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Certain Problem for Starlike Functions with Respect to Other Points

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Abstract

In this work, a new class of analytic and univalent functions $Q_{ns}(\alpha,\gamma,\mu)$, $Q_{nc}(\alpha,\gamma,\mu)$, $Q_{nsc}(\alpha,\gamma,\mu)$ with respect to other points that include symmetrical and conjugats in the unit disk are studied. The estimated coefficients are calculated respectively for each class of functions. The fractional calculus techniques $I_{0,z}^{\beta,\eta,\delta} z^k = \frac{\Gamma(k+1)\Gamma(k-r+\delta+1)}{\Gamma(k-\eta+1)\Gamma(k+\beta+\delta+1)} z^{k-\delta}$ where $\beta > 0, k > \eta - \delta - 1$ were used to study the distortion theorem. The fractional integral operator $D_z^{-\lambda} f(z) = \frac{1}{\Gamma(\lambda)} \int_0^z \frac{f(\zeta)}{(z-\zeta)^{1-\lambda}} \mathrm{d}\zeta$ $(\lambda > 0)$ was used to satistfy the analytic function *f*(*z*) in a simply-connected region of the *z-plane* containing the origin on a class $Q_{ns}(\alpha,\gamma,\mu)$, and hence concluded to the analytic fonctions *f*(*z*) on the calsses $Q_{nc}(\alpha,\gamma,\mu)$ and $Q_{nsc}(\alpha,\gamma,\mu)$.

Subject Areas

Algebraic Geometry

Keywords

Univalent Functions, Distortion Theorem, Fractional Calculus Operators

1. Introduction

Let S denote the class of functions

$$f(z) = z + \sum_{k=2}^{\infty} a^k z^k \tag{1.1}$$

which is analytic and univalent in $U = \{z : |z| < 1\}$.

Let T denote the subclass of S consisting of functions of the form

$$f(z) = z - \sum_{k=2}^{\infty} a_k z^k, \quad (a_k \geq 0) \tag{1.2}$$

Let S_S^* be the subclass of S consisting of functions given by (1.1) satisfying

$$Re\left\{\frac{zf'(z)}{f(z)-f(-z)}\right\} > 0, \quad z \in U$$

These functions are called starlike with respect to symmetric points [1].

The aim of this work is to study the class $Q_{ns}(\alpha,\gamma,\mu)$ which consists of functions f of T, satisfies

$$\left|\frac{\dfrac{z\left(D^n f(z)\right)'}{D^n f(z)-D^n f(-z)}-1}{\dfrac{\alpha z\left(D^n f(z)\right)'}{D^n f(z)-D^n f(-z)}+(1-\alpha)}\right| < \mu, \quad z \in U \tag{1.3}$$

for $0 \leq \gamma < 1$, $0 \leq \alpha < 1$, $0 < \mu \leq 1$.

and a class $Q_{nc}(\alpha,\gamma,\mu)$ of functions f of T, satisfies

$$\left|\frac{\dfrac{z\left(D^n f(z)\right)'}{D^n f(z)-D^n \overline{f(\overline{z})}}-1}{\dfrac{\alpha z\left(D^n f(z)\right)'}{D^n f(z)-D^n \overline{f(\overline{z})}}+(1-\alpha)}\right| < \mu, \quad z \in U \tag{1.4}$$

for $0 \leq \gamma < 1$, $0 \leq \alpha < 1$, $0 < \mu \leq 1$.

and a class $Q_{nsc}(\alpha,\gamma,\mu)$ of functions f of T, satisfies

$$\left|\frac{\dfrac{z\left(D^n f(z)\right)'}{D^n f(z)-D^n \overline{f(-z)}}-1}{\dfrac{\alpha z\left(D^n f(z)\right)'}{D^n f(z)-D^n \overline{f(-z)}}+(1-\alpha)}\right| < \mu, \quad z \in U \tag{1.5}$$

for $0 \leq \gamma < 1$, $0 \leq \alpha < 1$, $0 < \mu \leq 1$.

Using fractional calculus to study distortions cahractaristics and estimated coefficients is obtained.

2. The Class $Q_{ns}(\alpha,\gamma,\mu)$

Theorem 2.1 Let the function f be defined by (1.2), then $f \in Q_{ns}(\alpha,\gamma,\mu)$ *if and only if*

$$\begin{aligned}&\sum_{k=2}^{\infty}\left[(-1)^{k+1}k(1-\mu\alpha)-\left(1-(-1)^k\right)\left(1+\mu(1-\gamma)\right)\right]\delta(n,k)a_k \\ &\leq \mu\left(\alpha+2(1-\alpha)\right)-1\end{aligned} \tag{2.1}$$

where $0 \leq \gamma < 1$, $0 \leq \alpha < \dfrac{1}{2(1-\gamma)}$, $\dfrac{1}{\alpha+2(1-\gamma)} < \mu \leq 1$ and

$$\delta(n,k)=\binom{n+k-1}{n}.$$

the result (2.1) is sharp for the function

$$f(z)=z-\frac{\mu(\alpha+2(1-\gamma))-1}{\left[(-1)^{k+1}k(1-\mu\alpha)-\left(1-(-1)^k\right)(1+\mu(1-\gamma))\right]\delta(n,k)}z^k \quad (k\geq 2)$$

Proof. Assume that the inequality (2.1) holds true and $|z|=1$. Then obtain

$$\begin{aligned}&\left|z\left(D^n f(z)\right)'-D^n f(z)+D^n f(-z)\right|\\&-\mu\left|\alpha\left(D^n f(z)\right)'+(1-\gamma)D^n f(z)-(1-\gamma)D^n f(-z)\right|\\&=\left|-z-\sum_{k=2}^{\infty}\left[k-\left(1-(-1)^k\right)\right]\delta(n,k)a_k z^k\right|\\&\quad-\mu\left|\alpha z-\sum_{k=2}^{\infty}\left[\alpha k-(1-\gamma)\right]\delta(n,k)a_k z^k+2(1-\gamma)z\right|\\&\leq\sum_{k=2}^{\infty}\left[(-1)^{k+1}k(1-\mu\alpha)-\left(1-(-1)^k\right)(1+\mu(1-\gamma))\right]\delta(n,k)a_k+1\\&\quad-\mu(\alpha+2(1-\gamma))+1<0\end{aligned}$$

Thus, by maximum modulus principle [2], $f\in Q_{ns}(\alpha,\gamma,\mu)$

Now assume that $f\in Q_{ns}(\alpha,\gamma,\mu)$ then

$$\left|\frac{\dfrac{z\left(D^n f(z)\right)'}{D^n f(z)-D^n f(-z)}-1}{\dfrac{\alpha z\left(D^n f(z)\right)'}{D^n f(z)-D^n f(-z)}+(1-\alpha)}\right|<\mu,\quad z\in U$$

Then

$$\begin{aligned}&\left|z\left(D^n f(z)\right)'-D^n f(z)+D^n f(-z)\right|\\&<\mu\left|\alpha z\left(D^n f(z)\right)'+(1-\gamma)D^n f(z)-(1-\gamma)D^n f(-z)\right|\end{aligned}$$

i.e.

$$\begin{aligned}&\left|-z-\sum_{k=2}^{\infty}\left[(-1)^{k+1}k-(1-(-1)^k\right]\delta(n,k)\right|\\&<\mu\left|\alpha z-\sum_{k=2}^{\infty}\left[(-1)^{k+1}\alpha k-(1-\gamma)\right]\delta(n,k)a_k z^k+2(1-\gamma)z\right|\end{aligned}$$

Thus

$$\sum_{k=2}^{\infty}\left[(-1)^{k+1}k(1-\mu\alpha)-\left(1-(-1)^k\right)\right]\delta(n,k)a_k<\mu\left(\alpha+2(1-\gamma)\right)-1$$

And the proof is complete.

3. The Class $Q_{nc}(\alpha,\gamma,\mu)$

Theorem 3.1 Let the function *f* be defined by (1.2), then $f\in Q_{nc}(\alpha,\gamma,\mu)$ *if* *and only if*

$$\sum_{k=2}^{\infty}\left[(-1)^{k-1}k(1+\mu\alpha)-\left(1-(-1)^{k}\right)\left(1+\mu(1-\gamma)\right)\right]\delta(n,k)a_k \le \mu\left(\alpha+2(1-\gamma)\right)-1 \tag{3.1}$$

where $0\le\gamma<1$, $0\le\alpha<\frac{1}{2(1-\gamma)}<1$, $\frac{1}{\alpha+2(1-\gamma)}<\mu\le 1$ and $\delta(n,k)=\binom{n+k-1}{n}$.

The result (3.1) is sharp for the function

$$f(z)=\frac{\mu\left(\alpha+2(1-\gamma)\right)-1}{\left[(-1)^{k-1}k(1+\mu\alpha)-\left(1-(-1)^{k}\right)\left(1+\mu(1-\gamma)\right)\right]\delta(n,k)}z^k \quad (k\ge 2)$$

***Proof*.** Assume that the inequality (3.1) holds true and $|z|=1$. Then obtain

$$\begin{aligned}&\left|z\left(D^n f(z)\right)'-D^n f(z)+D^n f(-z)\right|\\&-\mu\left|\alpha\left(D^n f(z)\right)'+(1-\gamma)D^n f(z)-(1-\gamma)D^n f(-z)\right|\\&=\left|-z-\sum_{k=2}^{\infty}\left[(-1)^{k-1}k-\left(1-(-1)^{k}\right)\right]\delta(n,k)a_k z^k\right|\\&\quad-\mu\left|\alpha z-\sum_{k=2}^{\infty}\left[(-1)^{k-1}\alpha k-(1-\gamma)\right]\delta(n,k)a_k z^k+2(1-\gamma)z\right|\\&\le\sum_{k=2}^{\infty}\left[(-1)^{k-1}k(1-\mu\alpha)-\left(1-(-1)^{k}\right)\left(1+\mu(1-\gamma)\right)\right]\delta(n,k)a_k\\&\quad-\mu\left(\alpha+2(1-\gamma)\right)+1<0\end{aligned}$$

Thus, by maximum modulus principle [2], $f\in Q_{nc}(\alpha,\gamma,\mu)$.

Now assume that $f\in Q_{nc}(\alpha,\gamma,\mu)$ then

$$\left|\frac{\dfrac{z\left(D^n f(z)\right)'}{D^n f(z)-D^n\overline{f(\overline{z})}}-1}{\dfrac{\alpha z\left(D^n f(z)\right)'}{D^n f(z)-D^n\overline{f(\overline{z})}}+(1-\alpha)}\right|<\mu,\quad z\in U$$

Then

$$\begin{aligned}&\left|z\left(D^n f(z)\right)'-D^n f(z)+D^n f(-z)\right|\\&<\mu\left|\alpha z\left(D^n f(z)\right)'+(1-\gamma)D^n f(z)-(1-\gamma)D^n f(-z)\right|\end{aligned}$$

i.e.

$$\begin{aligned}&\left|-z-\sum_{k=2}^{\infty}\left[(-1)^{k-1}k-\left(1-(-1)^{k}\right)\right]\delta(n,k)\right|\\&<\mu\left|\alpha z-\sum_{k=2}^{\infty}\left[(-1)^{k-1}\alpha k-(1-\gamma)\right]\delta(n,k)a_k z^k+2(1-\gamma)z\right|\end{aligned}$$

Thus

$$\sum_{k=2}^{\infty}\left[(-1)^{k-1}k(1-\mu\alpha)-\left(1-(-1)^{k}\right)\left(1+\mu(1-\gamma)\right)\right]\delta(n,k)a_k$$
$$<\mu\left(\alpha+2(1-\gamma)\right)-1$$

And the proof is complete.

4. The Class $Q_{nsc}(\alpha,\gamma,\mu)$

Theorem 4.1 Let the function f be defined by (1.2), then $f\in Q_{nc}(\alpha,\gamma,\mu)$ *if and only if*

$$\sum_{k=2}^{\infty}\left[(-1)^{k-1}k(1+\mu\alpha)-\left(1+\mu(1-\gamma)\right)\right]\delta(n,k)a_k \le \mu\left(\alpha+2(1-\gamma)\right)-1 \quad (4.1)$$

where $0\le\gamma<1$, $0\le\alpha<\frac{1}{2(1-\gamma)}<1$, $\frac{1}{\alpha+2(1-\gamma)}<\mu\le 1$ and $\delta(n,k)=\binom{n+k-1}{n}$.

The result (4.1) is sharp for the function

$$f(z)=\frac{\mu\left(\alpha+2(1-\gamma)\right)-1}{\left[(-1)^{k-1}k(1+\mu\alpha)-\left(1+\mu(1-\gamma)\right)\right]\delta(n,k)}z^k \quad (k\ge 2)$$

Proof. Assume that the inequality (4.1) holds true and $|z|=1$. Then obtain

$$\left|z\left(D^n f(z)\right)'-D^n f(z)+D^n f(-z)\right|$$
$$-\mu\left|\alpha\left(D^n f(z)\right)'+(1-\gamma)D^n f(z)-(1-\gamma)D^n f(-z)\right|$$
$$=\left|-z-\sum_{k=2}^{\infty}\left[(-1)^{k-1}k-1\right]\delta(n,k)a_k z^k\right|$$
$$-\mu\left|\alpha z-\sum_{k=2}^{\infty}\left[(-1)^{k-1}\alpha k-(1-\gamma)\right]\delta(n,k)a_k z^k+2(1-\gamma)z\right|$$
$$\le\sum_{k=2}^{\infty}\left[(-1)^{k+1}k(1-\mu\alpha)-\left(1+\mu(1-\gamma)\right)\right]\delta(n,k)a_k$$
$$-\mu\left(\alpha+2(1-\gamma)\right)+1<0$$

Now assume that $f\in Q_{nsc}(\alpha,\gamma,\mu)$ then

$$\left|\frac{\dfrac{z\left(D^n f(z)\right)'}{D^n f(z)-D^n\overline{f(-z)}}-1}{\dfrac{\alpha z\left(D^n f(z)\right)'}{D^n f(z)-D^n\overline{f(-z)}}+(1-\alpha)}\right|<\mu,\quad z\in U$$

Then

$$\left|z\left(D^n f(z)\right)'-D^n f(z)+D^n f(-z)\right|$$
$$<\mu\left|\alpha z\left(D^n f(z)\right)'+(1-\gamma)D^n f(z)-(1-\gamma)D^n f(-z)\right|$$

i.e.

$$\left|-z-\sum_{k=2}^{\infty}\left[(-1)^{k-1}k-1\right]\delta(n,k)\right|$$
$$<\mu\left|\alpha z-\sum_{k=2}^{\infty}\left[(-1)^{k-1}\alpha k-(1-\gamma)\right]\delta(n,k)a_k z^k+2(1-\gamma)z\right|$$

Thus

$$\sum_{k=2}^{\infty}\left[(-1)^{k-1}k(1-\mu\alpha)-(1+\mu(1-\gamma))\right]\delta(n,k)a_k$$
$$<\mu(\alpha+2(1-\gamma))-1$$

And the proof is complete.

5. Application of the Fractional Calculus

Several operators of fractional calculus (*i.e.*, fractional derivative and fractional integral) have been rather extensively studied by many researchers (c.f. [3] [4] [5]). Making use of the following Lemma (given by Srivastava *et al.* [6] and used by Gh. Esa and Darus [7]) stated as

Lemma 5.1 Let $\beta>0, k>\eta-\delta-1$, then

$$I_{0,z}^{\beta,\eta,\delta}z^k=\frac{\Gamma(k+1)\Gamma(k-r+\delta+1)}{\Gamma(k-\eta+1)\Gamma(k+\beta+\delta+1)}z^{k-\delta}.$$

to prove the following theorem:

Theorem 5.2 Let $\beta>0, \eta<2, \beta+\delta>-2, \eta(\beta+\delta)\le 3\beta$. If $f(z)$ defined by (1.2) in the class

$$Q_{ns}(\alpha,\gamma,\mu),$$

then

$$\left|I_{0,z}^{\beta,\eta,\delta}f(z)\right|$$
$$\ge\frac{\Gamma(2-\eta+\delta)|z|^{1-\eta}}{\Gamma(2-\eta)\Gamma(2+\beta+\delta)}\left(1-\frac{\left[\mu(\alpha+2(1-\gamma))-1\right](2-\eta+\delta)}{(-1)^{n+2}(n+1)(1-\mu\alpha)(2-\eta)(2+\beta+\delta)}|z|\right) \tag{5.1}$$

and

$$\left|I_{0,z}^{\beta,\eta,\delta}f(z)\right|$$
$$\le\frac{\Gamma(2-\eta+\delta)|z|^{1-\eta}}{\Gamma(2-\eta)\Gamma(2+\beta+\delta)}\left(1-\frac{\left[\mu(\alpha+2(1-\gamma))-1\right](2-\eta+\delta)}{(-1)^{n+2}(n+1)(1-\mu\alpha)(2-\eta)(2+\beta+\delta)}|z|\right) \tag{5.2}$$

for $z\in U_0$, where

$$U_0=\begin{cases}U & \eta\le 1\\ U-\{0\} & \eta>1\end{cases}$$

the result is sharp and is given by

$$f(z)=z-\frac{\mu(\alpha+2(1-\gamma))-1}{(-1)^{n+2}(n+1)(1-\mu\alpha)}z^2 \tag{5.3}$$

Proof. By using Lemma 5.1, we have

$$\begin{aligned} I_{0,z}^{\beta,\eta,\delta} f(z) &= \frac{\Gamma(2-\eta+\delta)}{\Gamma(2-\eta)\Gamma(2+\beta+\delta)} z^{1-\eta} \\ &= \sum_{k=2}^{\infty} \frac{\Gamma(k+1)\Gamma(k-\eta+\delta+1)}{\Gamma(k-\eta+1)\Gamma(k+\beta+\delta+1)} a_k z^{k-\delta} \end{aligned} . \tag{5.4}$$

Setting

$$H(z) = \frac{\Gamma(2-\eta)\Gamma(2+\beta+\delta)}{\Gamma(2-\eta+\delta)} z^{\delta} I_{0,z}^{\beta,\eta,\delta} f(z) = z - \sum_{k=2}^{\infty} h(k) a_k z^k$$

where

$$h(z) = \frac{(2-\eta+\delta)_{k-1}(1)_k}{(2-\eta)_{k-1}(2+\beta+\delta)_{k-1}} \quad (k \ge 2). \tag{5.5}$$

It is easily verified that $h(k)$ is non-decreasing for $k \ge 2$, and thus we have

$$0 < h(z) \le h(2) = \frac{2(2-\eta+\delta)}{(2-\eta)(2+\beta+\delta)} \tag{5.6}$$

Now, noting that $\delta(n, 2)$ is increasing function of n, we have

$$\begin{aligned} & 2(-1)^{n+2}(1+\mu\alpha)\delta(n,2)\sum_{k=2}^{\infty} a_k \\ & \le \sum_{k=2}^{\infty} \delta(n,k) a_k (1-\mu\alpha) \le \mu(\alpha+2(1-\alpha))-1 \end{aligned}$$

or

$$\sum_{k=2}^{\infty} a_k \le \frac{\mu(\alpha+2(1-\gamma))-1}{2(-1)^{n+2}(n+1)(1-\mu\alpha)} \tag{5.7}$$

Hence, using (5.6) and (5.7), we have

$$\begin{aligned} |H(z)| &\ge |z| - h(z)|z|^2 \sum_{k=2}^{\infty} a_k \\ &\ge |z| - \frac{[\mu(\alpha+2(1-\gamma))-1](2-\eta+\delta)}{(-1)^{n+2}(1-\mu\alpha)(2-\eta)(2+\beta+\delta)} |z|^2, \end{aligned} \tag{5.8}$$

which proves (5.1), and other parts (5.2) we can find that

$$|H(z)| \le |z| - \frac{[\mu(\alpha+2(1-\gamma))-1](2-\eta+\delta)}{(-1)^{n+2}(1-\mu\alpha)(2-\eta)(2+\beta+\delta)} |z|^2, \tag{5.9}$$

and the prove is complete.

Using the same technique for the functions $f(z)$ in the classes $Q_{nc}(\alpha,\gamma,\mu)$ and $Q_{nsc}(\alpha,\gamma,\mu)$.

Now, taking $\eta = -\beta = -\lambda$ and $\eta = -\beta = \lambda$ in the Theorem 5.1, and using the definition given by Owa [8] which is stated as:

Definition 5.3 (*Fractional Integral Operator*) [8]. The fractional integral of order λ is defined, for a function $f(z)$, by

$$D_z^{-\lambda} f(z) = \frac{1}{\Gamma(\lambda)} \int_0^z \frac{f(\zeta)}{(z-\zeta)^{1-\lambda}} \mathrm{d}\zeta \quad (\lambda > 0) \tag{5.10}$$

where *f*(*z*) is an analytic function in a simply-connected region of the *z-plane* containing the origin, and the multiplicity of $(z-\zeta)^{\lambda-1}$ is removed by requiring log(*z* − *ζ*) to be real when

$$(z-\zeta)>0.$$

We get two seperated corollaries which are contained in;

Corollary 5.4 Let the fuction *f*(*z*) defined by (1.2) be in the class $Q_{ns}(\alpha,\gamma,\mu)$, then we have

$$\left|D_z^{-\lambda} f(z)\right| \geq \frac{|z|^{1+\lambda}}{\Gamma(2+\lambda)}\left(1-\frac{\mu(\alpha+2(1-\gamma))-1}{(-1)^{n+2}(1-\mu\alpha)(2+\lambda)(n+1)}|z|\right) \tag{5.11}$$

and

$$\left|D_z^{-\lambda} f(z)\right| \leq \frac{|z|^{1+\lambda}}{\Gamma(2+\lambda)}\left(1+\frac{\mu(\alpha+2(1-\gamma))-1}{(-1)^{n+2}(1-\mu\alpha)(2+\lambda)(n+1)}|z|\right) \tag{5.12}$$

for $\lambda>0$, $z\in E$. The result is sharp for the function

$$D_z^{-\lambda} f(z) = \frac{|z|^{1+\lambda}}{\Gamma(2+\lambda)}\left(1+\frac{\mu(\alpha+2(1-\gamma))-1}{(-1)^{n+2}(1-\mu\alpha)(2+\lambda)(n+1)}|z|\right) \tag{5.13}$$

Corollary 5.5 Let the fuction *f*(*z*) defined by (1.2) be in the class $Q_{ns}(\alpha,\gamma,\mu)$, then we have

$$\left|D_z^{\lambda} f(z)\right| \geq \frac{|z|^{1-\lambda}}{\Gamma(2+\lambda)}\left(1-\frac{\mu(\alpha+2(1-\gamma))-1}{(-1)^{n+2}(1-\mu\alpha)(2+\lambda)(n+1)}|z|\right) \tag{5.14}$$

and

$$\left|D_z^{\lambda} f(z)\right| \leq \frac{|z|^{1-\lambda}}{\Gamma(2+\lambda)}\left(1+\frac{\mu(\alpha+2(1-\gamma))-1}{(-1)^{n+2}(1-\mu\alpha)(2+\lambda)(n+1)}|z|\right) \tag{5.15}$$

for $0\leq\lambda<1, z\in U$. The result is sharp for the function

$$D_z^{\lambda} f(z) = \frac{|z|^{1-\lambda}}{\Gamma(2+\lambda)}\left(1+\frac{\mu(\alpha+2(1-\gamma))-1}{(-1)^{n+2}(1-\mu\alpha)(2+\lambda)(n+1)}|z|\right) \tag{5.16}$$

Again the same technique uses for the function in the classes $Q_{nc}(\alpha,\gamma,\mu)$ and $Q_{nsc}(\alpha,\gamma,\mu)$.

6. Conclusion

The classes $Q_{ns}(\alpha,\gamma,\mu)$, $Q_{nc}(\alpha,\gamma,\mu)$, $Q_{nsc}(\alpha,\gamma,\mu)$ of analytic and univalent functions are investigated. The estimated coefficients are studied and obtaind respectively and shown in the Equations (2.1), (3.1) and (4.1). The application of the fractional calculus is studied on the class $Q_{ns}(\alpha,\gamma,\mu)$ and obtained in Equations (5.1) and (5.2) and concluded for other classes $Q_{nc}(\alpha,\gamma,\mu)$ and $Q_{nsc}(\alpha,\gamma,\mu)$. Fractional Integral Operator is studied and obtained on the class $Q_{ns}(\alpha,\gamma,\mu)$ and concluded for other classes by using the same mathematical techniques.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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