable.

It is interesting to note that the above mentioned inherent problems of the second-order quantum theories are compatible with the Dirac lifelong objection to these theories [23].

Conflicts of Interest

The author declares no conflicts of interest.

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