Modified Newton's Gravitational Theory to Explain Mercury Precession and Light Deflection

Keli Kou

Beijing, China

Email: kelikou@163.com

Abstract

By introducing relative speed of two objects into Newton's gravitational theory, Newton's gravitational formula can be modified as: $F = GMm/r^2(1 + V^2/C^2)$. V is the relative speed between the two objects and C is the light speed. In order to test this formula, the paper uses it to calculate the Perihelion Precession of Mercury and light deflection near the Sun. The calculation result shows that Modified Newton's Gravity theory and General Relative theory have same accuracy to calculate planets orbit and light deflection. It also shows that the gravitational force between two objectives with high relative speed is a bit bigger than that with low relative speed. When this speed is up to light speed, the gravitational force is two times bigger than that with zero speed.

Subject Areas

Classical Mechanics, Classical Physics, Mechanics, Modern Physics, Theoretical Physics

Keywords

MOND Theories of Gravity, Mercury Precession, Light Deflection

1. Introduction

From Newton's gravitational force, the planets orbit can be calculated as an oval orbit [1]. But the Perihelion Precession of Mercury and light deflection near the Sun are the two examples which can't be explained from Newton's theory. The main reason is that Newton's theory did not consider objective relative speed in formula of Newton's gravitational force. This article tries to modify Newton's

gravitational formula by introducing object's relative speed into Newton's gravitational formula. The modified Newton's gravitational formula will be tested from the two experiments' observation, including the Perihelion Precession of Mercury and light deflection near the Sun. The following calculation approved that the modified Newton's gravitational formula can well explain the Perihelion Precession of Mercury and light deflection near the Sun, with same result from the calculation of general relativity theory.

2. Test the Modified Newton's Gravitational Theory from the Perihelion Precession of Mercury

2.1. To Calculate the Planet''s Orbit from Newton's Gravitational Theory

From Newton's dynamic theory, planets orbit can be calculated from the below formula [1] [2]:

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{L}{r^2} \tag{2.1}$$

$$\frac{1}{2}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 = E + \frac{GM}{r} - \frac{L^2}{2r^2} \tag{2.2}$$

Formula (2.1) shows the planet angular momentum conservation, and formula (2.2) shows the planet energy conservation. L is the angular momentum and E is the energy. r is the distance from planet to the center of polar coordinates. The φ is the angle of the planet in polar coordinates. To cancel t, then we have the planet orbit formula:

$$\frac{1}{2}\left[\frac{\mathrm{d}}{\mathrm{d}\varphi}\left(\frac{1}{r}\right)\right]^2 = \frac{E}{L^2} - \frac{1}{2r^2} + \frac{GM}{rL^2} \tag{2.3}$$

To derivative the above formula at both sides, then we have:

$$\frac{\mathrm{d}^2}{\mathrm{d}\varphi^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{GM}{L^2} \tag{2.4}$$

To replace the variable as below:

$$u = \frac{GM}{r}$$

Formula (2.3) can be modified as:

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\varphi^2} + u = \left(\frac{GM}{L}\right)^2 \tag{2.5}$$

This equation is from Newton's theory to describe the planet orbit [1] [2]. The solution of the equation is:

$$u = \left(\frac{GM}{L}\right)^2 (1 + e\cos\varphi) \tag{2.6}$$

$$r = \frac{L^2}{GM} * \frac{1}{1 + e\cos\varphi} \tag{2.7}$$

The above formula shows the planet orbit is a closed oval, according to Newton's theory.

But from the experiments' observation, the planet orbit is not a closed oval. At the perihelion, there is precession, especially for Mercury. Every 100 years, there is 43″ precession for Mercury, which can't be explained by Newton's theory.

This article will use the modified Newton's theory to explain this precession of 43″.

2.2. To Calculate the Perihelion Precession of Mercury from the Modified Newton's Gravitational Formula

The modified Newton's gravitational formula is as below [3] [4]:

$$F = \frac{GMm}{r^2}\left(1 + \frac{V^2}{C^2}\right) \tag{2.8}$$

F is the gravitational force between sun and planet; M is the mass of the sun; m is the mass of planet; r is the distance between sun and planet; V is the planet speed relative to the sun.

Based on Newton's dynamic theory and modified Newton's gravitational formula, the plant orbit comes from the two equations as below.

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{L}{r^2} \tag{2.9}$$

$$\frac{1}{2}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 = E + \frac{GM}{r}\left(1 + \frac{V^2}{C^2}\right) - \frac{L^2}{2r^2} \tag{2.10}$$

L is the angular momentum; E is the energy; V is the mercury speed to the sun.

According to dynamic theory, the planet speed can be resolved as speed of tangential direction and radial direction.

$$V^2 = V_{\text{tangential}}^2 + V_{\text{radial}}^2 = \left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 + \left(r\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right)^2 = \left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 + \left(\frac{L}{r}\right)^2$$

From the above equation, formula (2.10) can be written as:

$$\frac{1}{2}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 = E + \frac{GM}{r} + \frac{GM}{rC^2}\left[\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 + \left(\frac{L}{r}\right)^2\right] - \frac{L^2}{2r^2}$$

$$\left(\frac{1}{2} - \frac{GM}{rC^2}\right)\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 = E + \frac{GM}{r} + \frac{GM}{rC^2}\frac{L^2}{r^2} - \frac{L^2}{2r^2} \tag{2.11}$$

In formula (2.11), $\frac{1}{2} \gg \frac{GM}{rC^2}$, hence the part $\frac{GM}{rC^2}$ can be ignored and be deleted.

Then formula (2.11) can be simplified and be written as:

$$\frac{1}{2}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 = E + \frac{GM}{r} + \frac{GM}{r}\frac{L^2}{(rC)^2} - \frac{L^2}{2r^2} \tag{2.12}$$

To combine formula (2.9) and (2.12), and delete t in formula, then we have planet orbit equation:

$$\frac{1}{2}\frac{L^2\left(\mathrm{d}r\right)^2}{r^4\left(\mathrm{d}\varphi\right)} = E - \frac{L^2}{2r^2} + \frac{GM}{r} + \frac{GM}{r^3}\frac{L^2}{C^2} \tag{2.13}$$

To derivative the Equation (2.13) at both sides, then we have:

$$\frac{L^2}{2}\frac{\mathrm{d}^2}{\mathrm{d}\varphi^2}\left(\frac{1}{r}\right) + \frac{L^2}{r} = GM + \frac{3GML^2}{r^2C^2}$$

To make replacement as below:

$$u = \frac{GM}{r}$$

The above equation can be written as:

$$\frac{\mathrm{d}^2u}{\mathrm{d}\varphi^2} + u = \left(\frac{GM}{L}\right)^2 + \frac{3u^2}{c^2} \tag{2.14}$$

This equation is same as the equation from general relative theory [5] [6].

According to Newton's theory, the planet orbit should be as below equation [5] [6]:

$$\frac{\mathrm{d}^2u}{\mathrm{d}\varphi^2} + u = \left(\frac{GM}{L}\right)^2 \tag{2.5}$$

The resolution of Equation (2.5) is:

$$u = \left(\frac{GM}{L}\right)^2\left(1 + e\cos\theta\right) \tag{2.6}$$

$$r = \frac{L^2}{GM} * \frac{1}{1 + e\cos\theta} \tag{2.7}$$

Obviously, it is a closed oval orbit. And e is the eccentricity of the orbit.

According to modified Newton's gravitational formula (2.14), it shows that Mercury orbit is not a closed oval. Compared with Equation (2.5), it has an additional part $3u^2/C^2$. This additional part is from the modification for Newton's Theory. For sun, there is $GM = 1.5 \times 10^3$ m. The radial of the mercury is $r = 5 \times 10^{10}$ m. The order of the u is:

$$u = \frac{GM}{r} \sim 10^{-7}$$

From Equation (2.6), we can find $\left(\frac{GM}{L}\right)^2$ and u has the save order:

$$\left(\frac{GM}{L}\right)^2 \sim 10^{-7}$$

From the above discussion, the additional part of the equation has much lower order:

$$\frac{3u^2}{C^2} \sim 10^{-30}$$

The additional part is so small, that we can treat resolution of Equation (2.6) as the zero order resolution of Equation (2.14).

To change the nonlinear Equation (2.14) to linear equation, we take zero order solution into Equation (2.14). Then we have the equation as below:

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\varphi^2} + u = \left(\frac{GM}{L}\right)^2 + \frac{3}{c^2}\left[\left(\frac{GM}{L}\right)^2 (1 + e\cos\varphi)\right]^2$$

The above equation can be written as:

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\varphi^2} + u = \left(\frac{GM}{L}\right)^2 + \frac{3}{c^2}\left(\frac{GM}{L}\right)^4 + \frac{6}{c^2}\left(\frac{GM}{L}\right)^4 e\cos\varphi$$
$$+ \frac{3}{C^2}\left(\frac{GM}{L}\right)^4 (e\cos\varphi)^2 \tag{2.15}$$

As the eccentricity of Mercury orbit is very small, $e^2 \ll 1$.

To compare the two constants, we have: $\left(\frac{GM}{L}\right)^2 \gg \frac{3}{C^2}\left(\frac{GM}{L}\right)^4$.

To ignore the small constant,

$$\frac{3}{C^2}\left(\frac{GM}{L}\right)^4 \quad \text{and} \quad \frac{3}{C^2}\left(\frac{GM}{L}\right)^4 (e\cos\varphi)^2$$

Equation (2.15) can be written as:

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\varphi^2} + u \approx \left(\frac{GM}{L}\right)^2 + \frac{6}{C^2}\left(\frac{GM}{L}\right)^4 e\cos\varphi \tag{2.16}$$

We separate the solution of the linear equation as two parts: $u = u_1 + u_2$.

And take it into Equation (2.16). Then there comes:

$$\frac{\mathrm{d}^2 u_1}{\mathrm{d}\varphi^2} + u_1 = \left(\frac{GM}{L}\right)^2 \tag{2.17}$$

$$\frac{\mathrm{d}^2 u_2}{\mathrm{d}\varphi^2} + u_2 = \frac{6}{C^2}\left(\frac{GM}{L}\right)^4 e\cos\varphi \tag{2.18}$$

The solution of Equation (2.17) is:

$$u_1 = \left(\frac{GM}{L}\right)^2 (1 + e\cos\theta)$$

The solution of Equation (2.18) is:

$$u_2 = \frac{3}{C^2}\left(\frac{GM}{L}\right)^4 e\varphi\sin\varphi$$

Hence the solution of Equation (2.16) can be written as:

$$u = \left(\frac{GM}{L}\right)^2 \left[1 + e\cos\varphi + \frac{3}{C^2}\left(\frac{GM}{L}\right)^2 e\varphi\sin\varphi\right] \tag{2.19}$$

By use the auxiliary angle formula as below:

$$a\sin x + b\cos x = \sqrt{a^2 + b^2}\cos(x - \theta)$$

$$\theta = \arctan\frac{a}{b} \approx \frac{a}{b}$$

To ignore part $\left(\dfrac{GM}{L}\right)^4$, Equation (2.19) can be written as:

$$u \approx \left(\frac{GM}{L}\right)^2 \left\{1 + e\cos\left[1 - 3\left(\frac{GM}{cL}\right)^2\right]\varphi\right\} \tag{2.20}$$

(2.20) is the solution of the planet orbit equation, which is conducted from the modified Newton's theory.

From Equation (2.20), we can see that after running one turning $\varphi = 2\pi$, the planet can't go back to the base point. It will turn a small angle. The reason is that the cycle T of the orbit is no longer $\varphi = 2\pi$. It is:

$$T = \frac{2\pi}{1 - \dfrac{3}{C^2}\left(\dfrac{GM}{L}\right)^2}$$

For the continuous running, at each point, the planet will turn an angle φ_n to return the base point:

$$\varphi_n = \frac{2n\pi}{1 - \dfrac{3}{C^2}\left(\dfrac{GM}{L}\right)^2} \approx 2n\pi\left[1 + \frac{3}{C^2}\left(\frac{GM}{L}\right)^2\right] \tag{2.21}$$

It means, the orbit will have precession. For the orbit with precession, the meaning of base point is not clear. And this precession is not easy to be observed. It can only be observed at perihelion point. From Equation (2.21), we have

$$\Delta\varphi_n = \left(\varphi_{n+1} - \varphi_n\right) - 2\pi = 6\pi\left(\frac{GM}{CL}\right)^2 = 6\pi\left(\frac{GM}{C*r*V}\right)^2 \tag{2.22}$$

The distance from Mercury to the Sun is: $r = 4.6 \times 10^{10}$ m; The Mercury speed at perihelion is: $V = 5.8976 \times 10^5$ m; the gravitational constant is: $G = 6.67 \times 10^{-11}$ N·m²/kg²; the light speed is: $C = 2.9979 \times 10^8$ m/s. Take above date into (2.22), then we have:

$$\Delta\varphi = 0.1''$$

It means, for every turn, the orbit will have 0.1" as precession angle. In 100 years, the total precession angle is:

$$\sum\Delta\varphi \approx 43''$$

This result is same as the observation result, and is same as the result calculated from General Relativity Theory [5] [6].

3. To Calculate the Light Deflection Angle

Assume light, like particles, will be attracted by the sun and will be deflected from strait line. According to the modified Newton's theory, the orbit equation of light can be conducted from Equation (2.9), (2.10).

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{L}{r^2} \tag{2.9}$$

$$\frac{1}{2}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 = E + \frac{GM}{r}\left(1+\frac{V^2}{C^2}\right) - \frac{L^2}{2r^2} \tag{2.10}$$

Take light speed C into Equation (2.10), then there is:

$$\frac{1}{2}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 = E + \frac{2GM}{r} - \frac{L^2}{2r^2} \tag{3.1}$$

Combine Equation (2.9) and (3.1) and take $\mathrm{d}r = -r^2\mathrm{d}\left(\frac{1}{r}\right)$ into (3.1) then there is:

$$\frac{L^2}{2}\left[\frac{\mathrm{d}\left(\frac{1}{r}\right)}{\mathrm{d}\varphi}\right]^2 = E - \frac{L^2}{2r^2} + \frac{2GM}{r} \tag{3.2}$$

To derivative the above formula at both sides, then we have

$$L^2\frac{\mathrm{d}^2\left(\frac{1}{r}\right)}{\mathrm{d}\varphi^2} + \frac{L^2}{r} = 2GM \tag{3.3}$$

To replace the variable as below:

$$u = \frac{GM}{r}$$

Equation (3.3) can be simplified as:

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\varphi^2} + u = 2\left(\frac{GM}{L}\right)^2$$

$L = RC$. R is the radial of the sun; C is the light speed.
To replace the variable again as below:

$$\tilde{u} = \frac{1}{r} \quad \text{and} \quad u = \frac{GM}{r} = GM\tilde{u}$$

$$\frac{\mathrm{d}^2\tilde{u}}{\mathrm{d}\varphi^2} + \tilde{u} = \frac{2GM}{L^2} = \frac{2GM}{(RC)^2} \tag{3.4}$$

To ignore the small quantity $\dfrac{2GM}{(RC)^2}$.

The approximate solution for Equation (3.4) is a strait line.

$$\tilde{u}_0 = R^{-1}\cos\varphi \tag{3.5}$$

The solution of Equation (3.4) is a line with deflection:

$$\tilde{u} = \tilde{u}_0 + \frac{2GM}{(RC)^2}$$

$$\tilde{u} = R^{-1}\cos\varphi + \frac{2GM}{(RC)^2} \tag{3.6}$$

Strait line (3.5) can be extended to far away.

$$r \to \infty; u \to 0$$

The azimuth angle of strait line is $\left(\pm \dfrac{\pi}{2} \right)$.

At infinite point ($u \to 0$), let line (3.6) have azimuth angle as:

$$\pm \left(\frac{\pi}{2} + \theta \right)$$

Take it into Equation (3.6)

$$0 = R^{-1} \cos \left[\pm \left(\frac{\pi}{2} + \theta \right) \right] + \frac{2GM}{C^2 R^2}$$

Then we can calculate the difference of azimuth angle

$$\theta \approx \sin \theta = \frac{2GM}{C^2 R}$$

The light deflection angle is

$$\Delta \varphi = 2\theta \approx \frac{4GM}{C^2 R} \tag{3.7}$$

Take the mass of the sun $M = 1.988 \times 10^{30}$ kg, and radial $R = 6.955 \times 10^8$ m into Equation (3.7), then we have:

$$\Delta \varphi = 1.75''$$

This result is same as observation data, and also same as the result from general relativity theory [5] [6].

4. The Gravitational Force for the Two Objectives with Zero Related Speed

According to Equation (2.8)

$$F = \frac{GMm}{r^2} \left(1 + \frac{V^2}{C^2} \right) \tag{2.8}$$

When the related speed of the two objectives is $V = 0$, this equation will be same as Newton's gravitational force theory.

$$F = \frac{GMm}{r^2} \left(1 + \frac{0^2}{C^2} \right) = \frac{GMm}{r^2}$$

5. Conclusions

According to the above 3 examples, we tested the modified Newton's theory for 3 different relative speed, *i.e.* for objectives with relative zero speed, with light speed and with low speed (Mercury running orbit).

When introducing speed V into Newton's gravitational theory, we can conduct the same orbit equation for mercury orbit and light deflection. And the modified Newton's theory will become same as Newton's gravitational theory, when the objectives speed is zero. These results are also the same as the results calculated from the general relativity theory.

It is concluded that Modified Newton's Gravity theory and General Relative theory have the same accuracy to calculate planets orbit and light deflection.

Conflicts of Interest

The author declares no conflicts of interest.

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