e-Abacus Diagram Rows Rearranging Technology

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Abstract

In our ordinary lives, we need to re-arrange our priorities so that we present and delay things according to the appropriateness of the circumstance we are going through. Also when we write something, after its completion we ask what if we made some adjustments to it in terms of presentation and delay. The question here is, can we use this technique to replace the rows in the *e*-abacus diagram, and know the representation of the new partition from the original partition?

Subject Areas

Mathematics, Partition Theory

Keywords

Partition, *e*-Abacus Diagram, Rows Rearrange

1. Introduction

Let r be a non-negative integer. A **partition** $\delta = (\delta_1, \delta_2, \cdots, \delta_n)$ of r is a sequence of non-negative integers such that $|\delta| = \sum_{j=1}^{n} \delta_j = r$, $\forall j \geq 1$ and $\forall j \geq 1$. Fix δ is a partition of r, choosing an integer b greater than or equal to the number of parts of δ and defining $\beta_i = \delta_i + b - i$, $1 \leq i \leq b$. The set $\{\beta_1, \beta_2, \cdots, \beta_b\}$ is said to be the set of β-numbers for δ, see [1]. Let e be a positive integer number greater than or equal to 2, we can represent β-numbers by a diagram called ***e*-abacus diagram:** (Table 1).

Where every β will be represented by a bead (■) and the rest of the sites by (-), which takes its location *e*-abacus diagram, [2]. For example, if we take $\delta = (6^2, 3^4, 2, 1^3)$ and $b = 10$ then

$$\beta_1 = 15, \beta_2 = 14, \beta_3 = 10, \beta_4 = 9, \beta_5 = 8, \beta_6 = 7, \beta_7 = 5, \beta_8 = 3, \beta_9 = 2, \beta_{10} = 1.$$

Table 1. *e*-abacus diagram.

Runner 1	Runner 2	…	Runner *e*
0	1	…	$e - 1$
e	$e + 1$	…	$2e - 1$
$2e$	$2e + 1$	…	$3e - 1$
⋮	⋮	⋮	⋮

If we choosing $e = 4$ then we have **Table 2**.

Mahmood & Ali in [3] [4] and [5] introduced the idea of the e-abacus diagram which rotates at certain angles (90, 180, 270, 360), and it is quite clear that the new diagram is completely different in its representation from the original partition. Also, Mahmood & Sami in [6], presented the idea of changing the rows together, and there were hypotheses and rules that were carefully studied. Each rule there is a representation of the diagram that is completely different of original. In this paper, we present a very fun and easy technique for solving the dilemma that was mentioned in the above two papers, which we will learn directly about the representation of the partition after the replacement process from the original without the need for a lot of rules and relationships that appeared there.

2. Replacement Technology

We will introduce the technique of switching rows according to the following steps:

1) We switch the partition sites from small to large, so that it becomes a shape:

$$\delta = \left(\delta_n, \delta_{n-1}, \cdots, \delta_1 \right).$$

2) We put in front of each number of δ a number of (-) which is of the same value. So if two or more numbers of δ are equal, then we do not repeat the (-) mode only at the first value of them only. Now, if we come to other different δ, we put a number of (-) in front of it, taking into account that a portion of it already exists so only the difference between them will be added this time. Repeat the steps with all δ.

3) According to the chosen *e*, we will put brackets on what we did in the 2nd step and in it exactly inside the number of *e* and then repeat the process until all of what is found in the 2nd step is completed. So if the last bracket has a number equal to *e* then we stand, if not, we will add the difference from the value of *e* by -.

4) Now, any bracket we want to place in the beginning we take it as it is, noting that the bracket is either -, all numbers or a combination of the two. The first and the second have no problem, except that the second case will completely cancel its effect from the later partition representation. As for the third case, we add the number of consecutive -, so this addition is the sum of the first number that comes after the -. By repeating this process with the rest of the brackets, we always add the number of—with the ones before it to give us the representation of partition after the change.

Table 2. 4-abacus diagram for a partition $\delta = \left(6^2, 3^4, 2, 1^3\right)$.

0	1	2	3	-	■	■	■
4	5	6	7	-	■	-	■
8	9	10	11	■	■	■	-
12	13	14	15	-	-	■	■

From the above example, when $\delta = \left(6^2, 3^4, 2, 1^3\right)$, then:

1$^{\text{st}}$ step: $\delta = \left(1^3, 2, 3^4, 6^2\right) = \left(1,1,1,2,3,3,3,3,6,6\right)$.

2$^{\text{nd}}$ step: $\left(1,1,1,2,3,3,3,3,6,6\right) = \left(-,1,1,1,-,2,-,3,3,3,3,-,-,-,6,6\right)$

3$^{\text{rd}}$ step: If we $e = 2$, then

$\left(\{-,1\},\{1,1\},\{-,2\},\{-,3\},\{3,3\},\{3,-\},\{-,-\},\{6,6\}\right)$, and if we take $e = 3$, then

$\left(\{-,1,1\},\{1,-,2\},\{-,3,3\},\{3,3,-\},\{-,-,6\},\{6,-,-\}\right)$.

4$^{\text{th}}$ step: When $e = 2$, we have 8 brackets, if we want to consider the 8$^{\text{th}}$ as the 1$^{\text{st}}$, 7$^{\text{th}}$ as to 2$^{\text{nd}}$, and so on the rest of the classes then we will be in front of the following:

$$\left(\{6,6\}\right) \rightarrow \left(\{\text{has no effect}, \text{has no effect}\}\right) = \left(\{0,0\}\right).$$

$$\left(\{-,-\}\right) \rightarrow \left(\{0,0\},\{-,-\}\right),$$

$$\left(\{3,-\}\right) \rightarrow \left(\{0,0\},\{-,-\},\{2,-\}\right),$$

$$\left(\{3,3\}\right) \rightarrow \left(\{0,0\},\{-,-\},\{2,-\},\{3,3\}\right),$$

$$\left(\{-,3\}\right) \rightarrow \left(\{0,0\},\{-,-\},\{2,-\},\{3,3\},\{-,4\}\right),$$

$$\left(\{-,2\}\right) \rightarrow \left(\{0,0\},\{-,-\},\{2,-\},\{3,3\},\{-,4\},\{-,5\}\right),$$

$$\left(\{1,1\}\right) \rightarrow \left(\{0,0\},\{-,-\},\{2,-\},\{3,3\},\{-,4\},\{-,5\},\{5,5\}\right),$$

$$\left(\{-,1\}\right) \rightarrow \left(\{0,0\},\{-,-\},\{2,-\},\{3,3\},\{-,4\},\{-,5\},\{5,5\},\{-,6\}\right).$$

Then the new partition is $\left(6,5^3,4,3^2,2\right)$. If we choose $e = 3$, then we have:

$$\left(\{6,-,-\}\right) \rightarrow \left(\{0,-,-\}\right),$$

$$\left(\{-,-,6\}\right) \rightarrow \left(\{0,-,-\},\{-,-,4\}\right),$$

$$\left(\{3,3,-\}\right) \rightarrow \left(\{0,-,-\},\{-,-,4\},\{4,4,-\}\right),$$

$$\left(\{-,3,3\}\right) \rightarrow \left(\{0,-,-\},\{-,-,4\},\{4,4,-\},\{-,6,6\}\right),$$

$$\left(\{1,-,2\}\right) \rightarrow \left(\{0,-,-\},\{-,-,4\},\{4,4,-\},\{-,6,6\},\{6,-,7\}\right),$$

$$\left(\{-,1,1\}\right) \rightarrow \left(\{0,-,-\},\{-,-,4\},\{4,4,-\},\{-,6,6\},\{6,-,7\},\{-,8,8\}\right)$$

The new partition is $\left(8^2,7,6^3,4^3\right)$.

With the same mechanism, if we want the arrangement of the brackets in the case $e = 3$, to be $\{6\}$, $\{3\}$, $\{5\}$, $\{2\}$, $\{4\}$, $\{1\}$ then we will be in front of the following:

Table 3. *e*-abacus diagram by exchange.

No. of rows			
1	-	■	■
2	■	-	■
3	-	■	■
4	■	■	-
5	-	-	■
6	■	-	-
6	■	-	-
3	-	■	■
5	-	-	■
2	■	-	■
4	■	■	-
1	-	■	■

$$\left(\{6,-,-\}\right) \rightarrow \left(\{0,-,-\}\right),$$

$$\left(\{-,3,3\}\right) \rightarrow \left(\{0,-,-\},\{-,3,3\}\right),$$

$$\left(\{-,-,6\}\right) \rightarrow \left(\{0,-,-\},\{-,3,3\},\{-,-,5\}\right),$$

$$\left(\{1,-,2\}\right) \rightarrow \left(\{0,-,-\},\{-,3,3\},\{-,-,5\},\{5,-,6\}\right),$$

$$\left(\{3,3,-\}\right) \rightarrow \left(\{0,-,-\},\{-,3,3\},\{-,-,5\},\{5,-,6\},\{6,6,-\}\right),$$

$$\left(\{-,1,1\}\right) \rightarrow \left(\{0,-,-\},\{-,3,3\},\{-,-,5\},\{5,-,6\},\{6,6,-\},\{-,8,8\}\right).$$

Therefore the new partition is $\left(8^2,6^3,5^2,3^2\right)$.

To represent this last part and to ensure the accuracy of the results that we obtained (**Table 3**).

It is very clear; after using the method of **Table 2** the new partition is $\left(8^2,6^3,5^2,3^2\right)$.

It is natural to point out that the changes taking place here are different in style from Mahmood presented in [7], as he depended on a change in the shape of what was already present in the same site.

3. Conclusion

This technique will make an important difference for us in coding any partition that depends on its representation on the concept of the *e*-abacus diagram and therefore we only have to use the same method used here to find the new representation of the new partition of the original partition. Research can be completely used [8]-[14].

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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