



On Study of Some Concepts in Nano Continuity via Graph Theory

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Abstract

The purpose of this paper is to introduce a nano-topological space via graph theory which depends on a neighborhood between the vertices based on undirected graph with example. The Concept of Continuity was generalized via graph in order to obtain more of characterization which is application in the graph, we also introduced the concept of nano-dense via graph theory with an example and theorem. At last the concept of homeomorphism was introduced with some examples and theorem.

Subject Areas

Computer Graphics and Visualization

Keywords

Nano-Topology Graph, Nano-Continuity Graph, Nano-Homeomorphism Graph, Nano-Dense Graph

1. Introduction

L. Thivagar [1] introduced the concept of nano topological spaces which was defined in terms of approximations and boundary region of a subset of a universe using an equivalence relation on it and also defined nano closed sets, nano interior and nano closure. L. Thivagar [2] introduced a new class of functions on nano-topological spaces called nano continuous functions and derived their characterizations in terms of nano closed sets, nano closure and nano-interior. Atef [3] and Ibrahim [4] studied the relationship between topology and the graph. The two scientists Thivagar [5] and Abd El-Fattah [6] also studied nano-topological via graph theory, where he studied neighborhood between the vertices based on directed graph. In our paper we presented a definition of the

nano-topological via graph theory, where he studied is neighborhood between the vertices and we studied in our paper an undirected graph theory, also, new class nano continuous function via graph theory.

2. Preliminaries

Definition 2.1: [7] [8]

“A graph G is a pair (V, E) , where V is nonempty set called vertices or nodes and E is 2-element subsets of V called Edges or links”.

Definition 2.2: [7] [8]

“Let $G = (V, E)$ be a graph; we call H a subgraph of G if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$, in which case we write $H \subseteq G$ ”. A simple graph G in which each pair of distinct vertices is a complete graph we denote the complete graph on vertices by Kn and has $n(n-1)/2$ edges.

Definition 2.3: [7] [8]

Two graphs G_1 and G_2 are isomorphic if there is a one-one correspondence between the vertices of G_1 and those of G_2 such that the number of edges joining any two vertices of G_1 is equal to the number of edges joining the corresponding vertices of G_2 .

Definition 2.4: [1] [9]

“Let U be a non empty finite set of objects called the universe and R be an equivalence relations on U named as indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one other. The pair (U, R) is said to be approximation space. Let $X \subseteq U$.

1) The lower approximation of X with respect to R is the set of all objects, which can be certain classified as X with respect to R and is defined by $L_R[X] = \bigcup \{R(x) : R(x) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by X .

2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and is defined by

$$U_R[V(H)] = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}.$$

3) The boundary region of X with respect to R is the set of all objects, which can be classified as X neither as X nor as not- X with respect to R and is defined by $B_R(x) = U_R(x) - L_R(x)$ ”.

Definition 2.5: [1] [9]

“Let U be the universe and R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(x), U_R(x), B_R(x)\}$. Where $X \subseteq U$, then $\tau_R(X)$ satisfies the following axioms:

- 1) U and $\emptyset \in \tau_R(X)$.
- 2) The union of elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
- 3) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ forms a topology on U called as the nano topology on U with respect to X . We call $(U, \tau_R(X))$ as the nano-topological space. The elements

of $\tau_R(X)$ are called as Nano open set”.

Definition 2.6: [1] [9]

“If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $A \subseteq U$, then the nano interior of A is defined as the union of all nano open subsets of A and it is denoted by $NInt(A)$. That is, $NInt(A)$ is the largest nano open subset of A . The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by $NCl(A)$. That is, $NCl(A)$ is the smallest nano closed set containing A ”.

Definition 2.7: [5]

Let $G = (V, E)$ be a graph, $v \in V(G)$. Then the neighborhood of v and denote that $N(v)$ were defined by $N(v) = \{v\} \cup \{u \in V(G) : \overline{vu} \in E(G)\}$.

Definition 2.8: [5]

Let $G = (V, E)$ be a graph, H be a subgraph from G , $N(v)$ is neighborhood of v in $V(G)$; Then

1) The lower approximation $L : P(V(G)) \rightarrow P(V(G))$ is

$$L_N[V(H)] = \bigcup_{v \in V(G)} \{v : N(v) \subseteq V(H)\}.$$

2) The upper approximation $U : P(V(G)) \rightarrow P(V(G))$ is

$$U_N[V(H)] = \bigcup_{v \in V(G)} \{N(v) : N(v) \cap V(H) \neq \emptyset\}.$$

3) The boundary region is $B_N[V(H)] = U_N[V(H)] - L_N[V(H)]$.

3. Nano Topological Space via Graph Theory

In this section we have studied the concept of nano topological space via graph theory with example.

Definition 3.1:

“Let $G = (V, E)$, $N(v)$ is neighborhood of v in V , H be a subgraph from G $\tau_N[V(H)] = \{V(G), \emptyset, L_N[V(H)], U_N[V(H)], B_N[V(H)]\}$. Forms a topology on $V(G)$ called nano-topology on $V(G)$ with respect to $V(H)$. $(V(G), \tau_N[V(H)])$ is called nano-topological graph”.

Example 3.2:

“Let $G = (V, E)$ be a graph (see **Figure 1**). Then, $N(v_1) = \{v_1, v_4\}$, $N(v_2) = \{v_2, v_3\}$, $N(v_3) = \{v_2, v_3, v_4\}$, $N(v_4) = \{v_1, v_3, v_4\}$. H is a subgraph with vertices $V(H) = \{v_2, v_3\}$, then $L_N[V(H)] = \{v_1\}$, $U_N[V(H)] = \{v_1, v_3, v_4\}$, $B_N[V(H)] = \{v_3, v_4\}$. Therefore, the nano-topology from G will be $\tau_N[V(H)] = \{V(G), \emptyset, \{v_2\}, \{v_2, v_3, v_4\}, \{v_3, v_4\}\}$.”

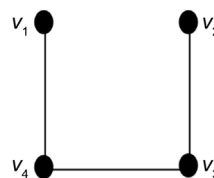


Figure 1. Simple graph (nano-topology test of connected graph without cycle).

4. Nano Continuity via Graph Theory

In this section we studied the concept nano continuity via graph theory with some characterization, examples and proofs.

Definition 4.1:

Let $G = (V, E)$ and $G_1 = (V_1, E_1)$ be any two isomorphic graphs with nano topological graph $(V(G), \tau_N(V(H))), (V(G_1), \tau_N(f(V(H))))$. Then the mapping $f : (V(G), \tau_N(V(H))) \rightarrow (V(G_1), \tau_N(f(V(H))))$ is called nano-continuous on $V(G_1)$ if the inverse image from each nano-open set in $V(G)$ is nano-open in $V(G_1)$.

Example 4.2:

Let $G_1 = (V_1, E_1), G_2 = (V_2, E_2)$ be two isomorphic graphs (see **Figure 2**). Then there exists a function $f : V(G_1) \rightarrow V(G_2)$ as $f(v_1) = u_2, f(v_2) = u_3, f(v_3) = u_4, f(v_4) = u_1$. We a construct nano-topology on G_1 . Assume that H is a subgraph from G_1 with vertices $V(H) = \{v_1, v_2, v_4\}$.

Then $N(v_1) = \{v_1, v_2, v_4\}, N(v_2) = V(G), N(v_3) = \{v_2, v_3, v_4\}, N(v_4) = V(G)$. Since, $L_N[V(H)] = \{v_1\}, U_N[V(H)] = V(G), B_N[V(H)] = \{v_2, v_3, v_4\}$. Then the nano-topological graph from $V(H)$ are $\tau_N(V(H)) = \{V(G_1), \emptyset, \{v_1\}, \{v_2, v_3, v_4\}\}$.

We a construct nano-topology on G_2 generated by $f(V(H)) = \{u_1, u_2, u_3\}$.

Assume that W is a subgraph from G_2 with vertices $V(W) = \{u_2, u_4\}$. Then, $N(u_1) = V(G), N(u_2) = \{u_1, u_2, u_3\}, N(u_3) = V(G), N(u_4) = \{u_1, u_3, u_4\}$. Since, $L_N[V(H)] = \{u_2\}, U_N[V(H)] = V(G), B_N[V(H)] = \{u_1, u_3, u_4\}$. Then, the nano-topological graph from $f(V(H))$ are

$\tau_N(f(V(H))) = \{V(G_2), \emptyset, \{u_2\}, \{u_1, u_3, u_4\}\}$. Then, $f^{-1}\{u_1, u_3, u_4\} = \{v_2, v_3, v_4\}, f^{-1}\{u_2\} = \{v_1\}$ Therefore, f is nano continuous.

Theorem 4.3:

“Let $G = (V, E)$ and $G_1 = (V_1, E_1)$ be any two isomorphic graphs with nano topological graph $(V(G), \tau_N(V(H))), (V(G_1), \tau_N(f(V(H))))$.”

A function $f : (V(G), \tau_N(V(H))) \rightarrow (V(G_1), \tau_N(f(V(H))))$ is nano continuous if and only if the inverse image of every nano closed set in $V(G_1)$ is nano closed in $V(G)$ ”.

Proof:

“Let f be nano continuous and $V(F)$ be nano closed in $V(G_1)$. That is, $V(G_1) - V(F)$ is nano-open in $V(G_1)$. Since f is nano continuous, $f^{-1}(V(G_1) - V(F))$ is nano-open in $V(G)$. That is, $V(G) - f^{-1}(V(F))$ is

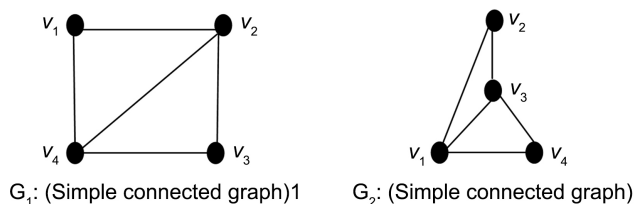


Figure 2. Two isomorphic graphs (the continuity test of nano topological space via graph theory).

nano-open in $V(G)$. Therefore, $f^{-1}(V(F))$ is nano closed in $V(G)$. Thus, the inverse image of every nano closed set in $V(G_1)$ is nano closed in, if f is nano continuous on $V(G)$. Conversely, let the inverse image of every nano closed set be nano closed. Let $V(H)$ be nano-open in $V(G_1)$. Then $V(G_1) - V(H)$ is nano closed in $V(G_1)$. Then, $f^{-1}(V(G_1) - V(H))$ is nano closed in $V(G)$. That is, $V(G) - f^{-1}(V(H))$ is nano closed in $V(G)$. Therefore, $f^{-1}(V(H))$ is nano-open in $V(G)$. Thus, the inverse image of every nano-open set in $V(G_1)$ is nano-open in $V(G)$. That is, f is nano continuous on $V(G)$ ”.

Theorem 4.4:

Let $G = (V, E)$ and $G_1 = (V_1, E_1)$ be any two isomorphic graphs with nano topological graph $(V(G), \tau_N(V(H))), (V(G_1), \tau_N(f(V(H))))$. A function $f : (V(G), \tau_N(V(H))) \rightarrow (V(G_1), \tau_N(f(V(H))))$ is nano continuous if and only if $f(NCl_G(V(H))) \subseteq NCl_{G_1}(f(V(H)))$ for every subgraph H of G .

Proof:

“Let f be nano continuous and $V(H) \subseteq V(G)$. Then $f(V(H)) \subseteq V(G_1)$. $NCl_{G_1}(f(V(H)))$ is nano closed in $V(G_1)$. Since f is nano continuous, $f^{-1}(NCl_{G_1}(f(V(H))))$ is nano closed in $V(G)$. Since $f(V(H)) \subseteq NCl_{G_1}(f(V(H)))$, $V(H) \subseteq f^{-1}(NCl_{G_1}(f(V(H))))$. Thus $f^{-1}(NCl_{G_1}(f(V(H))))$ is a nano closed set containing $V(H)$. But, $NCl_G(V(H))$ is the smallest nano closed set containing $V(H)$. Therefore, $NCl_G(V(H)) \subseteq f^{-1}(NCl_{G_1}(f(V(H))))$. That is, $f(NCl_G(V(H))) \subseteq NCl_{G_1}(f(V(H)))$.”

Conversely, let $f(NCl_G(V(H))) \subseteq NCl_{G_1}(f(V(H)))$ for every subgraph $V(H)$ of $V(G)$. If $V(F)$ is nano closed in $V(G_1)$, since $f^{-1}(V(F)) \subseteq V(G)$, $f(NCl_G(f^{-1}(V(F)))) \subseteq NCl_{G_1}(f(f^{-1}(V(F)))) \subseteq NCl_{G_1}(V(F))$.

That is, $NCl_G(f^{-1}(V(F))) \subseteq f^{-1}(NCl_{G_1}(V(F))) = f^{-1}(V(F))$, since $V(F)$ is nano closed. Thus $NCl_G(f^{-1}(V(F))) \subseteq f^{-1}(V(F))$. But, $f^{-1}(V(F)) \subseteq NCl_G(f^{-1}(V(F)))$. Therefore, $NCl_G(f^{-1}(V(F))) = f^{-1}(V(F))$. Therefore, $f^{-1}(V(F))$ is nano closed in $V(G)$ for every nano closed set $V(F)$ in $V(G_1)$. That is, f is nano continuous”.

Theorem 4.5:

“Let $G = (V, E)$ and $G_1 = (V_1, E_1)$ be any two isomorphic graphs with nano topological graph $(V(G), \tau_N(V(H))), (V(G_1), \tau_N(f(V(H))))$. A function $f : (V(G), \tau_N(V(H))) \rightarrow (V(G_1), \tau_N(f(V(H))))$ is nano continuous if and only if $NCl_G(f^{-1}(V(H))) \subseteq f^{-1}(NCl_{G_1}(V(H)))$ for every subgraph H of G ”.

Proof:

“If f is nano continuous and $V(H) \subseteq V(G_1)$, $NCl_{G_1}(V(H))$ is nano closed in $V(G_1)$ and hence $f^{-1}(NCl_{G_1}(V(H)))$ is nano closed in $V(G)$. Therefore, $NCl_G(f^{-1}(NCl_{G_1}(V(H)))) = f^{-1}(NCl_{G_1}(V(H)))$. Since $V(H) \subseteq NCl_G(V(H)) \Rightarrow f^{-1}(V(H)) \subseteq f^{-1}(NCl_G(V(H)))$. Therefore, $NCl_G(f^{-1}(V(H))) \subseteq NCl_G(f^{-1}(NCl_G(V(H)))) = f^{-1}(NCl_G(V(H)))$. That is, $NCl_G(f^{-1}(V(H))) \subseteq f^{-1}(NCl_G(V(H)))$.”

Conversely, assume that $NCl_G(f^{-1}(V(H))) \subseteq f^{-1}(NCl_G(V(H)))$ for every $V(H) \subseteq V(G_1)$. Let $V(H)$ be nano closed in $V(G_1)$. Then $NCl_G(V(H)) = V(H)$. By Assumption, $NCl_G(f^{-1}(V(H))) \subseteq f^{-1}(NCl_G(V(H))) = f^{-1}(V(H))$. Thus, $NCl_G(f^{-1}(V(H))) \subseteq f^{-1}(V(H))$. But, $f^{-1}(V(H)) \subseteq NCl_G(f^{-1}(V(H)))$. Therefore, $NCl_G(f^{-1}(V(H))) = f^{-1}(V(H))$. That is, $f^{-1}(V(H))$ is nano closed in $V(G)$ for every nano closed set $V(H)$ in $V(G_1)$. Therefore, f is nano continuous on $V(G)$.

Theorem 4.6:

Let $G = (V, E)$ and $G_1 = (V_1, E_1)$ be any two isomorphic graphs with nano topological graph $(V(G), \tau_N(V(H))), (V(G_1), \tau_N(f(V(H))))$. A function $f: (V(G), \tau_N(V(H))) \rightarrow (V(G_1), \tau_N(f(V(H))))$ is nano continuous on $V(G)$ if and only if $f^{-1}(NInt_G(V(H))) \subseteq NInt_G(f^{-1}(V(H)))$ for every subgraph H of G_1 .

Proof:

Assume that f is nano continuous and $V(H) \subseteq V(G_1)$, $NInt_G(V(H))$ is nano open in $(V(G_1), \tau_N(f(V(H))))$ and hence $f^{-1}(NInt_G(V(H)))$ is nano open in $(V(G), \tau_N(V(H)))$. Therefore,

$f^{-1}(NInt_G(V(H))) = NInt_G(f^{-1}(NInt_G(V(H))))$. Since $NInt_G(V(H)) \subseteq V(H) \Rightarrow f^{-1}(NInt_G(V(H))) \subseteq f^{-1}(V(H))$. Therefore, $NInt_G(f^{-1}(NInt_G(V(H)))) \subseteq NInt_G(f^{-1}(V(H))) = f^{-1}(NCl_G(V(H)))$. That is $f^{-1}(NInt_G(V(H))) \subseteq NInt_G(f^{-1}(V(H)))$.

Conversely, assume that $f^{-1}(NInt_G(V(H))) \subseteq NInt_G(f^{-1}(V(H)))$ for every $V(H) \subseteq V(G_1)$. If $V(H)$ be nano open in $V(G_1)$. $NInt_G(V(H)) = V(H)$. By assumption, $NInt_G(f^{-1}(V(H))) \subseteq NInt_G(f^{-1}(V(H)))$. That is, $f^{-1}(V(H)) \subseteq NInt_G(f^{-1}(V(H)))$. But, $NInt_G(f^{-1}(V(H))) \subseteq f^{-1}(V(H))$. Therefore, $f^{-1}(V(H)) = NInt_G(f^{-1}(V(H)))$. That is, $f^{-1}(V(H))$ is nano open in $V(G)$ for every nano open set $V(H)$ in $V(G_1)$. Therefore, f is nano continuous.

Definition 4.7:

Let $G = (V, E)$ be a graph with nano-topological graph $(V(G), \tau_N(V(H)))$. If $W \subseteq G$ is said to be nano-dense if $NCl_G(V(W)) = V(G)$.

Example 4.8:

From example 4.2. Assume that H is a subgraph from G_1 with vertices $V(H) = \{v_1, v_2, v_4\}$. Then $N(v_1) = \{v_1, v_2, v_4\}$, $N(v_2) = V(G)$, $N(v_3) = \{v_2, v_3, v_4\}$, $N(v_4) = V(G)$. Since, $C(N(v_1)) = \{v_3\}$, $C(N(v_2)) = \emptyset$, $C(N(v_3)) = \{v_1\}$, $C(N(v_4)) = \emptyset$. Then, $C(N(v_3)) \cap V(H) \neq \emptyset$. Therefore, $NCl_G(V(H)) = U_N[V(H)] \cup \{v_3\} = V(G_1)$. Hence $NCl_G(V(H)) = V(G_1)$.

Theorem 4.9:

Let $G = (V, E)$ and $G_1 = (V_1, E_1)$ be any two isomorphic graphs with nano topological graph $(V(G), \tau_N(V(H))), (V(G_1), \tau_N(f(V(H))))$ and $f: (V(G), \tau_N(V(H))) \rightarrow (V(G_1), \tau_N(f(V(H))))$ be an onto and nano continuous function, $V(H) \subseteq V(G)$. If $V(H)$ is nano dense in $V(G)$, then

$f(V(H))$ is nano dense in $V(G_1)$.

Proof:

“Since $V(H)$ is nano dense in $V(G)$, $NCl_G(V(H)) \subseteq V(G)$. Then $f(NCl_G(V(H))) \subseteq f(V(G)) = V(G_1)$, since f is onto and nano continuous on $V(G)$, $f(NCl_G(V(H))) \subseteq NCl_G(f(V(H)))$. Therefore, $V(G_1) \subseteq NCl_G(V(H))$ but $NCl_G(V(H)) \subseteq V(G_1)$. Therefore, $NCl_G(V(H)) = V(G_1)$. That is $f(V(H))$ is dense in $V(G_1)$. Thus, a nano continuous function maps nano dense sets into nano dense sets, provided it is onto”.

5. Nano Homeomorphism via Graph Theory

In this section we studied of nano homeomorphism in the graph with some examples and proofs.

Definition 5.1:

Let $G = (V, E)$ and $G_1 = (V_1, E_1)$ be any two isomorphic graphs with nano topological graph $(V(G), \tau_N(V(H))), (V(G_1), \tau_N(f(V(H))))$. A function $f : (V(G), \tau_N(V(H))) \rightarrow (V(G_1), \tau_N(f(V(H))))$ is nano-open map if the image of every nano-open set in $V(G)$ is nano open in $V(G_1)$. The mapping f is said to be a nano closed set map if the image of every nano-closed set in $V(G)$ is nano closed in $V(G_1)$.

Theorem 5.2:

Let $G = (V, E)$ and $G_1 = (V_1, E_1)$ be any two isomorphic graphs with nano topological graph $(V(G), \tau_N(V(H))), (V(G_1), \tau_N(f(V(H))))$. A mapping $f : (V(G), \tau_N(V(H))) \rightarrow (V(G_1), \tau_N(f(V(H))))$ is nano closed map if and only if $NCl_G(f(V(H))) \subseteq f(NCl_G(V(H)))$, for every subgraph $H \subseteq G$.

Proof:

If f is nano closed, $f(NCl_G(V(H)))$ is nano closed in $V(G_1)$, since $NCl_G(V(F))$ is nano closed $V(G)$. Then, $V(H) \subseteq NCl_G(V(H))$, $f(V(H)) \subseteq f(NCl_G(V(H)))$. Thus, $f(NCl_G(V(H)))$ is a nano closed set containing $f(V(H))$. Therefore, $NCl_G(f(V(H))) \subseteq f(NCl_G(V(H)))$.

Conversely, if $V(F)$ is nano closed in $V(G)$, then $NCl_G(V(F)) = V(F)$ and hence $f(V(F)) \subseteq NCl_G(f(V(F))) \subseteq f(NCl_G(V(F))) = f(V(F))$. Thus, $V(F) = NCl_G(V(F))$. That is, $f(V(F))$ is nano closed in $V(G_1)$. Therefore, f is nano closed map.

Theorem 5.3:

Let $G = (V, E)$ and $G_1 = (V_1, E_1)$ be any two isomorphic graphs with nano topological graph $(V(G), \tau_N(V(H))), (V(G_1), \tau_N(f(V(H))))$. A mapping $f : (V(G), \tau_N(V(H))) \rightarrow (V(G_1), \tau_N(f(V(H))))$ is nano open map if and only if $f(NInt_G(V(H))) \subseteq NInt_G(f(V(H)))$, for every subgraph $H \subseteq G$.

Proof: It's clear by theorem

Definition 5.4:

Let $G = (V, E)$ and $G_1 = (V_1, E_1)$ be any two isomorphic graphs with nano topological graph $(V(G), \tau_N(V(H)))$. Then the mapping

$f: (V(G), \tau_N(V(H))) \rightarrow (V(G_1), \tau_N(f(V(H))))$ is called nano-homeomorphism if

- 1) f is one to one and onto.
- 2) f is nano-continuous.
- 3) f is nano-open.

Example 5.5:

From Example 4.2. Assume that H is a subgraph from G with vertices $V(H) = \{v_1, v_2, v_4\}$. Then the nano-topological graph from $V(H)$ and $f(V(H))$ are $\tau_N(V(H)) = \{V(G_1), \emptyset, \{v_1\}, \{v_2, v_3, v_4\}\}$. $\tau_N(f(V(H))) = \{V(G_2), \emptyset, \{u_2\}, \{u_1, u_3, u_4\}\}$. Then the function $f: (V(G_1), \tau_N(V(H))) \rightarrow (V(G_2), \tau_N(f(V(H))))$ is nano-homeomorphism.

Theorem 5.6:

“Let $G = (V, E)$ and $G_1 = (V_1, E_1)$ be any two isomorphic graphs with nano topological graph $(V(G), \tau_N(V(H)))$ and $f: (V(G), \tau_N(V(H))) \rightarrow (V(G_1), \tau_N(f(V(H))))$ is one-one and onto. Then f is a nano homeomorphism if and only if f is nano closed and nano continuous”.

Proof:

“Let f is a nano homeomorphism. Then f is nano continuous. Let $V(F)$ be an arbitrary nano closed set in $(V(G), \tau_N(V(H)))$. Then $V(G) - V(F)$ is nano open, since f is nano open, $f(V(G) - V(F))$ is nano open $V(G_1)$. That is, $V(G_1) - f(V(F))$ is nano open in $V(G_1)$. Therefore, $f(V(F))$ is nano closed in $V(G_1)$. Thus, the image of every nano closed set in $V(G)$ is nano closed in $V(G_1)$. That is f is nano closed.

Conversely, let f is nano closed and continuous. Let $V(H)$ be a nano open set in $(V(G), \tau_N(V(H)))$. Then $V(G) - V(H)$ is nano closed in $V(G)$, since f is nano closed, $f(V(G) - V(H)) = V(G_1) - f(V(H))$ is nano closed in $V(G_1)$. Therefore, $f(V(H))$ is nano open in $V(G_1)$. Thus, f is nano open and hence f is a nano homeomorphism”.

Theorem 5.7:

Let $G = (V, E), G_1 = (V_1, E_1)$ with nano topological graph $(V(G), \tau_N(V(H)))$. A one-one f of $(V(G), \tau_N(V(H)))$ onto $(V(G_1), \tau_N(f(V(H))))$ is a nano homeomorphism iff $f(NCl_G(V(H))) \subseteq NCl_{G_1}(f(V(H)))$ for every subgraph H of G .

Proof:

If f is a nano homeomorphism, f is nano continuous and nano closed. If $V(H) \subseteq V(G)$, $f(NCl_G(V(H))) \subseteq NCl_{G_1}(f(V(H)))$, since f is nano continuous. Then $NCl_G(V(H))$ is nano closed in $V(H)$ and f is nano closed, $f(NCl_G(V(H)))$ is nano closed in $V(G_1)$. $NCl_{G_1}(f(NCl_G(V(H)))) = f(NCl_G(V(H)))$. Since $V(H) \subseteq NCl_G(V(H))$, $f(V(H)) \subseteq f(NCl_G(V(H)))$ and hence $NCl_{G_1}(f(V(H))) \subseteq NCl_{G_1}(f(NCl_G(V(H)))) = f(NCl_G(V(H)))$. Therefore, $NCl_{G_1}(f(V(H))) \subseteq f(NCl_G(V(H)))$. Thus,

$f(NCl_G(V(H))) = NCl_G(f(V(H)))$ if f a nano homeomorphism.

Conversely, if $V(H)$ is nano-closed in $V(G)$. $NCl_G(V(H)) = V(H)$ which implies $f(NCl_G(V(H))) = f(V(H))$. Therefore, $NCl_G(f(V(H))) = f(V(H))$, thus $V(H)$ is nano closed in $V(G_1)$, for every nano closed set $V(H)$ in $V(G)$. That is f is nano closed. Also f is nano continuous. Thus, f is a nano homeomorphism.

6. Conclusion

In this paper, the relationship between nano topology and graph theory was studied to show how nano topology is deduced from any graph as explained in Section 3. We also deduced the concepts of continuity and homeomorphism in nano topology and their relationship with two isomorphic graphs as explained in Section 4 and Section 5.

Conflicts of Interest

The authors declare no conflicts of interest.

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