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A Three-Dimensional Chaotic System and Its New Proposed Electronic Circuit

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Abstract

In this paper, new electronic circuit was designed with two-equilibrium as an engineering application on a three-dimensional chaotic system. The circuit consists of resistors, capacitors, voltages and operational amplifiers TL032CN. The adopted continuous-time chaotic dynamical system is with quadratic cross-product nonlinear terms and parameters. The basic characteristics of the proposed circuit model were analyzed in detail by equilibrium points, stability analysis, Lyapunov exponents and Kaplan-Yorke dimension. The results were simulated theoretically using MultiSIM 10, and it was well consistent with the results obtained from the Matlab program.

Subject Areas

Dynamical System

Keywords

Hyperchaotic System, Chaotic Attractors, Kaplan-Yorke Dimension, Circuit Simulation

1. Introduction

Chaos theory describes nonlinear dynamical systems that are very sensitive to initial conditions. Since the experimental discovery of a chaotic system by Lorenz, chaos theory has found applications in several areas in science and engineering [1]. Chaos is a phenomenon caused by the sensitivity of perturbation structural parameters and the initial conditions of some categories of dynamic systems [2] [3].

In last decades, chaotic circuit has received considerable interest in the researches due to the fact that they have been applied in abundant areas like in secure communications, simulating economical models, design of electronic circuits, robotics, image processing, and neural networks [4] [5]. For this reason, a large number of chaotic systems, which are implemented in circuit, are reported in the literature [6].

Electrical laws are necessary to analyze any electrical circuit effectively and efficiently by determining different circuit parameters such as current, voltage power and resistance. These laws include Ohms law, Kirchhoff's current and voltage laws etc. [7].

This paper contains: Section 2, we description 3-D chaotic system; it is mainly consisted of six simple terms including two nonlinear terms. Section 3, an electronic circuit is designed to implement chaotic system (1). Section 4, we simulated the designed circuit by electronic simulation MultiSIM 10 program. And Section 5, we presented the conclusions.

2. Description of Chaotic System with Two Equilibrium Points

An autonomous 3-D dynamical system [8] with two quadratic cross-product nonlinear terms

$$\begin{aligned} \dot{x}_1 &= \rho\left(x_2 - x_1\right) \\ \dot{x}_2 &= a x_1 - \delta x_1 x_3 \\ \dot{x}_3 &= \varphi x_1 x_2 - x_3 \end{aligned} \tag{1}$$

where $x_1 x_3$, $x_1 x_2$ are the quadratic cross-product nonlinear terms in the dynamical system. The system (1) is chaotic when the parameters values (ρ, a, δ and φ) are taken as

$$\rho = 10,\ \ \delta = 40,\ \ a = 296.5,\ \ \varphi = 10 \tag{2}$$

So for the given values of parameters, the Lyapunov exponents of system (1) are determined as

$$L_1 = 2.509426,\ \ L_2 = 0.132019 \ \text{ and } \ L_3 = -11.818787 .$$

Also, Kaplan-Yorke dimension of system (1) is calculated as

$$D_{KY} = 2 + \frac{L_1 + L_2}{\left|L_3\right|} = 2.22349544$$

which shows hyper-chaotisity of system (1).

Therefore, the hyperchaotic system (1) has a strange attractor. For graphical results, we used Matlab and take initial states $x\big|_{x_1(0), x_2(0), x_3(0)} = \left[-2, 7, 12\right]$.

Figure 1 shows chaotic attractor for system (1) in $\boldsymbol{R}^3$.

Figures 2(a)-(c) show the system (1) exhibit chaotic attractors in (a): $\left(x_1, x_2\right)$ plane, (b): $\left(x_2, x_3\right)$ plane, (c): $\left(x_1, x_3\right)$ plane.

Now, the wave-form $x_1\left(t\right), x_2\left(t\right), x_3\left(t\right)$ for the system (1) has a non-periodic shape, shown in **Figures 3(a)-(c)** which is one of the basic characteristic behaviors of chaotic dynamical system.

3. The Proposed Circuit of 3-D Chaotic System (1)

An electronic circuit is designed to implement chaotic system (1). The circuit consists of electronic elements: capacitors, multipliers, resistors and operational amplifiers TL032CN.

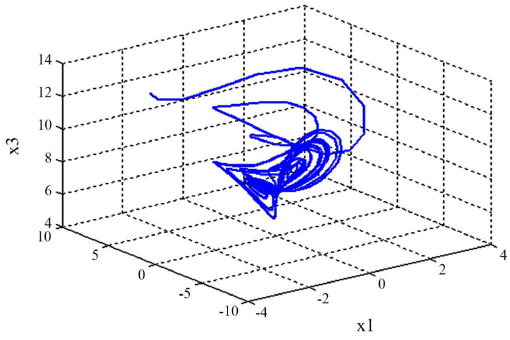

Figure 1. Chaotic attractor of system (1) with (ρ, a, δ, ϕ) = (10, 296.5, 40, 10) in (x_1-x_2-x_3) space.

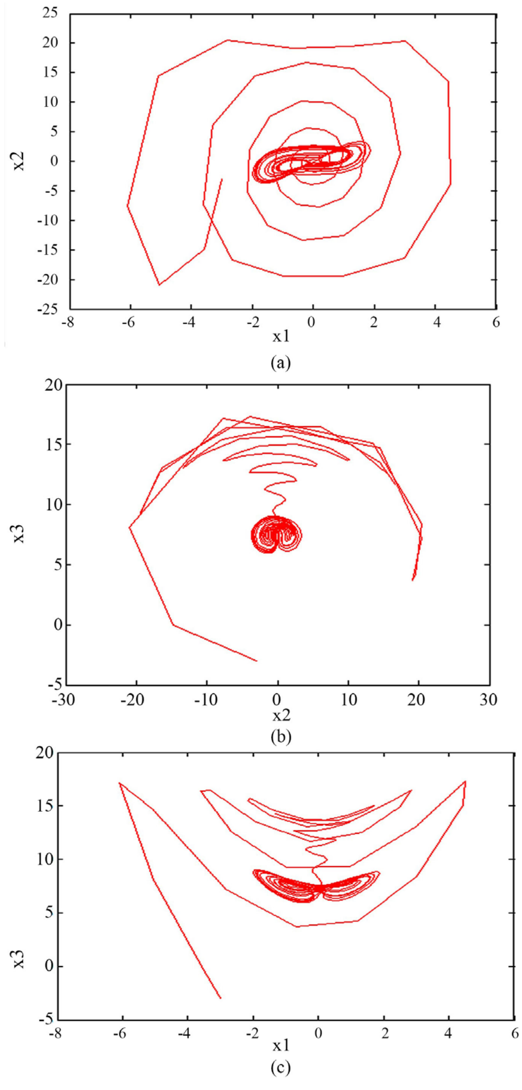

Figure 2. Chaotic attractor of system (1) with (ρ, a, δ, ϕ) = (10, 296.5, 40, 10) in (a) (x_1-x_2) plane, (b) (x_2-x_3) plane, (c) (x_1-x_3) plane.

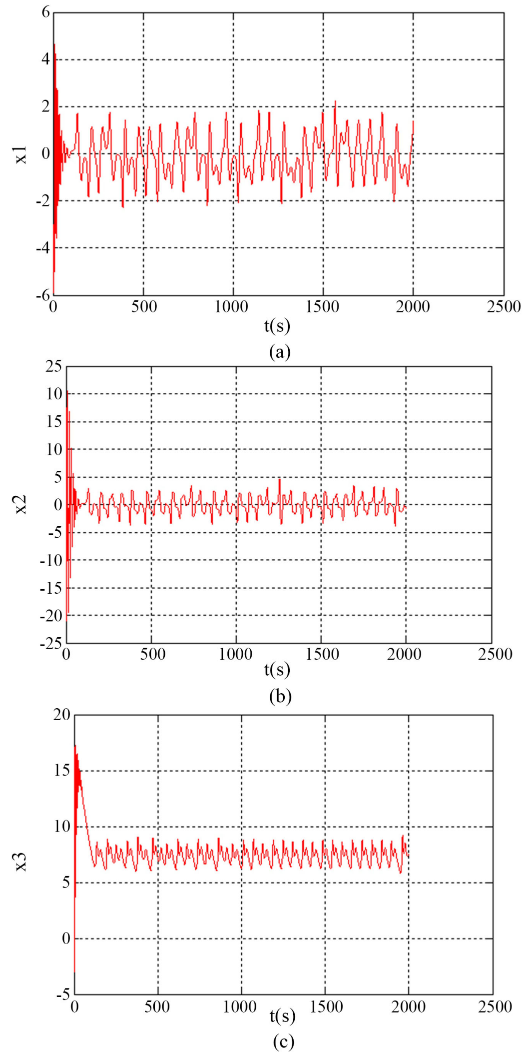

Figure 3. The wave-form of system (1). (a): Time versus x_1; (b): Time versus x_2; (c): Time versus x_3.

By applying Kirchhoff's laws [7], the corresponding circuit equations described as:

$$\begin{aligned}\frac{\mathrm{d}V_{x_1}}{\mathrm{d}t} &= \frac{1}{R_1C_1}\left(V_{x_2}-V_{x_1}\right)\\ \frac{\mathrm{d}V_{x_2}}{\mathrm{d}t} &= \frac{1}{R_2C_2}V_{x_1}-\frac{1}{R_3C_2}V_{x_1}V_{x_3}\\ \frac{\mathrm{d}V_{x_3}}{\mathrm{d}t} &= \frac{1}{R_4C_3}V_{x_1}V_{x_2}-\frac{1}{R_5C_3}V_{x_3}\end{aligned} \tag{3}$$

where $V_{x_1}, V_{x_2}, V_{x_3}$ are the output voltages and $k_m = 10\ \mathrm{V}$ is the fixed multipliers constant, hence the outputs are $V_{x_1x_3} = V_{x_1}V_{x_3}/k_m$ and $V_{x_1x_2} = V_{x_1}V_{x_2}/k_m$.

Voltages and time normalized by dimensionless states variables

$$V_{x_1} = 1\,\mathrm{V}\cdot x_1,\ V_{x_2} = 1\,\mathrm{V}\cdot x_2,\ V_{x_3} = 1\,\mathrm{V}\cdot x_3,\ t' = \tau\cdot t = 100\ \mu\mathrm{s}\cdot t \tag{4}$$

Substitute (4) in equations of system (3) we get:

$$\begin{aligned}\frac{dx_1}{dt'} &= \frac{\tau}{R_1C_1}(x_2 - x_1)\\ \frac{dx_2}{dt'} &= \frac{\tau}{R_2C_2}x_1 - \frac{\tau}{R_3C_2}x_1x_3\\ \frac{dx_3}{dt'} &= \frac{\tau}{R_4C_3}x_1x_2 - \frac{\tau}{R_5C_3}x_3\end{aligned} \tag{5}$$

Comparing system (1) with system (5) gives following conditions:

$$\frac{\tau}{R_1C_1} = \rho,\ \frac{\tau}{R_2C_2} = a,\ \frac{\tau}{R_3C_2} = \delta,\ \frac{\tau}{R_4C_3} = \varphi,\ \frac{\tau}{R_5C_3} = 1 \tag{6}$$

Take convenient values for capacitances and resistances as

$$\begin{aligned}&C_1 = C_2 = C_3 = 1\,\text{mF},\ R_1 = R_4 = 10\,\Omega,\ R_2 = 337.268\,\text{m}\Omega,\\ &R_3 = 2.5\,\Omega,\ R_5 = 100\,\Omega.\end{aligned} \tag{7}$$

We obtained the experimental electronic circuit (5) for system (1) with parameters $\rho = 10$, $a = 296.5$, $\delta = 40$, $\varphi = 10$.

3.1. Equilibrium Points

To find equilibrium points we need to solve the nonlinear equations as follows:

$$\begin{aligned}&\frac{\tau}{R_1C_1}(x_2 - x_1) = 0\\ &\frac{\tau}{R_2C_2}x_1 - \frac{\tau}{R_3C_2}x_1x_3 = 0\\ &\frac{\tau}{R_4C_3}x_1x_2 - \frac{\tau}{R_5C_3}x_3 = 0\end{aligned} \tag{8}$$

We get two equilibrium points

$$E_1 = \left(\frac{\sqrt{\frac{593}{2}}}{20}, \frac{\sqrt{\frac{593}{2}}}{20}, \frac{593}{80}\right),\quad E_2 = \left(-\frac{\sqrt{\frac{593}{2}}}{20}, -\frac{\sqrt{\frac{593}{2}}}{20}, \frac{593}{80}\right)$$

3.2. Stability Analysis

3.2.1. Characteristic Equation Roots

The Jacobian matrix of system (5) is:

$$J = \begin{bmatrix} -\frac{\tau}{R_1C_1} & \frac{\tau}{R_1C_1} & 0\\ \frac{\tau}{R_2C_2} - \frac{\tau}{R_3C_2}x_3 & 0 & -\frac{\tau}{R_3C_2}x_1\\ \frac{\tau}{R_4C_3}x_2 & \frac{\tau}{R_4C_3}x_1 & -\frac{\tau}{R_5C_3}\end{bmatrix}$$

$$J_{E_1} = \begin{bmatrix} -10-\lambda & 10 & 0\\ 266.85 & -\lambda & -34.43835072\\ 8.609587679 & 8.609587679 & -1-\lambda\end{bmatrix}$$

And

$$J_{E_2} = \begin{bmatrix} -10-\lambda & 10 & 0 \\ 266.85 & -\lambda & 34.43835072 \\ -8.609587679 & -8.609587679 & -1-\lambda \end{bmatrix}$$

Now, find characteristic equation by setting $\det(J-\lambda I)=0$, we get the same equation at E_1 and E_2:

$$\lambda^3 + 11\lambda^2 - 2362\lambda + 3261.5 = 0 \quad (9)$$

We obtain the same eigenvalues at equilibrium points E_1 and E_2:

$$\lambda_1 = 1.39097,\ \lambda_2 = 42.622,\ \lambda_3 = -55.013$$

Since, there are positive eigenvalues, so system (5) is unstable.

3.2.2. Routh Stability Criteria

From characteristic Equation (9) we get

$$a_0 = 3261.5$$

$$a_1 = -2362$$

$$a_2 = 11$$

$$a_3 = 1$$

$$b_1 = a_1 - \frac{a_3 a_0}{a_2} = -5634.5$$

Since, there is a negative element in the first column of Table 1. Therefore, System (5) is unstable.

3.2.3. Hurwitz Stability Criteria

We use determinants formed from coefficients of the characteristic Equation (9) we get:

$$\Delta_1 = a_2 = 11 > 0$$

$$\Delta_2 = \begin{vmatrix} a_2 & a_0 \\ a_3 & a_1 \end{vmatrix} = -29243.5 < 0$$

$$\Delta_3 = \begin{vmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{vmatrix} = -95377675.25 < 0$$

Since there is values of minors are less than zero, so the system (5) is unstable.

Table 1. Routh array of system (5).

λ^3	1	−2362
λ^2	11	3261.5
λ^1	−5634.5	0
λ^0	3261.5	0

3.2.4. Lapiynuov Function

Lapiynuov function and its derivatives for system (5) yield (10) & (11).

We assume that

$$V(x_1,x_2,x_3)=\frac{1}{2}\left(x_1^2+x_2^2+x_3^2\right) \tag{10}$$

$$\dot{V}(x_1,x_2,x_3)=\frac{\partial v}{\partial x_1}\frac{\mathrm{d}x_1}{\mathrm{d}t'}+\frac{\partial v}{\partial x_2}\frac{\mathrm{d}x_2}{\mathrm{d}t'}+\frac{\partial v}{\partial x_3}\frac{\mathrm{d}x_3}{\mathrm{d}t'}$$

$$\dot{V}(x_1,x_2,x_3)=x_1\dot{x}_1+x_2\dot{x}_2+x_3\dot{x}_3 \tag{11}$$

By substituting (5) in Equation (11) we get:

$$\dot{V}(x_1,x_2,x_3)=-10x_1^2+306.5x_1x_2-30x_1x_2x_3-x_3^2$$

Since $\dot{V}(x_1,x_2,x_3)>0$, therefore the system (5) is unstable.

3.3. Lapiynuov Exponent and Lapiynuov Dimension

The values of Lapiynuov exponents are:

$$L_1=2.509426,\ L_2=0.132019,\ L_3=-11.818787$$

Therefore, the Lapiynuov dimension "Kaplan-Yorke dimension" is:

$$D_L=2+\frac{L_1+L_2}{|L_3|}=2.22349544$$

So the system is hyperchaotic system, as shown in Figure 4.

4. The Simulation Results

In this section, the designed circuit to implement the chaotic system (1) was simulated by electronic simulation MultiSIM 10 program; Figure 5 shows sketch of the designed circuit of chaotic system (1)

The outputs voltages signals V_{x_1},V_{x_2},V_{x_3} versus time, and phase portraits of the attractors are presented, in Figure 6 which shows the chaotic behaviors of the circuit.

By comparing Figure 6 with Figure 2 & Figure 3 obtained by Matlab numerically, we conclude that between numerical simulation and experimental achievements there is a well qualitative agreement.

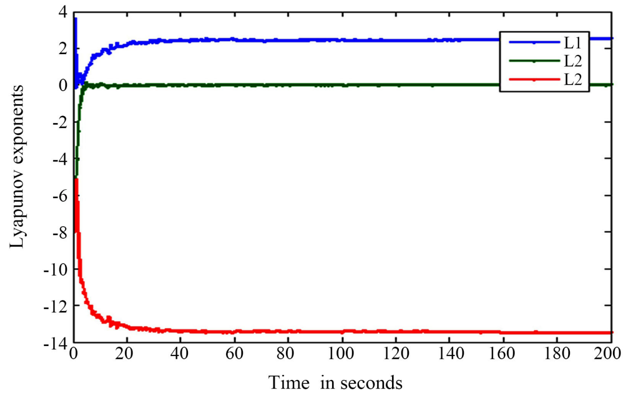

Figure 4. Lapiynuov exponent of system (5).

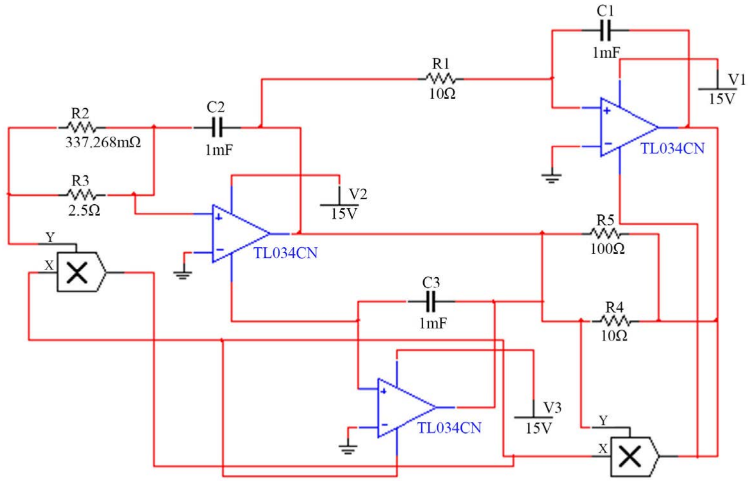

Figure 5. Sketch of the designed circuit of chaotic system (1).

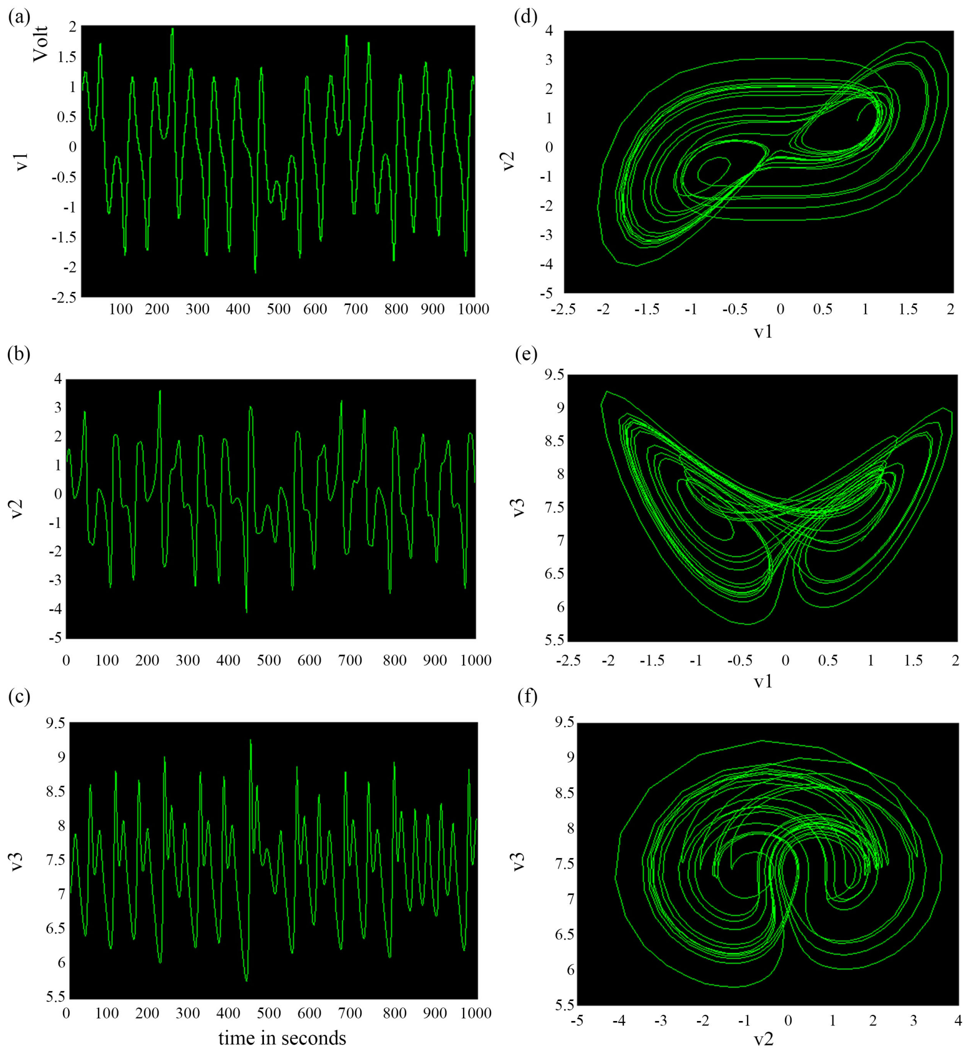

Figure 6. The left side shows the output voltage signals in (a) (V_1 versus time), (b) (V_2 versus time), (c) (V_3 versus time); While right side shows the phase portraits of the designed electronic circuit in (d) (V_1, V_2) plane, (e) (V_1, V_3) plane, (f) (V_2, V_3) plane.

5. Conclusion

A three-dimensional chaotic system with two equilibrium points is analyzed. The Lyapunov dimension of the chaotic system is computed as $D_L = 2.223495$, which shows that the system is hyperchaotic system. An electronic circuit is designed to implement chaotic system (1). Then, the basic characteristics of the proposed circuit model were analyzed by equilibrium points, stability analysis methods (such as characteristic equation roots, Routh criterion and Lapiynuov function); all these methods proved the instability of the new designed circuit (3), Lyapunov exponents and Kaplan-Yorke dimension that shows chaotisity of the designed circuit. The designed circuit is simulated by electronic simulation MultiSIM 10 program; the results show that there is a well qualitative agreement between the experimental achievements and numerical simulation which is obtained by using Matlab.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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