



A Four-Dimensional Chaotic System with Hidden Attractor and Its New Proposed Electronic Circuit

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Abstract

This research deals with designing a new electronic circuit as an engineering application on a four-dimensional chaotic system. The adopted chaotic dynamical system with a hidden attractor consists of two quadratic nonlinearities and parameters. The electronic circuit model is obtained by applying Kirchhoff's laws. The electronic circuit consists of seven resistors, four capacitors, four voltages and four operational amplifiers. The chaotic motions of the four-Dimensional system are investigated through Lyapunov exponents, Kaplan-Yorke dimension, phase portraits, and diagrams. Using MultiSIM12, the theoretical results were simulated and found to be well consistent with the results obtained from MATLAB.

Subject Areas

Mathematics

Keywords

Four-Dimensional Chaotic System, Kaplan-Yorke Dimension, Hidden Attractor, Circuit Simulation, MultiSIM12

1. Introduction

Electronic systems rule our modern world, bridging the gap between software and physical reality. From communication to industrial process control to transportation to entertainment, the field of electronics continues to grow in reach and complexity. A working knowledge of basic electrical engineering concepts is now a powerful tool in many fields, and this value is likely to grow in the coming decades [1].

In recent years, the chaotic circuit has received a lot of research interest due to the fact that it has been applied in abundant areas such as economic model simulation, secure communication, electronic circuits design, robotics, image processing, and neural networks [2]-[8].

In Section 2, we describe a 4-dimensional chaotic system with a hidden attractor; it consists of eight simple terms involving two quadratic nonlinearities. In Section 3, we proposed an electronic circuit to implement system (1.1). In Section 4, simulate the designed circuit by MultiSIM12. Finally Section 5 presents the main conclusions of this paper.

2. Description of Chaotic System with Hidden Attractor

First of all, let us review the autonomous four dimensional dynamical system [9] which consists of eight simple terms involving two nonlinear terms.

The equations are shown as follow:

$$\begin{aligned}\dot{x} &= \rho(y - x) \\ \dot{y} &= ax - \delta xz + w \\ \dot{z} &= \phi xy - z \\ \dot{w} &= -kx\end{aligned}\tag{1.1}$$

Given initial values $[x_0, y_0, z_0, w_0] = [4, 1, 4, 2]$ and parameters with the following values $\rho = 10, \delta = 40, a = 296.5, \phi = 10, k = 8$.

After solving the equations of the system:

$$\begin{aligned}10(y - x) &= 0 \\ 396.5x - 40xz + w &= 0 \\ 10xy - z &= 0 \\ -8x &= 0\end{aligned}\tag{1.2}$$

We note that the chaotic system (1.1) only has a zero equilibrium point. Therefore, system (1.1) has a hidden attractor.

Now, By applying Wolf's algorithm [10], The Lypanuov exponents are determined as: ($L_1 = 1.660748$, $L_2 = 0.149599$, $L_3 = -0.068474$ and $L_4 = -12.144118$). Also, the Lypanuov dimension Kaplan-Yorke dimension of this system is calculated as

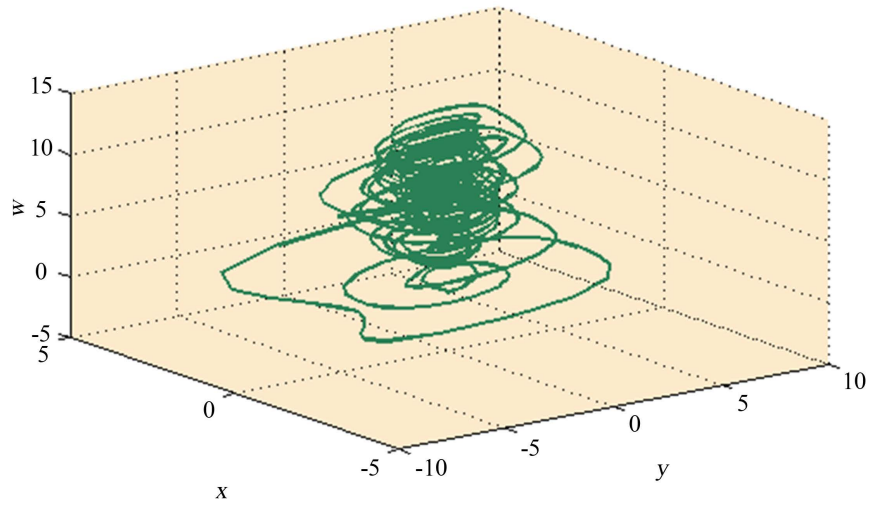
$$D_L = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3.143433471\tag{1.3}$$

So the chaotic behavior of system (1.1) in \mathbf{R}^3 is shown in **Figures 1(a)-(c)**, **Figure 1(a)**: (y, x, w) space, **Figure 1(b)**: (z, w, x) space, **Figure 1(c)**: (y, z, w) space. **Figures 2(a)-(c)** shows the Phase portraits of system (1.1), **Figure 2(a)**: (x, w) plane, **Figure 2(b)**: (y, w) plane, **Figure 2(c)**: (w, z) plane.

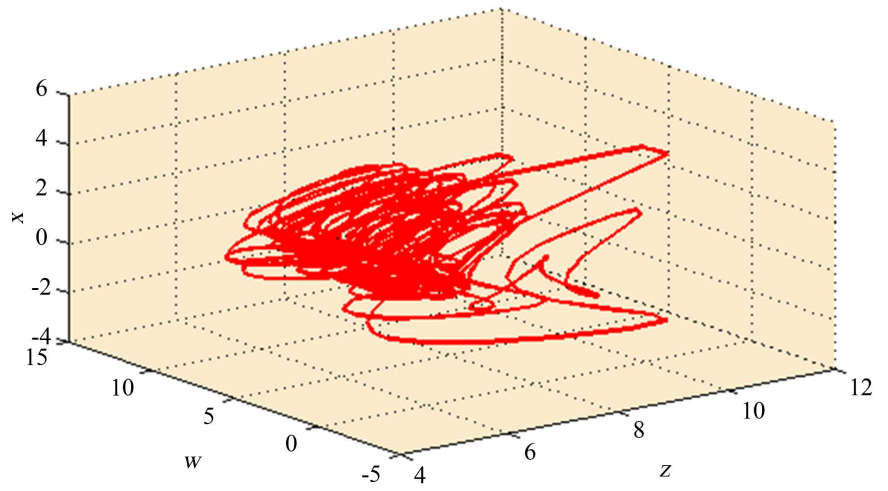
In additional, one of the basic behaviors of chaotic dynamical systems is shown in **Figure 3** which is the wave-form $x(t), y(t), z(t), w(t)$ for system (1.1).

3. Proposed Electronic Circuit

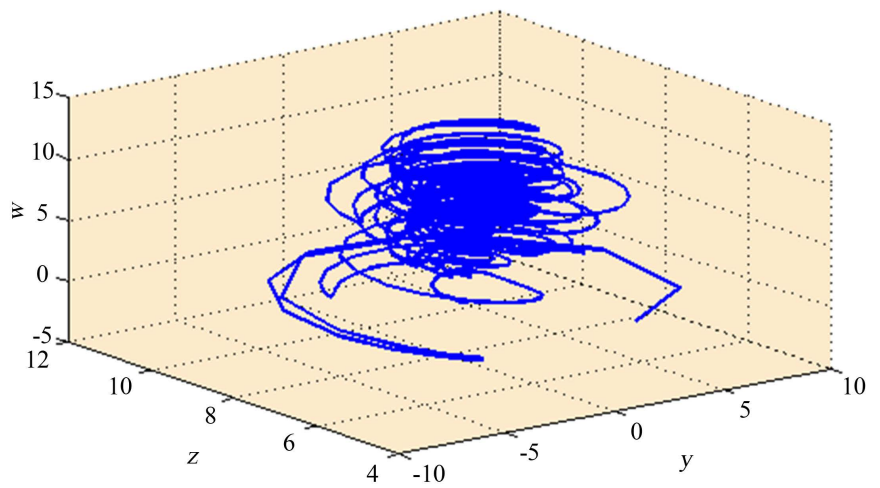
In this section we design an electronic circuit to implement the 4-D chaotic



(a)

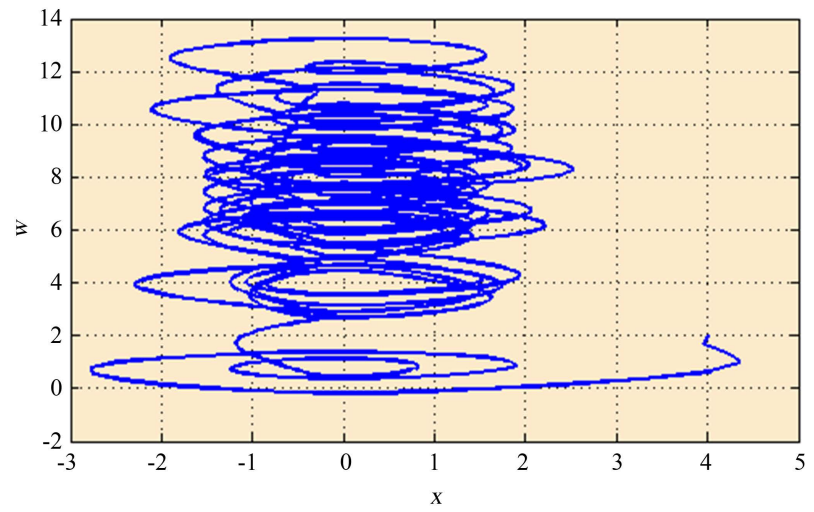


(b)

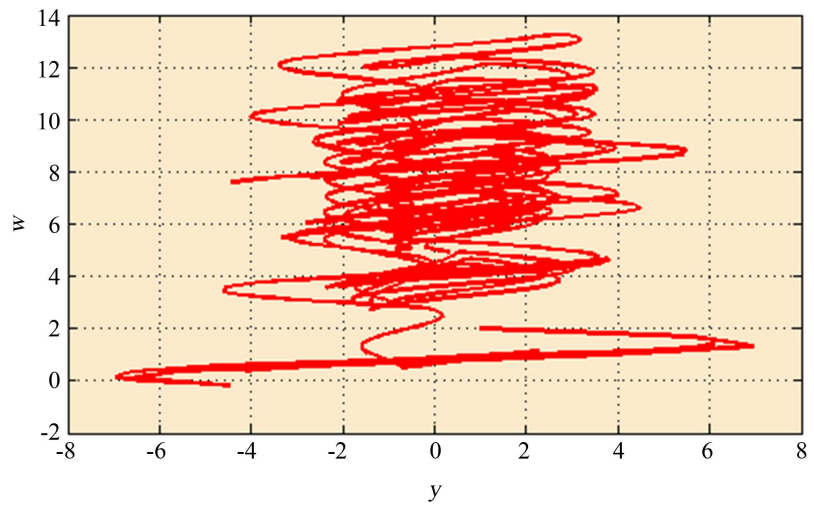


(c)

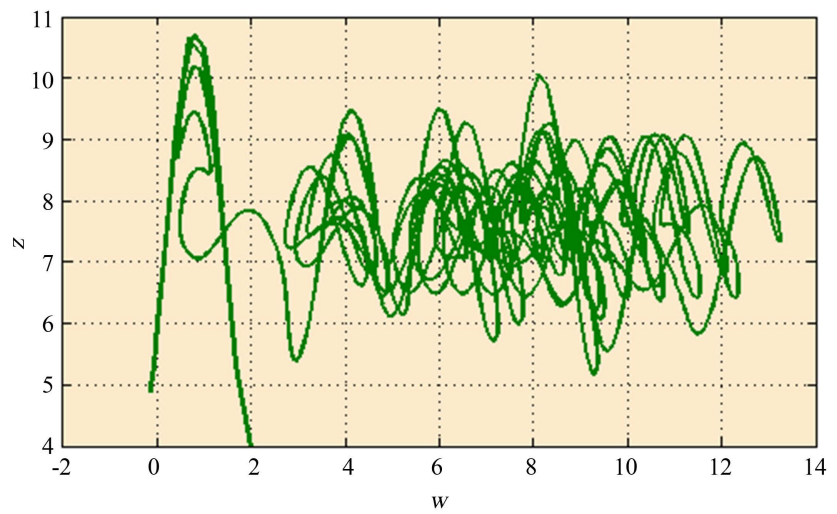
Figure 1. (a)-(c) The chaotic behavior in \mathbf{R}^3 .



(a)



(b)



(c)

Figure 2. (a)-(c) The phase portraits of system (1.1).

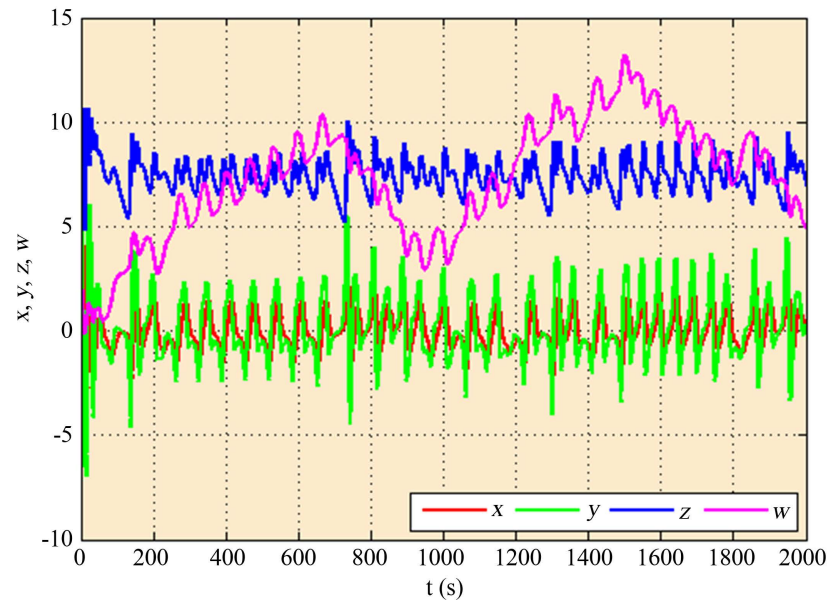


Figure 3. The wave form $x(t), y(t), z(t), w(t)$ for system (1.1).

system (1.1). It consists of the following electronic elements: resistors, capacitors, multipliers and operational amplifiers TL034CN.

Applying Kirchhoff's laws [7], we can obtain the equations of the corresponding circuit as follows:

$$\begin{aligned}
 \frac{dV_x}{dt} &= \frac{1}{R_1 C_1} (V_y - V_x) \\
 \frac{dV_y}{dt} &= \frac{1}{R_2 C_2} V_x - \frac{1}{R_3 C_2} V_x V_z + \frac{1}{R_4 C_2} V_w \\
 \frac{dV_z}{dt} &= \frac{1}{R_5 C_3} V_x V_y - \frac{1}{R_6 C_3} V_z \\
 \frac{dV_w}{dt} &= -\frac{1}{R_7 C_4} V_x
 \end{aligned} \tag{1.4}$$

where V_x, V_y, V_z, V_w are the output voltages, the fixed multipliers constant is $k_m = 10 \text{ V}$, consequently the outputs are $V_{xz} = V_x V_z / k_m$ and $V_{xy} = V_x V_y / k_m$.

$$V_x = 1 \text{ V} \cdot x, \quad V_y = 1 \text{ V} \cdot y, \quad V_z = 1 \text{ V} \cdot z, \quad t' = \tau \cdot t = 100 \mu\text{s} \cdot t \tag{1.5}$$

Substitute (1.5) in equations of system (1.4) we get:

$$\begin{aligned}
 \frac{dx}{dt'} &= \frac{\tau}{R_1 C_1} (y - x) \\
 \frac{dy}{dt'} &= \frac{\tau}{R_2 C_2} x - \frac{\tau}{R_3 C_2} xz + \frac{\tau}{R_4 C_2} w \\
 \frac{dz}{dt'} &= \frac{\tau}{R_5 C_3} xy - \frac{\tau}{R_6 C_3} z \\
 \frac{dw}{dt'} &= -\frac{\tau}{R_7 C_4} x
 \end{aligned} \tag{1.6}$$

Set side by side system (1.1) with system (1.6) gives posterior conditions:

$$\begin{aligned} \frac{\tau}{R_1 C_1} = \rho, \quad \frac{\tau}{R_2 C_2} = a, \quad \frac{\tau}{R_3 C_2} = \delta, \quad \frac{\tau}{R_4 C_2} = 1, \\ \frac{\tau}{R_5 C_3} = \varphi, \quad \frac{\tau}{R_6 C_3} = 1, \quad \frac{\tau}{R_7 C_4} = k \end{aligned} \quad (1.7)$$

Use convenient values for resistances and capacitances as

$$\begin{aligned} R_1 = 10 \Omega, \quad R_2 = 337.268 \text{ m}\Omega, \quad R_3 = 2.5 \Omega, \quad R_4 = 100 \Omega, \quad R_5 = 10 \Omega, \\ R_6 = 100 \Omega, \quad R_7 = 12.5 \Omega, \quad C_1 = C_2 = C_3 = C_4 = 1 \text{ mF} \end{aligned} \quad (1.8)$$

We got the empirical electronic circuit (1.6) for system (1.1) with parameters $\rho = 10, a = 296.5, \delta = 40, \varphi = 10, k = 8$.

4. The Simulation Results

In this section, we simulate the circuit designed to implement the chaotic system (1.1) electronically by MultiSIM12; where we show a circuit diagram of the chaotic system (1.1) in **Figure 4**.

The phase portraits of electronic circuit and the outputs voltages signals V_x, V_y, V_z, V_w versus time are presented in **Figure 5**.

From comparing **Figure 5** which was obtained from MultiSIM 12 with **Figure 2** and **Figure 3** which was obtained from MATLAB, we note that between experimental achievements and numerical simulation there is a good qualitative agreement.

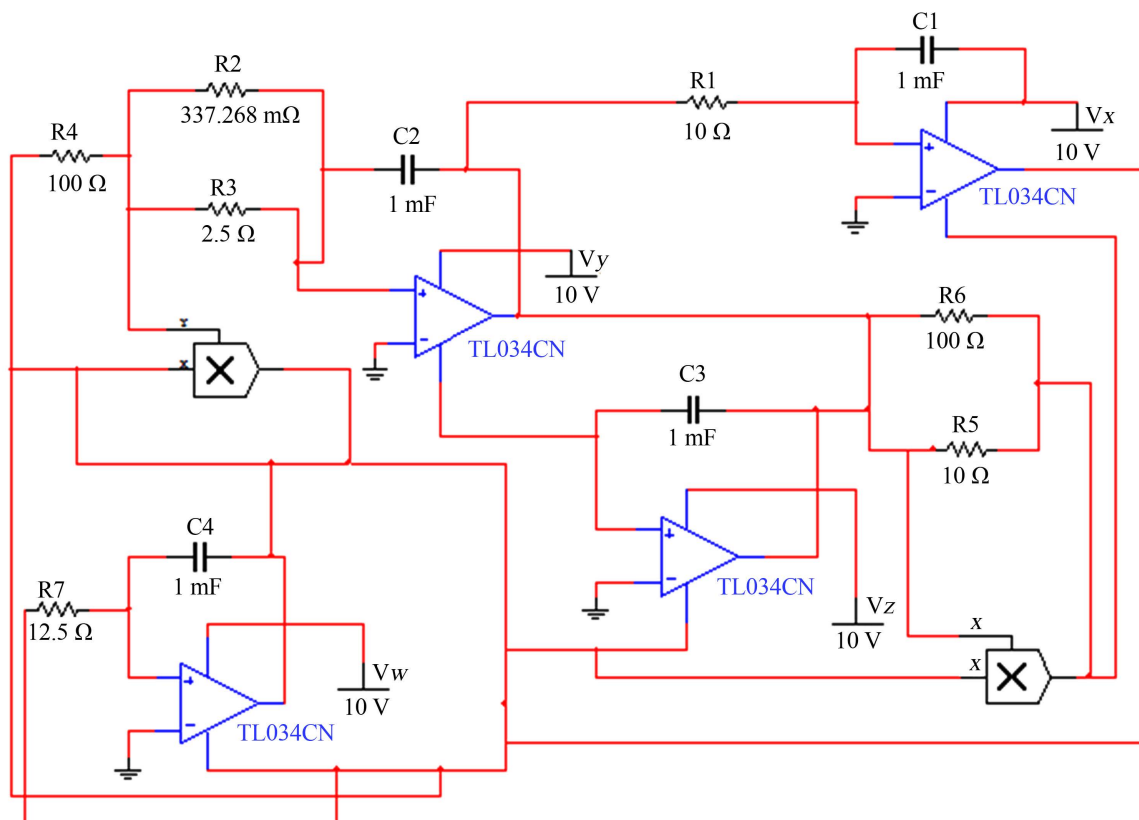
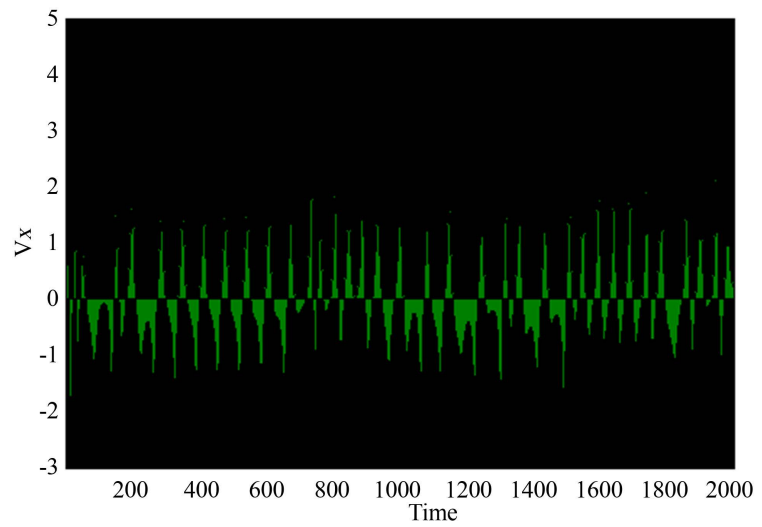
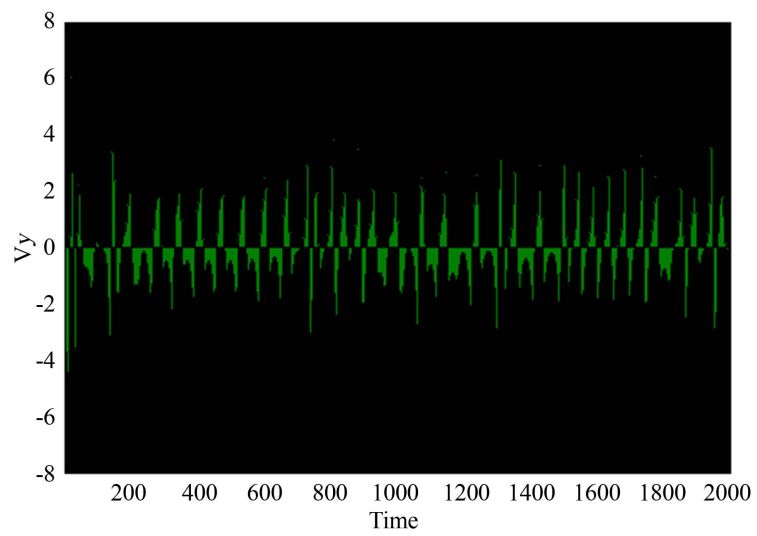


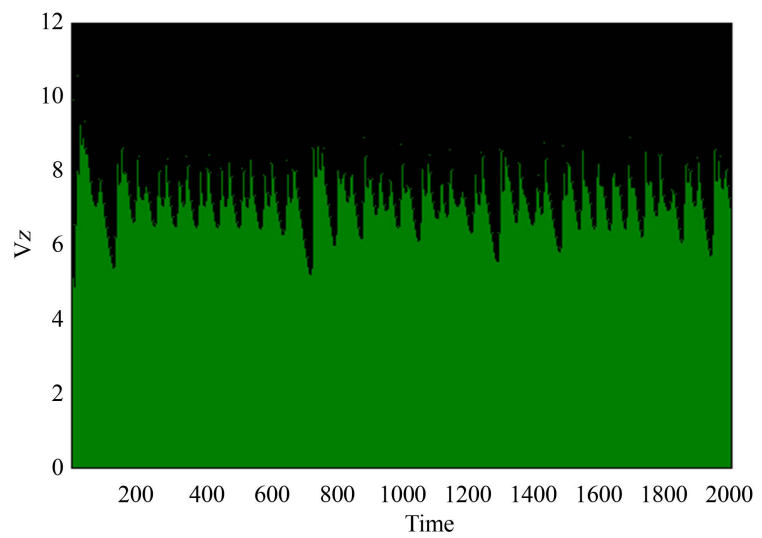
Figure 4. Diagram of the electronic circuit designed for chaotic system (1.1).



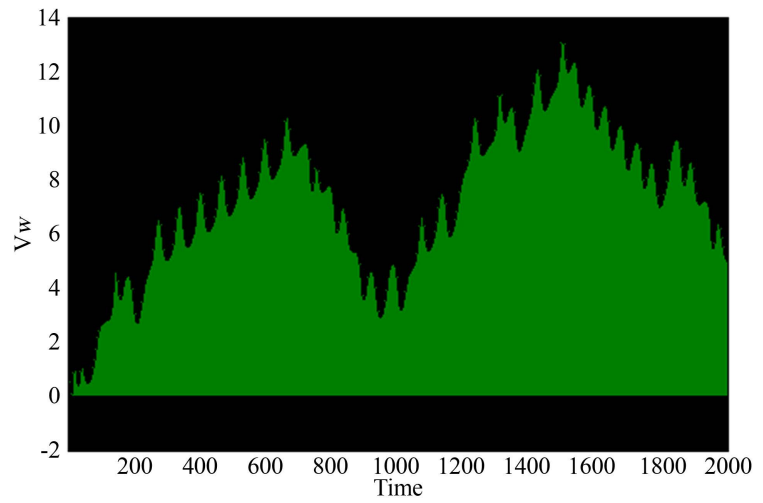
(a)



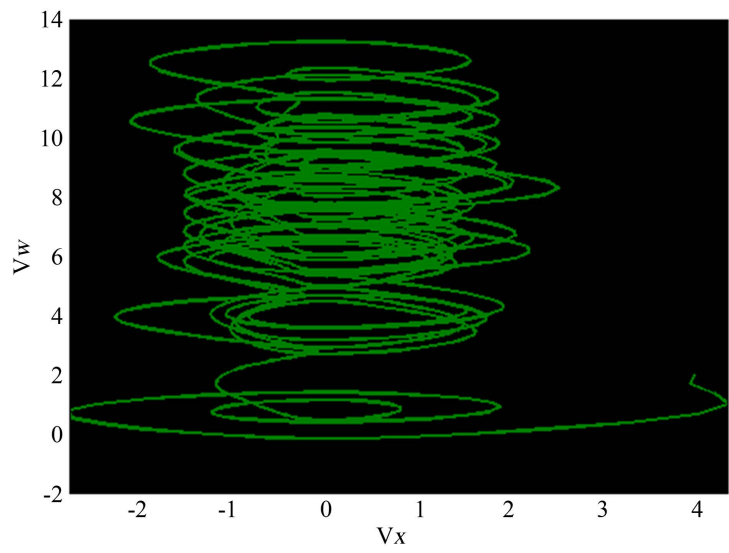
(b)



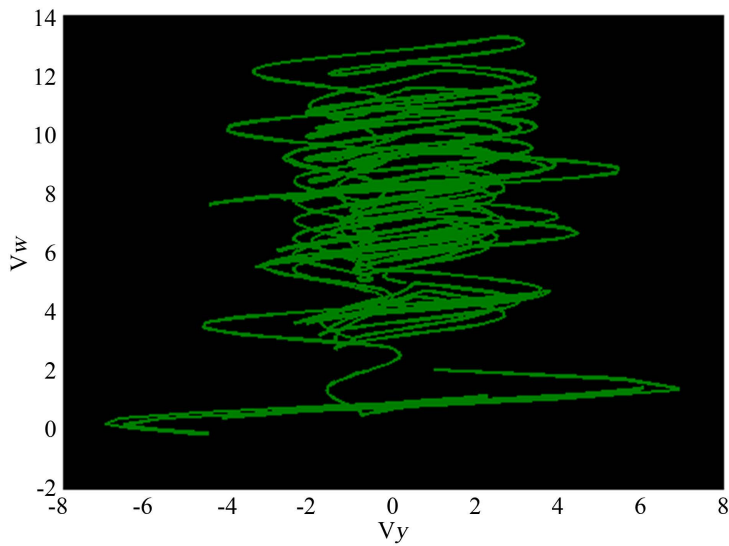
(c)



(d)



(e)



(f)

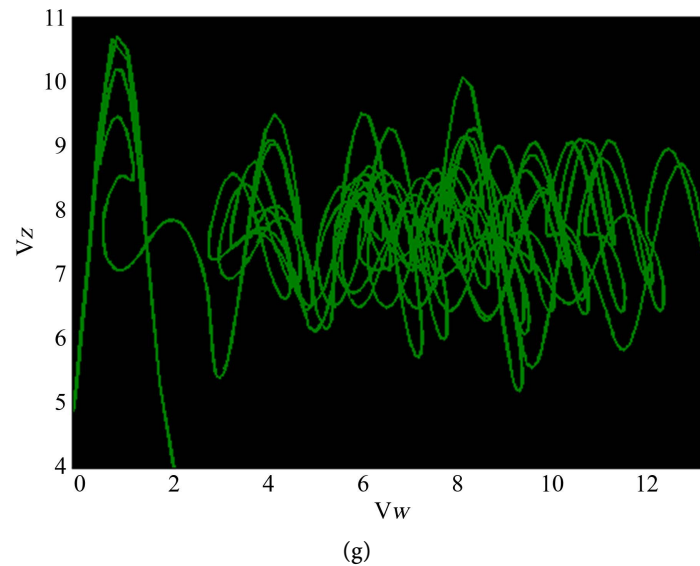


Figure 5. The right-hand side shows phase portraits of the proposed electronic circuit in (e) (V_x, V_w) plane, (f) (V_y, V_w) plane, (g) (V_w, V_z) plane; While The left-hand side shows the output voltage signals as seen in (a) (V_x) versus time, (b) (V_y) versus time, (c) (V_z) versus time, (d) (V_w) versus time).

5. Conclusion

In this paper, the variety chaotic motions of a four-dimensional dynamical system with hidden attractor are investigated through Lyapunov exponents, waveform and phase portraits. The Lyapunov exponents are determined as $(L_1 = 1.660748, L_2 = 0.149599, L_3 = -0.068474$ and $L_4 = -12.144118)$. Also, Kaplan-Yorke dimension is calculated as $D_L = 3.143433471$. Then, an electronic circuit is proposed to implement a chaotic system (1.1). The electronic circuit is designed by applying Kirchhoff's laws, Voltages and time normalized well by dimensionless states variables. The designed circuit is simulated by the MultiSIM12 program; after observing the results, it becomes clear that the numerical results obtained from MATLAB are well in agreement with the experimental results obtained from the Multisim12 program, meaning that the simulation was done well.

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Conflicts of Interest

The authors declare no conflicts of interest.

References

- [1] Robbins, M.F. (2017) Ultimate Electronics: Practical Circuit Design and Analysis.

- CircuitLab, Inc. <https://www.circuitlab.com/>
- [2] Aziz, M.M. and Hamid, M.A. (2019) The Possibility of Increasing the Predictability Indices after Control of 3D-Continuous-Time System. 2019 *International Conference on Computing and Information Science and Technology and Their Applications*, Kirkuk, 3-5 March 2019, 1-5.
 - [3] Aziz, M.M. and Merie, D.M (2020) Stability and Adaptive Control with Synchronization of 3-D Dynamical System. *Open Access Library Journal*, **7**, 1-18. <https://doi.org/10.4236/oalib.1106075>
 - [4] Aziz, M.M. and Al-Nuaimi, Z.A. (2013) Stability and Hop-Bifurcation for Diabetes Model. *International Journal of Electronics Communication and Computer Engineering*, **4**, 805-809.
 - [5] Aziz, M.M. and Faraj, M. (2012) Numerical and Chaotic Analysis of Chua's Circuit. *Journal of Emerging Trends in Computing and Information Science*, **3**, 783-791.
 - [6] Aziz, M.M. (2018) Stability Analysis of Mathematical Model. *International Journal of Science and Research (IJSR)*, **7**, 147-148.
 - [7] Salam, Md.A. and Rahman, Q.M. (2018) Fundamentals of Electrical Circuit Analysis. Springer Nature Singapore Private Ltd., Singapore.
 - [8] Aziz, M.M. (2021) Applied Mathematical Model for a Heat Transfer. *Periodical Engineering and Natural Science*, **9**, 940-945.
 - [9] Aziz, M.M. and Merie, D.M. (2020) Stability and Chaos with Mathematical Control of 4-D Dynamical System. *Indonesian Journal of Electrical Engineering and Computer Science*, **20**, No. 3. <https://doi.org/10.11591/ijeecs.v20.i3.pp1242-1251>
 - [10] Azar, A.T. and Vaidyanathan, S. (2016) *Advances in Chaos Theory and Intelligent Control*. Springer, Berlin. <https://doi.org/10.1007/978-3-319-30340-6>