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Schultz and Modified Schultz Polynomials of Vertex Identification Chain for Square and Complete Square Graphs

Mahmood M. Abdullah, Ahmed M. Ali

College of Computer Sciences and Mathematics: Mathematics, University of Mosul, Mosul, Iraq
Email: Mmadain7@gmail.com, ahmedgraph@uomosul.edu.iq

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Abstract

In this paper, we find the polynomials, indices and average distance for Schultz and modified Schultz of vertex identification chain for 4-cycle and 4-cycle complete.

Subject Areas

Discrete Mathematics, Graph Theory, Combinatory

Keywords

Schultz, Modified Schultz, Polynomials, Indices, Chain of Vertex Identification Graphs

1. Introduction

Given the importance of topological evidence, which can be deduced from the polynomial by finding the derivative with respect to a specific variable and then compensating for this variable by one value, therefore we have in this paper found the polynomial of Schultz and modified Schultz for a chain of special graphs which is the 4-cycle and 4-cycle complete by identification symmetrical vertices.

In this paper, we can refer to the basic concepts in graph theory and topological indices to the references [1] [2] [3]. Let $G=(V,E)$ be a simple connected graph without loop and multi-edges, where $V=V(G)$ and $E=E(G)$ be a set of vertices and edges of respect to a graph G. The distance between any two vertices of $V(G)$ is the shortest path between them which denoted its by $d(u,v)$, $u,v \in V(G)$ and the maximum distance between any two vertices in G is called the diameter graph G, that is: $\delta = \max_{u,v\in V(G)}\{d(u,v)\}$. The degree of vertex u in a graph G is the number of the edges which incident on u and denoted by $degu$.

Introduced Schultz index by Schultz in 1989 [4] and in 1997 Klavžar and Gutman were defined the modified Schultz index [5]. The Schultz index $Sc(G)$ and the modified Schultz index $Sc^*(G)$ are have defined as:

$$Sc(G) = \sum_{\{u,v\}\subseteq V(G)}(degv + degu)d(u,v). \tag{1.1}$$

$$Sc^*(G) = \sum_{\{u,v\}\subseteq V(G)}(degv \cdot degu)d(u,v). \tag{1.2}$$

The Schultz and modified Schultz polynomials are have defined respectively as:

$$Sc(G;x) = \sum_{\{u,v\}\subseteq V(G)}(degv + degu)x^{d(u,v)}. \tag{1.3}$$

$$Sc^*(G;x) = \sum_{\{u,v\}\subseteq V(G)}(degv \cdot degu)x^{d(u,v)}. \tag{1.4}$$

From these polynomials can obtain:

1) The Schultz index:

$$Sc(G) = \frac{\mathrm{d}}{\mathrm{d}x}\left(Sc(G;x)\right)\Big|_{x=1}. \tag{1.5}$$

2) The modified Schultz index:

$$Sc^*(G) = \frac{\mathrm{d}}{\mathrm{d}x}\left(Sc^*(G;x)\right)\Big|_{x=1}. \tag{1.6}$$

3) The average distance of Schultz:

$$\overline{Sc}(G) = 2Sc(G)/p(G)\left(p(G)-1\right) \tag{1.7}$$

4) The average distance of modified Schultz:

$$\overline{Sc^*}(G) = 2Sc^*(G)/p(G)\left(p(G)-1\right). \tag{1.8}$$

where $\frac{\mathrm{d}}{\mathrm{d}x}$ is represent the derivative w.r.t. x and $p(G)$ is the order of G.

There are many recent papers on polynomials and indices for Schultz and modified Schultz, see to references [6] [7] [8] and there are applications on Schultz and modified Schultz in chemistry, see to references [9] [10] [11].

Let $D_k(r,h)$ be the set of all (u,v) of G which distance between u and v is k such that $degu = r$ and $degv = h$ and $|D_k(G)|$ is the number of pairs (u,v) of G that are distance k "$D(G,k)$".

From clearly that $\sum_{k=1}^{diam(G)}|D_k(G)| = p(G)\left(p(G)-1\right)/2$.

2. The Vertex—Identification Chain (VIC)—Graphs

Let $\{G_1, G_2, \cdots, G_n\}$ be a set of pairwise disjoint graphs with vertices $u_i, v_i \in V(G_i)$, $i = 1,2,\cdots,n$, $n \geq 2$, then the vertex-identification chain graph $C_v(G_1, G_2, \cdots, G_n) \equiv C_v(G_1, G_2, \cdots, G_n : v_1 \cdot u_2; v_2 \cdot u_3; \cdots; v_{n-1} \cdot u_n)$ of $\{G_i\}_{i=1}^n$ with respect to the vertices $\{v_i, u_{i+1}\}_{i=1}^{n-1}$ is the graph obtained from the graphs $G_1, G_2, \cdots, G_n$ by identifying the vertex v_i with the vertex u_{i+1} for all $i = 1,2,\cdots,n-1$. (See **Figure 1**):

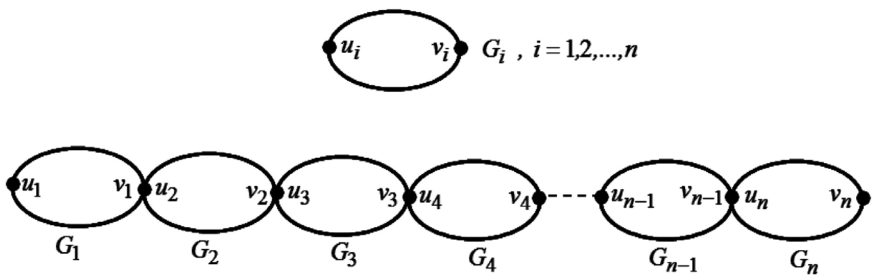

Figure 1. The graph $C_v(G_1, G_2, \cdots, G_n)$.

Some Properties of Graph $C_v(G_1, G_2, \cdots, G_n)$:

1) $p\left(C_v(G_1, G_2, \cdots, G_n)\right) = \sum_{i=1}^{n} p(G_i) - (n-1)$.

2) $q\left(C_v(G_1, G_2, \cdots, G_n)\right) = \sum_{i=1}^{n} q(G_i)$.

3) $n \le diam\left(C_v(G_1, G_2, \cdots, G_n)\right) \le \sum_{i=1}^{n} diam(G_i)$.

The equality of lower bound is satisfied at K_2 but the upper bound is satisfied at path graph.

If $G_i \equiv H_p$, for all $1 \le i \le n$, where H_p is a connected graph of order *p*, we denoted $C_v(H_p, H_p, \cdots, H_p)$ by $C_v(H_p)_n$.

2.1. Schultz and Modified Schultz of $C_v(C_4)_p$

From **Figure 2**, we note that $p\left(C_v(C_4)_p\right) = 3p+1$, $q\left(C_v(C_4)_p\right) = 4p$ and $diam\left(C_v(C_4)_p\right) = 2p$, for all $1 \le i, j \le p$, $i \ne j$, $2 \le h, k \le p$, $h \ne k$, then we have:

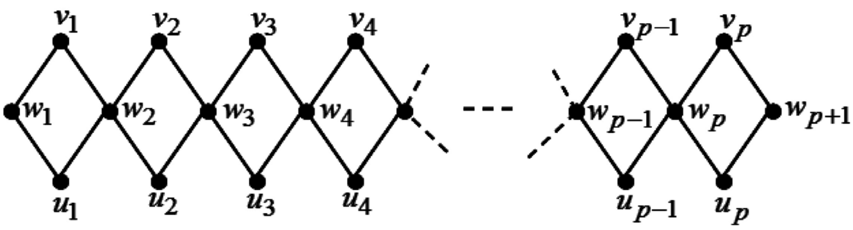

Figure 2. The graph $C_v(C_4)_p$.

Table 1. The vertices degrees of the graph $C_v(C_4)_p$.

+ / ×	$degu_i = 2$	$degv_i = 2$	$degw_1 = 2$	$degw_h = 4$	$degw_{p+1} = 2$
$degu_j = 2$	4 / 4	4 / 4	4 / 4	6 / 8	4 / 4
$degv_j = 2$	4 / 4	4 / 4	4 / 4	6 / 8	4 / 4
$degw_1 = 2$	4 / 4	4 / 4		6 / 8	4 / 4
$degw_k = 4$	6 / 8	6 / 8	6 / 8	8 / 16	6 / 8
$degw_{p+1} = 2$	4 / 4	4 / 4	4 / 4	6 / 8	

Theorem 2.1.1: For $p \geq 2$, then we have:

1) $Sc\left(C_v\left(C_4\right)_p;x\right) = 8(3p-1)x + 4(7p-5)x^2 + 4\sum_{k=3}^{2p}(6p-3k+1)x^k$.

2) $Sc^*\left(C_v\left(C_4\right)_p;x\right) = 16(2p-1)x + 4(9p-8)x^2 + 16\sum_{k=3}^{2p-1}(2p-k)x^k + 4x^{2p}$.

Proof: For all $p \geq 4$ and every two vertices $u,v \in V\left(C_v\left(C_4\right)_p\right)$, there is $d(u,v) = k$, $1 \leq k \leq 2p$, then obviously, $\sum_{i=1}^{2p}|D_i| = \frac{3p(3p+1)}{2}$. We will have six partitions for proof:

P1. If $d(u,v) = 1$, then $|D_1| = 4p = q\left(C_v\left(C_4\right)_p\right)$ and we have two subsets of it:

P1.1. $|D_1(2,2)| = \left|\left\{(u_1,w_1),(u_p,w_{p+1}),(v_1,w_1),(v_1,w_p)\right\}\right| = 4$.

P1.2. $|D_1(2,4)| = \left|\left\{(u_{i-1},w_i),(v_{i-1},w_i),(u_i,w_i),(v_i,w_i) : 2 \leq i \leq p\right\}\right| = 4(p-1)$.

P2. If $d(u,v) = 2$, then $|D_2| = 6p-4$ and we have three subsets of it:

P2.1. $|D_2(2,2)|$

$= \left|\left\{(u_i,u_{i+1}),(u_i,v_{i+1}),(v_i,v_{i+1}),(v_i,u_{i+1}) : 1 \leq i \leq p-1\right\} \cup \left\{(u_i,v_i) : 1 \leq i \leq p\right\}\right|$

$= 5p-4.$

P2.2. $|D_2(2,4)| = \left|\left\{(w_1,w_2),(w_{p+1},w_p)\right\}\right| = 2$.

P2.3. $|D_2(4,4)| = \left|\left\{(w_i,w_{i+1}) : 2 \leq i \leq p-1\right\}\right| = p-2$.

P3. If $d(u,v) = k$, then $|D_k| = 9p + \frac{1}{2}k + 3$, and we have:

1) If k is an odd, $3 \leq k \leq 2p-3$, then $|D_k| = 4p-2k+2$, and we have two subsets of it:

P3.1. $|D_k(2,2)| = \left|\left\{\left(w_1,u_{k-\frac{k-1}{2}}\right),\left(w_1,v_{k-\frac{k-1}{2}}\right),\left(w_1,u_{k-\frac{k-1}{2}}\right),\left(w_1,v_{k-\frac{k-1}{2}}\right)\right\}\right| = 4$.

P3.2.

$|D_k(2,4)| = \left|\left\{\left(u_{i+\frac{k-1}{2}},w_i\right),\left(v_{i+\frac{k-1}{2}},w_i\right),\left(u_{i-1},w_{i+\frac{k-1}{2}}\right),\left(v_{i-1},w_{i+\frac{k-1}{2}}\right) : 2 \leq i \leq p-\frac{k-1}{2}\right\}\right|$

$= 4\left(p - \frac{k-1}{2} - 1\right).$

2) If k is an even, $4 \leq k \leq 2p-4$, then $|D_k| = 5p + \frac{5}{2}k + 1$, and we have three subsets of it:

P3.3. $|D_k(2,2)| = \left|\left\{\left(u_i,u_{i+\frac{k}{2}}\right),\left(u_i,v_{i+\frac{k}{2}}\right),\left(v_i,v_{i+\frac{k}{2}}\right),\left(v_i,u_{i+\frac{k}{2}}\right) : 1 \leq i \leq p-\frac{k}{2}\right\}\right|$

$= 4\left(p - \frac{k}{2}\right).$

P3.4. $|D_k(2,4)| = \left|\left\{\left(w_1,w_{1+\frac{k}{2}}\right),\left(w_{p+1},w_{p-\frac{k}{2}+1}\right)\right\}\right| = 2$.

P3.5. $|D_k(4,4)| = \left|\left\{\left(w_i,w_{i+\frac{k}{2}}\right) : 2 \leq i \leq p-\frac{k}{2}\right\}\right| = p - \frac{k}{2} - 1$.

P4. If $d(u,v)=2p-2$, then $|D_{2p-2}|=6$, and we have two subsets of it:

P4.1. $|D_{2p-2}(2,2)|=|\{(u_1,u_p),(v_1,v_p),(u_1,v_p),(v_1,u_p)\}|=4$.

P4.2. $|D_{2p-2}(2,4)|=|\{(w_1,w_p),(w_{p+1},w_2)\}|=2$.

P5. If $d(u,v)=2p-1$, then $|D_{2p-1}|=4$, and we have one subset of it:

$$|D_{2p-1}(2,2)|=|\{(w_1,u_p),(w_1,v_p),(w_{p+1},u_1),(w_{p+1},v_1)\}|=4.$$

P6. If $d(u,v)=2p$, then $|D_{2p}|=1$, and we have one subset of it:

$$|D_{2p}(2,2)|=|\{(w_1,w_{p+1})\}|=1.$$

Hence, from **P1** to **P6** and **Table 1**, we get:

$$\begin{aligned}
Sc\left(C_v(C_4)_p;x\right)&=\{4(4)+6(4(p-1))\}x+\{4(5p-4)+6(2)+8(p-2)\}x^2\\
&+\begin{cases}\sum_{k=3}^{2p-3}\left\{4(4)+6\left(4\left(p-\frac{k-1}{2}-1\right)\right)\right\}x^k, & k \text{ is an odd}\\ \sum_{k=4}^{2p-4}\left\{4\left(4\left(p-\frac{k}{2}\right)\right)+6(2)+8\left(p-\frac{k}{2}-1\right)\right\}x^k, & k \text{ is an even}\end{cases}\\
&+\{4(4)+6(2)\}x^{2p-2}+\{4(4)\}x^{2p-1}+\{4(1)\}x^{2p}\\
&=8(3p-1)x+4(7p-5)x^2+4\sum_{k=3}^{2p}(6p-3k+1)x^k.
\end{aligned}$$

And,

$$\begin{aligned}
Sc^*\left(C_v(C_4)_p;x\right)&=\{4(4)+8(4(p-1))\}x+\{4(5p-4)+8(2)+16(p-2)\}x^2\\
&+\begin{cases}\sum_{k=3}^{2p-3}\left\{4(4)+8\left(4\left(p-\frac{k-1}{2}-1\right)\right)\right\}x^k, & k \text{ is an odd,}\\ \sum_{k=4}^{2p-4}\left\{4\left(4\left(p-\frac{k}{2}\right)\right)+8(2)+16\left(p-\frac{k}{2}-1\right)\right\}x^k, & k \text{ is an even}\end{cases}\\
&+\{4(4)+8(2)\}x^{2p-2}+\{4(4)\}x^{2p-1}+\{4(1)\}x^{2p}\\
&=16(2p-1)x+4(9p-8)x^2+16\sum_{k=3}^{2p-1}(2p-k)x^k+4x^{2p}.
\end{aligned}$$

By simple, we can calculate:

1) $Sc\left(C_v(C_4)_2;x\right)=40x+36x^2+16x^3+4x^4$.

$Sc^*\left(C_v(C_4)_2;x\right)=48x+40x^2+16x^3+4x^4$.

2) $Sc\left(C_v(C_4)_3;x\right)=64x+64x^2+40x^3+28x^4+16x^5+4x^6$.

$Sc^*\left(C_v(C_4)_3;x\right)=80x+76x^2+48x^3+32x^4+16x^5+4x^6$. ■

Corollary 2.1.2: For $p\ge 2$, then we have:

1) $Sc\left(C_v(C_4)_p\right)=8p\left(2p^2+p+1\right)$.

2) $Sc^*\left(C_v(C_4)_p\right)=\frac{32}{3}p\left(2p^2+1\right)$. ■

Corollary 2.1.3: For $p\ge 2$, then we have:

1) $\overline{Sc}\left(C_v(C_4)_p\right)=\frac{16}{27}\left(6p+1+8/(3p+1)\right)$.

2) $\overline{Sc^*}\left(C_v\left(C_4\right)_p\right)=\frac{128}{81}\left(3p-1+11/2\left(3p+1\right)\right)$. ■

2.2. Schultz and Modified Schultz of $C_v\left(K_4\right)_p$

From **Figure 3**, we note that $p\left(C_v\left(K_4\right)_p\right)=3p+1$, $q\left(C_v\left(K_4\right)_p\right)=6p$ and $daim\left(C_v\left(K_4\right)_p\right)=p$, for all $1\le i,j\le p$, $i\ne j$, $2\le h,k\le p$, $h\ne k$, then we have:

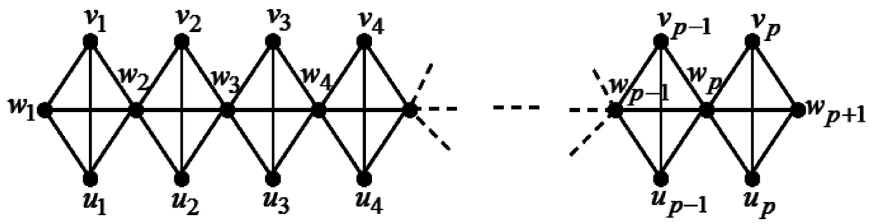

Figure 3. The graph $C_v\left(K_4\right)_p$.

Table 2. The vertices degrees of the graph $C_v\left(K_4\right)_p$.

× / +	$degu_i=3$	$degv_i=3$	$degw_1=3$	$degw_h=6$	$degw_{p+1}=3$
$degu_j=3$	9 / 6	9 / 6	9 / 6	18 / 9	9 / 6
$degv_j=3$	9 / 6	9 / 6	9 / 6	18 / 9	9 / 6
$degw_1=3$	9 / 6	9 / 6		18 / 9	9 / 6
$degw_k=6$	18 / 9	18 / 9	18 / 9	36 / 12	18 / 9
$degw_{p+1}=3$	9 / 6	9 / 6	9 / 6	18 / 9	

Theorem 2.2.1: for $p\ge 3$, then we have:

1) $Sc\left(C_v\left(K_4\right)_p;x\right)=18\left(3p-1\right)x+18\sum_{k=2}^{p}\left(4p-4k+3\right)x^k$.

2) $Sc^*\left(C_v\left(K_4\right)_p;x\right)=9\left(13p-8\right)x+72\sum_{k=2}^{p-1}\left(2p-2k+1\right)x^k+81x^p$.

Proof: For all $p\ge 4$, and every vertex $u,v\in V\left(C_v\left(K_4\right)_p\right)$ there is $d\left(u,v\right)=k$, $1\le k\le p$, then obviously, $\sum_{i=1}^{2p}\left|D_i\right|=\frac{3p\left(3p+1\right)}{2}$. We will have four partitions for proof:

P1. If $d\left(u,v\right)=1$, then $\left|D_1\right|=6p=q\left(C_v\left(K_4\right)_p\right)$ and we have three subsets of it:

P1.1. $\left|D_1\left(3,3\right)\right|=\left|\left\{\left(u_i,v_i\right):1\le i\le p\right\}\cup\left\{\left(u_1,w_1\right),\left(v_1,w_1\right),\left(u_p,w_{p+1}\right),\left(v_p,w_{p+1}\right)\right\}\right|$
$=p+4$.

P1.2. $\left|D_1(3,6)\right| = \left|\{(u_i,w_i),(u_i,w_{i+1}),(v_i,w_i),(v_i,w_{i+1}) : 2 \le i \le p-1\}\right.$
$\left.\cup\{(u_p,w_p),(u_1,w_2),(v_p,w_p),(v_1,w_2),(w_1,w_2),(w_p,w_{p+1})\}\right|$
$= 4p-2.$

P1.3. $\left|D_1(6,6)\right| = \left|\{(w_i,w_{i+1}) : 2 \le i \le p-1\}\right| = p-2$.

P2. If $d(u,v)=k$, $2 \le k \le p-2$, then $\left|D_k\right| = 9(p-k+1)$ and we have three subsets of it:

P2.1

$$\left|D_k(3,3)\right| = \left|\{(u_i,u_{i+k-1}),(v_i,v_{i+k-1}),(u_i,v_{i+k-1}),(v_i,u_{i+k-1}) : 1 \le i \le p-k+1\}\right.$$
$$\left.\cup\{(u_k,w_1),(u_{p-k+1},w_{p+1}),(v_k,w_1),(v_{p-k+1},w_{p+1})\}\right| = 4(p-k+2).$$

P2.2.

$$\left|D_k(3,6)\right|$$
$$= \left|\{(u_i,w_{i+k}),(u_{i+k-1},w_i),(v_i,w_{i+k}),(v_{i+k-1},w_i) : 2 \le i \le p-k\}\right.$$
$$\left.\cup\{(u_1,w_{1+k}),(u_p,w_{p-k+1}),(v_1,w_{1+k}),(v_p,w_{p-k+1}),(w_1,w_{1+k}),(w_{p+1},w_{p-k+1})\}\right|$$
$$= 2(2p-2k+1).$$

P2.3. $\left|D_k(6,6)\right| = \left|\{(w_i,w_{i+k}) : 2 \le i \le p-k\}\right| = p-k-1$.

P3. If $d(u,v)=p-1$, then $\left|D_{p-1}\right| = 18$ and we have two subsets of it:

P3.1. $\left|D_{p-1}(3,3)\right| = \left\{\{(u_i,u_{i+p-2}),(u_i,v_{i+p-2}),(v_i,v_{i+p-2}),(v_i,u_{i+p-2}) : i=1,2\}\right.$
$\left.\cup\{(u_2,w_{p-1}),(u_{p-1},w_1),(v_2,w_{p+1}),(v_{p-1},w_1)\}\right| = 12.$

P3.2. $\left|D_{p-1}(3,6)\right| = \left|\{(u_1,w_p),(u_p,w_2),(v_p,w_2),(v_1,w_p),(w_1,w_p),(w_1,w_p)\}\right| = 6$.

P4. If $d(u,v)=p$, then $\left|D_p\right| = 9$, and we have one subset of it:

$$\left|D_p(3,3)\right| = \left|\{(u_1,u_p),(u_1,w_{p+1}),(u_1,v_p),(v_1,w_{p+1}),(v_1,v_p),\right.$$
$$\left.(u_p,w_1),(u_p,v_1),(w_1,w_{p+1}),(w_1,v_p)\}\right| = 9$$

Hence, from P1 to P4 and Table 2, we get:

$$Sc\left(C_v(K_4)_p;x\right) = \{6(p+4)+9(4p-2)+12(p-2)\}x$$
$$+\sum_{k=2}^{p-2}\{6(4p-4k+8)+9(4p-4k+2)+12(p-k-1)\}x^k$$
$$+\{6(12)+9(6)\}x^{p-1}+\{6(9)\}x^p$$
$$=18(3p-1)x+18\sum_{k=2}^{p}(4p-4k+3)x^k.$$

And,

$$Sc^*\left(C_v(K_4)_p;x\right) = \{9(p+4)+18(4p-2)+36(p-2)\}x$$
$$+\sum_{k=2}^{p-2}\{9(4p-4k+8)+18(4p-4k+2)+36(p-k-1)\}x^k$$
$$+\{9(12)+18(6)\}x^{p-1}+\{9(9)\}x^p$$
$$=9(13p-8)x+72\sum_{k=2}^{p-1}(2p-2k+1)x^k+81x^p.$$

By simply we can calculate:

$$Sc\left(C_v\left(K_4\right)_3;x\right)=144x+126x^2+54x^2.$$

$$Sc\left(C_v\left(K_4\right)_3;x\right)=279x+216x^2+81x^2.$$

This completes the proof. ■

Remark:

1) $Sc\left(C_v\left(K_4\right)_2;x\right)=90x+54x^2$.

2) $Sc^*\left(C_v\left(K_4\right)_2;x\right)=162x+81x^2$.

Corollary 2.2.2: For $p\geq 2$, then we have:

1) $Sc\left(C_v\left(K_4\right)_p\right)=3p\left(4p^2+9p-1\right)$.

2) $Sc^*\left(C_v\left(K_4\right)_p\right)=6p\left(4p^2+6p-1\right)$. ■

Corollary 2.2.3: For $p\geq 2$, then we have:

1) $\overline{Sc}\left(C_v\left(K_4\right)_p\right)=\frac{2}{9}\left(12p+23-32/(3p+1)\right)$.

2) $\overline{Sc^*}\left(C_v\left(K_4\right)_p\right)=\frac{8}{9}\left(6p+7-23/2(3p+1)\right)$. ■

3. Some Properties of the Coefficients of Schultz and Modified Schultz Polynomials

A finite sequence $(a_1,a_2,\cdots,a_h)$ of h positive integers is coefficients of polynomial $P(x)=\sum_{i=1}^{h}a_ix^i$. Then:

1) The polynomial $P(x)$ is called *j*-unimodal if, for some index *j*, $a_1\leq a_2\leq\cdots\leq a_j\geq a_{j+1}\geq\cdots\geq a_h$ and it is strictly *j*-unimodal if the inequality holds without equalities.

2) The polynomial $P(x)$ is called monotonically increasing (or monotonically decreasing) if, $a_i\leq a_{i+1}$ or $a_i\geq a_{i+1}$, respectively, for all $1\leq i\leq h$ and it is strictly-increasing or strictly-decreasing respectively if the inequalities hold without equalities.

3) The polynomial $P(x)$ is called palindromic if $a_i=a_{h-i+1}$, for all $1\leq i\leq h$ and is called semi-palindromic if $a_j=a_{h-j+1}$, $1+i\leq j\leq h-i$ and for all $1\leq i\leq h-2$.

4) The polynomial $P(x)$ is called troubled if $a_i\neq a_{i+1}$, for all $1\leq i\leq h$.

5) The polynomial $P(x)$ is called equality if $a_i=a_{i+1}$, for all $1\leq i\leq h$ and is called semi-equality if $a_i=a_{i+1}$ for some values *i*.

The following **Table 3** shows the properties of polynomials coefficients of Schultz and modified Schultz of $C_v\left(C_4\right)_p$ and $C_v\left(K_4\right)_p$.

4. Conclusion

In this paper, we obtained the general formulas of polynomials for Schultz and modified Schultz polynomials of operation vertex identification chain for square and complete square graphs and indices of its. Also, we discuss some properties of the coefficients of these polynomials.

Table 3. The some properties of the coefficients of $C_v(C_4)_p$ and $C_v(K_4)_p$, $p \geq 3$.

Polynomials of types graphs \ Some Properties	Property 1	Property 2	Property 3	Property 4	Property 5
$Sc\left(C_v(C_4)_p;x\right)$	Satisfy at $j=2$	Not satisfy	Not satisfy	Satisfy	Not satisfy
$Sc^*\left(C_v(C_4)_p;x\right)$	Satisfy at $j=2$	Not satisfy	Not satisfy	Satisfy	Not satisfy
$Sc\left(C_v(K_4)_p;x\right)$	Satisfy at $j=2$	Not satisfy	Not satisfy	Satisfy	Not satisfy
$Sc^*\left(C_v(K_4)_p;x\right)$	Satisfy at $j=2$	Not satisfy	Not satisfy	Satisfy	Not satisfy

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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