



# Numerical Solution of Fluid Flow in Horizontal Tube under Effects of Radiation Field

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## Abstract

In this work, we studied the matter of heat transfer by the natural convection for a dissipatable fluid which flows in a tube, its walls composed from porous material, and a mathematical model was constructed, represented by a system of two-dimension non-linear partial differential equations, describing the flow behavior of the fluid through a horizontal tube and the distribution of temperatures inside the tube and under the influence of a vertical magnetic field at the tube level. The resulting differential equations were treated by numerical methods using the alternating directions implicit method (ADI), which is one of the finite differences methods, and in both cases: unsteady state and steady state. And we completed the study of the effect for each of: Prandtl number, Schmidt number, Gratshof number, and Radiation parameter.

## Subject Areas

Applied Mathematics

## Keywords

Heat Transfer, Porous Medium, Prandtl Number, Schmidt Number, Gratshof Number

## 1. Introduction

The flow and heat transfer of electrically conducting fluid in channels and circular tubes under the effect of a transverse magnetic field occurs in magnetohydrodynamic (MHD) generators, pumps and flow meters and has applications in nuclear reactors, filtration, geothermal systems and others [1].

In 2009, Singha studied an analytical solution to the problem of magnetohydrodynamic (MHD) free convection flow of an electrically conducting fluid be-

tween two heated parallel plates in the presence of an induced magnetic field [2].

In 2010, Cooky C., and Omubo-Pepple V., investigated the combined effects of radiative heat transfer and transverse magnetic field on steady flow of an electrically conducting optically thin fluid through a horizontal channel filled with porous medium and non-uniform temperature at walls; they concluded that increasing magnetic field, radiation and porosity parameters reduced the velocity and temperature profiles, and the shear stress at the wall while increasing radiation parameters causes an increase in the magnitude of the rate of heat transfer [3].

In 2011, Dalia S., and Ioan P., have studied the entropy generation production for a problem of mixed convection flow of a fluid saturated porous medium through an inclined channel with uniform heat walls; he used the analytical results obtained for the velocity and temperature profiles to obtain the entropy production [4].

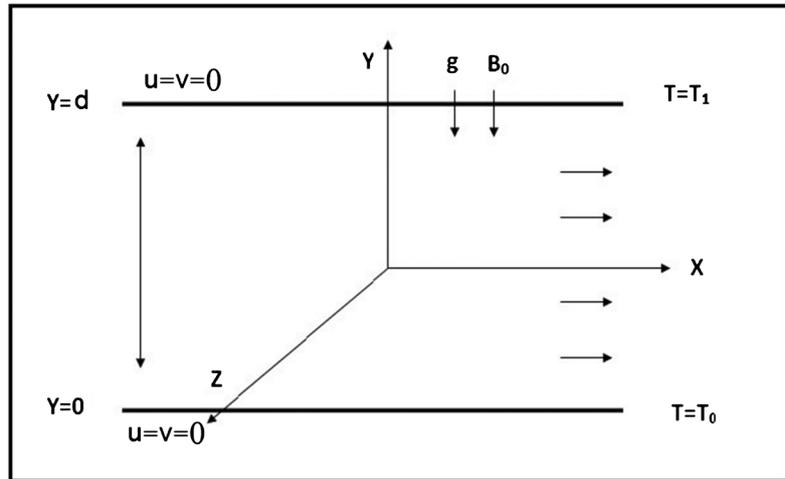
In 2012, Sudershan S., Reddy R., and Reddy J., have discussed the effect of radiation and chemical reaction on a free convection MHD flow through a porous medium boundary by vertical surface [5].

In 2015, Mukhopadhyay S. and Mandal I., studied the numerical solutions for steady MHD mixed convection boundary layer flow and heat transfer over a porous plate in the presence of the velocity and thermal slip; we found the effect of the magnetic field of non-compressible liquid on increasing fluid velocity, which in turn leads to a decrease in the temperature of the liquid [6].

In 2017, Kumari K. and Gayal M. presented the effect of mass transfer, viscous dissipative and suction parameter on two dimensional steady hydromagnetic viscous fluid flow between two parallel plates in the presence of thermal radiation and we found that velocity, temperature and concentration decrease with increase suction parameter and Reynolds number in addition to the relation among the different quality physicals [7]. The aim of this work, is to study the numerical solution of the equations from the heat transfer, diffusion and motion in a porous tube with the presences of magnetic field and radiation has been investigated. It's found that the parameters  $Pr$ ,  $Sc$ ,  $R$  and  $Gr$  have a significant effect on the solution of the equation.

## 2. Formulation of the Problem

Consider the unsteady flow of a dissipative fluid passing through a long horizontal tube with porous walls, if we assume that the temperature at the two horizontal walls is constant so that  $T_0, T_1$  represents the lower (upper) walls temperature of horizontal tube with  $T_1 > T_0$ . Let  $u, v$  be the velocity components in directions  $x$  and  $y$  respectively, we assume that all the components in  $z$  direction vanish. The magnetic field is applied perpendicular to the  $x$ -axis which in the direction of flow and apposite to the  $y$  direction as it is illustrated by **Figure 1**:



**Figure 1.** Geometry of the problem.

The governing continuity, momentum, energy and concentration equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] - \frac{\nu}{k} u \tag{2.2}$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] + \frac{1}{\rho} \sigma B_0^2 v - g\beta(T - T_0) - g\beta^*(C - C_0) \tag{2.3}$$

Derive Equation (2.2) with respect to  $y$ , and Equation (2.3) with respect to  $x$ , and subtract the resulting equations we get:

$$\begin{aligned} & \frac{\partial}{\partial t} \left[ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right] + \left[ \frac{\partial}{\partial x} \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) - \frac{\partial}{\partial y} \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) \right] \\ & = \nu \nabla^2 \left[ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right] + \frac{\sigma B_0^2}{\rho} \frac{\partial v}{\partial x} - g\beta \frac{\partial T}{\partial x} - g\beta^* \frac{\partial C}{\partial x} + \frac{\nu}{k} \frac{\partial u}{\partial y} \end{aligned} \tag{2.4}$$

And this called the general motion equation.

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k^*}{\rho C_p} \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] - \frac{1}{\rho C_p} \left[ \frac{\partial q}{\partial y} \right] + \frac{\nu}{C_p} \left[ \left( \frac{\partial u}{\partial y} \right)^2 \right] \tag{2.5}$$

By using the Roseland approximations consider the radiative heat flux for optically thick fluid is given by [8]:

$$q = - \left( \frac{4\sigma^*}{3k_1} \right) \frac{\partial T^4}{\partial y} \tag{2.6}$$

where  $\sigma^*$  is the Stefan-Boltzmann constant and  $k_1$  is the mean absorption coefficient. Assume that the difference in temperature within the flow is sufficiently small such as that  $T^4$  can be expressed as a linear function of the temperature, we expand  $T^4$  in a Taylor's series about  $T_1$  and neglected higher

order terms, thus [9].

$$T^4 \approx -3T_1^4 + 4T_1^3 T \tag{2.7}$$

Hence the equation of energy Equation (2.5) becomes:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k^*}{\rho C_p} \frac{\partial^2 T}{\partial x^2} + \left( \frac{k^*}{\rho C_p} + \frac{16\sigma^* T_1^3}{3k_1 \rho C_p} \right) \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p} \left[ \left( \frac{\partial u}{\partial y} \right)^2 \right] \tag{2.8}$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left[ \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right] \tag{2.9}$$

where  $u, v$  are the velocity component in  $x$  and  $y$  directions respectively,  $t$  is the time and  $T, C, \rho, \nu, p, \sigma, B_0, g, k, k^*, C_p, q$  are the temperature, concentration, density, kinematic viscosity, pressure, electrical conductivity, Boltzmann number, gravitational acceleration, permeability of medium, thermal conductivity, specific heat with constant pressure, radiation flux respectively.

The following boundary conditions of the Problem given by:

$$\left. \begin{aligned} u = v = 0 \\ T = T_0, T_1 \\ C = C_0, C_1 \end{aligned} \right\} \text{ at } y = 0, d \tag{2.10}$$

Let us introduce the following similarity transformation [3]

$$\left. \begin{aligned} X = \frac{x}{d}, Y = \frac{y}{d}, U = \frac{ud}{\nu\sqrt{Gr}}, V = \frac{vd}{\nu\sqrt{Gr}}, \tau = \frac{t\nu\sqrt{Gr}}{d^2} \\ \Theta = \frac{T - T_0}{T_1 - T_0}, \phi = \frac{C - C_0}{C_1 - C_0} \end{aligned} \right\} \tag{2.11}$$

And non-dimensional parameters:

$$\left. \begin{aligned} M = B_0 h \left( \frac{\sigma}{\mu} \right)^{\frac{1}{2}}, \alpha = \frac{k^*}{\rho C_p}, R = \frac{3k^* k_1 + 16\sigma^* T_1^3}{3k_1 \mu C_p} \\ N = \frac{\nu}{h^2 \Delta T}, Sc = \frac{\nu}{D}, Da = \frac{k}{h^2}, Pr = \frac{\nu}{\alpha} \\ Gr = \frac{g \beta h^3 (T_1 - T_0)}{\nu^2}, Gr^* = \frac{g \beta^* h^3 (C_1 - C_0)}{\nu^2}, \epsilon = \frac{\nu}{C_p} \end{aligned} \right\} \tag{2.12}$$

where  $M$  is Hartmann number,  $\alpha$  is Thermal diffusion,  $R$  is Radiation coefficient,  $N$  is new physical quantity,  $Sc$  is Schmidt number,  $Da$  is Darcy number,  $Pr$  is Prandtl number,  $Gr$  is Gratshof number for heat transfer,  $Gr^*$  is Gratshof number for mass transfer,  $\epsilon$  is dispersion parameter. The above Equations (2.11) and (2.12) reduce the Equations (2.1), (2.4), (2.8) and (2.9) into the following system of non-dimensional equations:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2.13}$$

$$\begin{aligned} & \frac{\partial}{\partial \tau} \left[ \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y} \right] + \left[ \frac{\partial}{\partial X} \left( U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) - \frac{\partial}{\partial Y} \left( U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) \right] \\ & = \frac{1}{\sqrt{Gr}} \nabla^2 \left( \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y} \right) + \frac{M^2}{\sqrt{Gr}} \frac{\partial V}{\partial X} - \frac{\partial \Theta}{\partial X} - \frac{Gr^*}{Gr} \frac{\partial \phi}{\partial X} + \frac{1}{Da} \frac{1}{\sqrt{Gr}} \frac{\partial U}{\partial Y} \end{aligned} \tag{2.14}$$

$$\begin{aligned} & \frac{\partial \Theta}{\partial \tau} + U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} \\ &= \frac{1}{Pr} \frac{1}{\sqrt{Gr}} \frac{\partial^2 \Theta}{\partial X^2} + R \frac{1}{\sqrt{Gr}} \frac{\partial^2 \Theta}{\partial Y^2} + \epsilon N \sqrt{Gr} \left( \frac{\partial U}{\partial Y} \right)^2 \end{aligned} \tag{2.15}$$

$$\frac{\partial \phi}{\partial \tau} + U \frac{\partial \phi}{\partial X} + V \frac{\partial \phi}{\partial Y} = \frac{1}{Sc} \frac{1}{\sqrt{Gr}} \left[ \frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Y^2} \right] \tag{2.16}$$

The boundary conditions (2.10) in the non-dimensional form become:

$$\left. \begin{aligned} U &= V = 0 \\ \Theta &= 0, 1 \\ \phi &= 0, 1 \end{aligned} \right\} \text{ at } Y = 0, 1 \tag{2.17}$$

### 3. Method of Solution

In order to solve the system of Equations (2.13)-(2.16) with the boundary conditions (2.17), we resort to ADI finite difference method [10], and to achieve this, we have to start with the last Equation (2.16) then Equation (2.15) and finally Equation (2.14) as following:

#### 3.1. Solution of Diffusion Equation

$$\frac{\phi_{i,j}^* - \phi_{i,j,n}}{\frac{\Delta \tau}{2}} = \omega \left[ \frac{\phi_{i+1,j}^* - 2\phi_{i,j}^* + \phi_{i-1,j}^*}{(\Delta X)^2} + \frac{\phi_{i,j+1,n} - 2\phi_{i,j,n} + \phi_{i,j-1,n}}{(\Delta Y)^2} \right] \tag{3.1.1}$$

$$\begin{aligned} & -U_{i,j,n} \frac{\phi_{i+1,j}^* - \phi_{i-1,j}^*}{2\Delta X} - V_{i,j,n} \frac{\phi_{i,j+1,n} - \phi_{i,j-1,n}}{2\Delta Y} \\ \frac{\phi_{i,j,n+1} - \phi_{i,j}^*}{\frac{\Delta \tau}{2}} &= \omega \left[ \frac{\phi_{i+1,j}^* - 2\phi_{i,j}^* + \phi_{i-1,j}^*}{(\Delta X)^2} + \frac{\phi_{i,j+1,n+1} - 2\phi_{i,j,n+1} + \phi_{i,j-1,n+1}}{(\Delta Y)^2} \right] \tag{3.1.2} \\ & -U_{i,j,n} \frac{\phi_{i+1}^* - \phi_{i-1}^*}{2\Delta X} - V_{i,j,n} \frac{\phi_{i,j+1,n+1} - \phi_{i,j-1,n+1}}{2\Delta Y} \end{aligned}$$

Equations (3.1.1) and (3.1.2) can be reduced to give:

$$A_i \phi_{i-1,j}^* + B_i \phi_{i,j}^* + C_i \phi_{i+1,j}^* = D_i \tag{3.1.3}$$

where

$$\left. \begin{aligned} A_i &= -1 \\ B_i &= \frac{f_1}{f_3} \\ C_i &= -\frac{f_4}{f_3} \\ D_i &= \frac{f_5}{f_3} \phi_{i,j-1,n} + \frac{f_2}{f_3} \phi_{i,j,n} + \frac{f_6}{f_3} \phi_{i,j+1,n} \end{aligned} \right\} i = 1, 2, \dots, N \tag{3.1.4}$$

Followed by:

$$A_i \phi_{i,j-1,n+1} + B_i \phi_{i,j,n+1} + C_i \phi_{i,j+1,n+1} = D_i \tag{3.15}$$

where

$$\left. \begin{aligned} A_i &= -1 \\ B_i &= \frac{f_1}{f_3} \\ C_i &= -\frac{f_6}{f_5} \\ D_i &= \frac{f_3}{f_5} \phi_{i-1}^* + \frac{f_2}{f_5} \phi_{i,j}^* + \frac{f_4}{f_5} \phi_{i+1}^* \end{aligned} \right\} i = 1, 2, \dots, N \quad (3.1.6)$$

where

$$\begin{aligned} f_1 &= 1 + \lambda\omega, \quad f_2 = 1 - \lambda\omega, \quad f_3 = \frac{\lambda\omega}{2} + \frac{\lambda h}{4} U_{i,j,n}, \\ f_4 &= \frac{\lambda\omega}{2} - \frac{\lambda h}{4} U_{i,j,n}, \quad f_5 = \frac{\lambda\omega}{2} + \frac{\lambda h}{4} V_{i,j,n}, \quad f_6 = \frac{\lambda\omega}{2} - \frac{\lambda h}{4} V_{i,j,n} \end{aligned}$$

### 3.2. Solution of Heat Equation

$$\begin{aligned} \frac{\Theta_{i,j}^* - \Theta_{i,j,n}}{\frac{\Delta\tau}{2}} &= \omega \left[ \frac{\Theta_{i+1,j}^* - 2\Theta_{i,j}^* + \Theta_{i-1,j}^*}{(\Delta X)^2} \right] + \varpi \left[ \frac{\Theta_{i,j+1,n} - 2\Theta_{i,j,n} + \Theta_{i,j-1,n}}{2\Delta Y} \right] \\ &\quad - U_{i,j,n} \frac{\Theta_{i+1,j}^* - \Theta_{i-1,j}^*}{2\Delta X} - V_{i,j,n} \frac{\Theta_{i,j+1,n} - \Theta_{i,j-1,n}}{2\Delta Y} + \Phi \end{aligned} \quad (3.2.1)$$

$$\begin{aligned} \frac{\Theta_{i,j,n+1} - \Theta_{i,j}^*}{\frac{\Delta\tau}{2}} &= \omega \left[ \frac{\Theta_{i+1,j}^* - 2\Theta_{i,j}^* + \Theta_{i-1,j}^*}{(\Delta X)^2} \right] + \varpi \left[ \frac{\Theta_{i,j+1,n+1} - 2\Theta_{i,j,n+1} + \Theta_{i,j-1,n+1}}{(\Delta Y)^2} \right] \\ &\quad - U_{i,j,n} \frac{\Theta_{i+1,j}^* - \Theta_{i-1,j}^*}{2\Delta X} - V_{i,j,n} \frac{\Theta_{i,j+1,n+1} - \Theta_{i,j-1,n+1}}{2\Delta Y} + \Phi \end{aligned} \quad (3.2.2)$$

Equations (3.2.1) and (3.2.2) can be reduced to give:

$$A_i \Theta_{i-1,j}^* + B_i \Theta_{i,j}^* + C_i \Theta_{i+1,j}^* = D_i \quad (3.2.3)$$

where

$$\left. \begin{aligned} A_i &= -1 \\ B_i &= \frac{2f_1}{f_3} \\ C_i &= -\frac{f_4}{f_3} \\ D_i &= \frac{f_5}{f_3} \Theta_{i,j-1,n} + \frac{2f_2}{f_3} \Theta_{i,j,n} + \frac{f_6}{f_3} \Theta_{i,j+1,n} + \frac{h^2\Phi}{f_3} \end{aligned} \right\} i = 1, 2, \dots, N$$

followed by

$$A_i \Theta_{i,j-1,n+1} + B_i \Theta_{i,j,n+1} + C_i \Theta_{i,j+1,n+1} = D_i \quad (3.2.5)$$

where

$$\left. \begin{aligned} A1_i &= -1 \\ B1_i &= \frac{2f_7}{f_5} \\ C1_i &= -\frac{f_6}{f_5} \\ D1_i &= \frac{f_3}{f_5} \Theta_{i-1,j}^* + \frac{2f_8}{f_5} \Theta_{i,j}^* + \frac{f_4}{f_5} \Theta_{i+1,j}^* + \frac{h^2 \Phi}{f_5} \end{aligned} \right\} i = 1, 2, \dots, N \quad (3.2.6)$$

where

$$\begin{aligned} f_1 &= \frac{1}{\lambda} + \omega, \quad f_2 = \frac{1}{\lambda} - \omega, \quad f_3 = \omega + \frac{h}{2} U_{i,j,n}, \quad f_4 = \omega - \frac{h}{2} U_{i,j,n}, \\ f_5 &= \omega + \frac{h}{2} V_{i,j,n}, \quad f_6 = \omega - \frac{h}{2} V_{i,j,n}, \quad f_7 = \frac{1}{\lambda} + \omega, \quad f_8 = \frac{1}{\lambda} - \omega \end{aligned}$$

### 3.3. Solution of the General Motion Equation

$$\frac{\xi_{i,j}^* - \xi_{i,j,n}}{\frac{\Delta \tau}{2}} = \omega \left[ \frac{\xi_{i+1,j}^* - 2\xi_{i,j}^* + \xi_{i-1,j}^*}{(\Delta X)^2} + \frac{\xi_{i,j+1,n} - 2\xi_{i,j,n} + \xi_{i,j-1,n}}{(\Delta Y)^2} \right] + \omega \xi_{i,j,n} \quad (3.3.1)$$

$$\frac{\xi_{i,j,n+1} - \xi_{i,j}^*}{\frac{\Delta \tau}{2}} = \omega \left[ \frac{\xi_{i+1,j}^* - 2\xi_{i,j}^* + \xi_{i-1,j}^*}{(\Delta X)^2} + \frac{\xi_{i,j+1,n+1} - 2\xi_{i,j,n+1} + \xi_{i,j-1,n+1}}{(\Delta Y)^2} \right] + \omega \xi_{i,j,n+1} \quad (3.3.2)$$

Equations (3.3.1) and (3.3.2) can be reduced to give:

$$A_i \xi_{i-1,j}^* + B_i \xi_{i,j}^* + C_i \xi_{i+1,j}^* = D_i \quad (3.3.3)$$

where

$$\left. \begin{aligned} A_i &= -1 \\ B_i &= \frac{f_1}{f_2} \\ C_i &= -1 \\ D_i &= \xi_{i,j-1,n} + \frac{f_3}{f_2} \xi_{i,j,n} + \xi_{i,j+1,n} \end{aligned} \right\} i = 1, 2, \dots, N \quad (3.3.4)$$

followed by

$$A1_i \xi_{i,j-1,n+1} + B1_i \xi_{i,j,n+1} + C1_i \xi_{i,j+1,n+1} = D1_i \quad (3.3.5)$$

where

$$\left. \begin{aligned} A1_i &= -1 \\ B1_i &= \frac{f_3}{f_2} \\ C1_i &= -1 \\ D1_i &= \xi_{i-1,j}^* + \frac{f_4}{f_2} \xi_{i,j}^* + \xi_{i+1,j}^* \end{aligned} \right\} i = 1, 2, \dots, N \quad (3.3.6)$$

where

$$f_1 = 1 + \lambda \omega, \quad f_2 = \frac{\lambda \omega}{2}, \quad f_3 = 1 - \lambda \omega + \frac{\lambda h^2 \omega}{2}, \quad f_4 = 1 - \lambda \omega$$

With boundary conditions:

$$\left. \begin{aligned} U = Constant, \quad U_{0,j,n} = 0, \quad U_{i,0,n} = 0 \\ V = Constant, \quad V_{0,j,n} = 0, \quad V_{i,0,n} = 0 \\ \phi_{i,0,n} = 0.0, \quad \phi_{i,N,n} = 1.0 \\ \Theta_{i,0,n} = 0.0, \quad \Theta_{i,N,n} = 10.0 \end{aligned} \right\} \quad (3.3.7)$$

The coefficients  $U, V$  are treated as constants during any one time-step of the computation, each of the equations (diffusion, heat, motion) creating a tridiagonal system which are solved by using Gauss elimination method, all are given in [5].

### 4. Conclusions

We present in this section some of the results obtained from the computation done on Equations (2.14), (2.15) and (2.16) for different points in the region of solution and this is because of the appearance of effect on this equations, and we find the following results (Figures 2-11):

- 1) With the increase of the radiation parameter, the faster steady state will be reached.
- 2) In energy equation, when  $Gr = 1.5, R = 2$  and  $Pr = 100$ , and the time-step increases, the faster steady state will be reached.
- 3) With the decrease of the Schmidt number  $Sc$ , the faster steady state will be reached.
- 4) In Darcy equation, when  $Gr = 1$  and  $Sc = 0.5$ , and the time-step increases, the faster steady state will be reached.
- 5) With the decrease of the Gratshof number  $Gr$ , the faster steady state will be reached.
- 6) With the decrease of the Prandtl number  $Pr$ , the faster steady state will be reached.

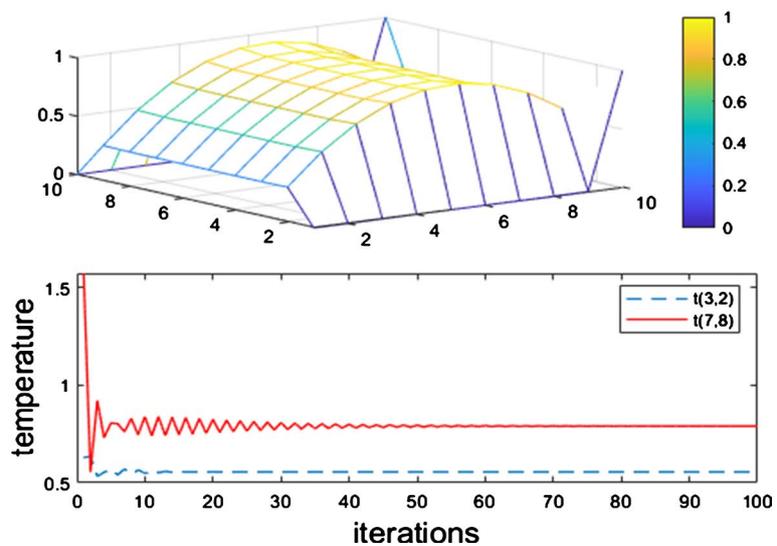


Figure 2. The temperature behavior.

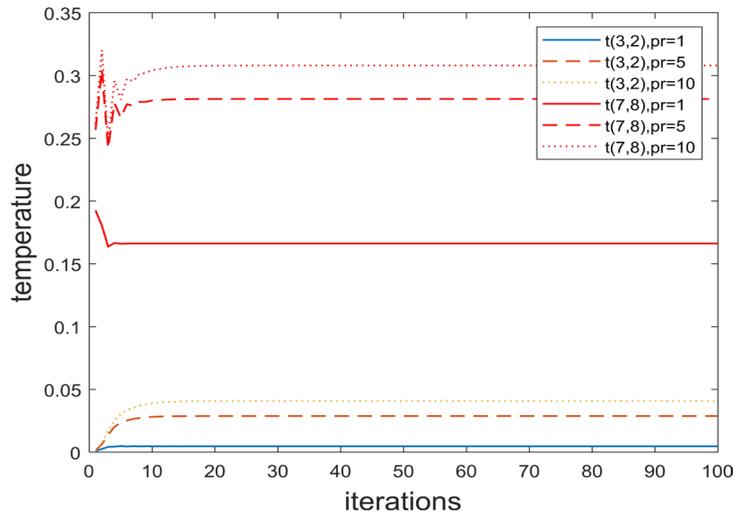


Figure 3. The effect of  $Pr$  on the temperature.

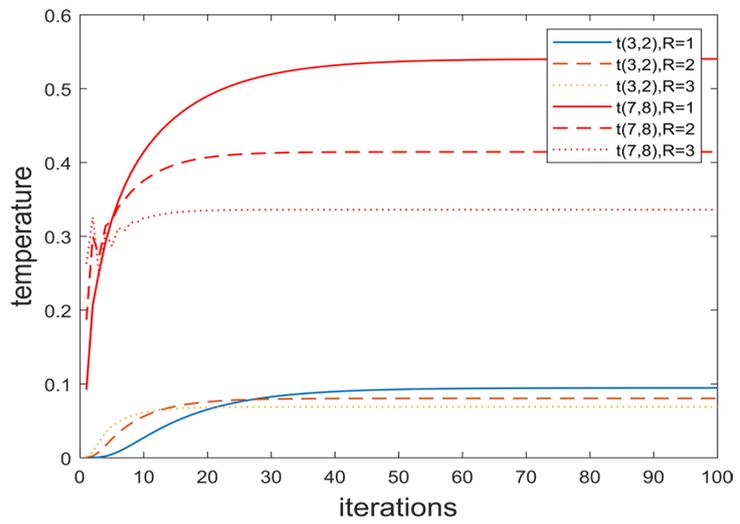


Figure 4. The effect of  $R$  on the temperature.

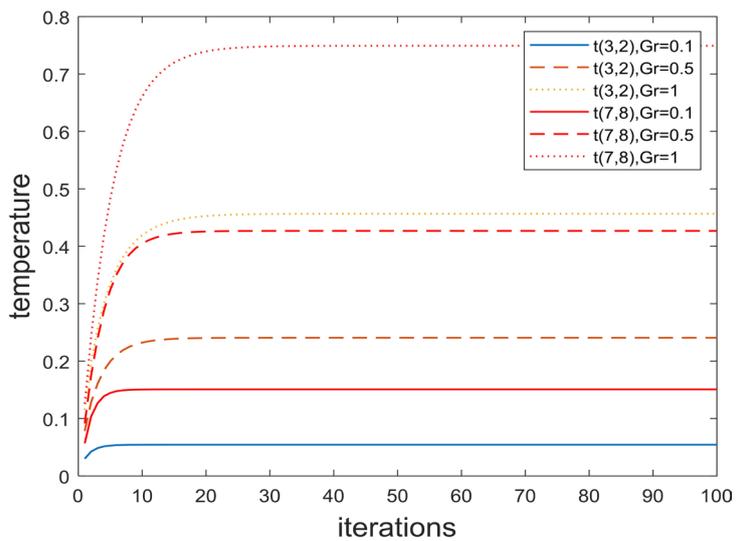


Figure 5. The effect of  $Gr$  on the temperature.

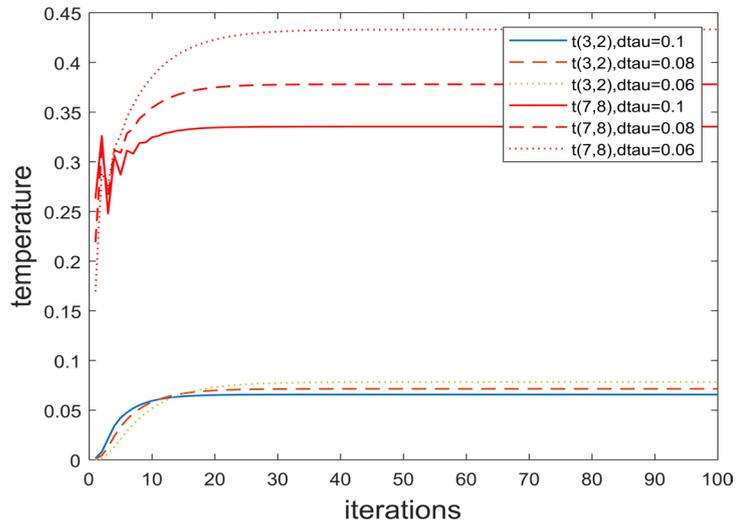


Figure 6. The effect of the time-step on the temperature.

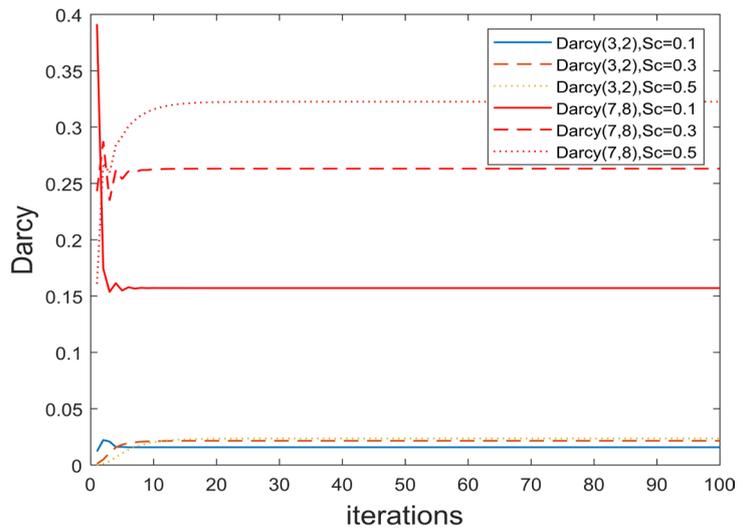


Figure 7. The effect of  $Sc$  on the Darcy.

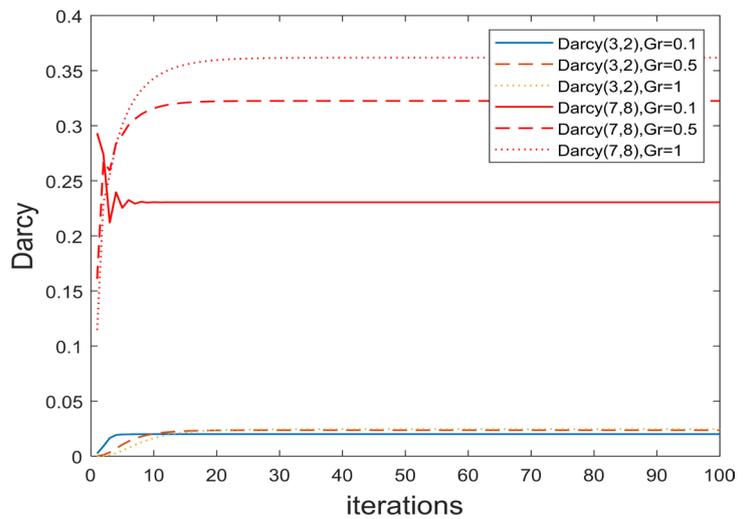


Figure 8. The effect of  $Gr$  on the Darcy.

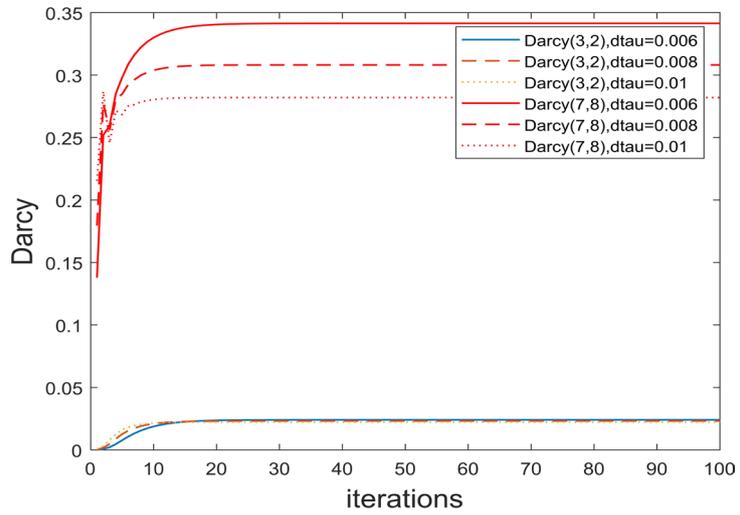


Figure 9. The effect of the time-step on Darcy.

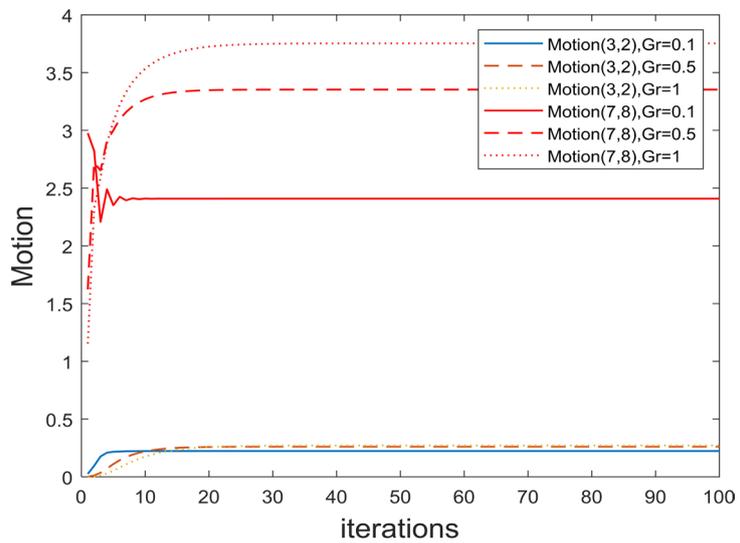


Figure 10. The effect of  $Gr$  on the motion.

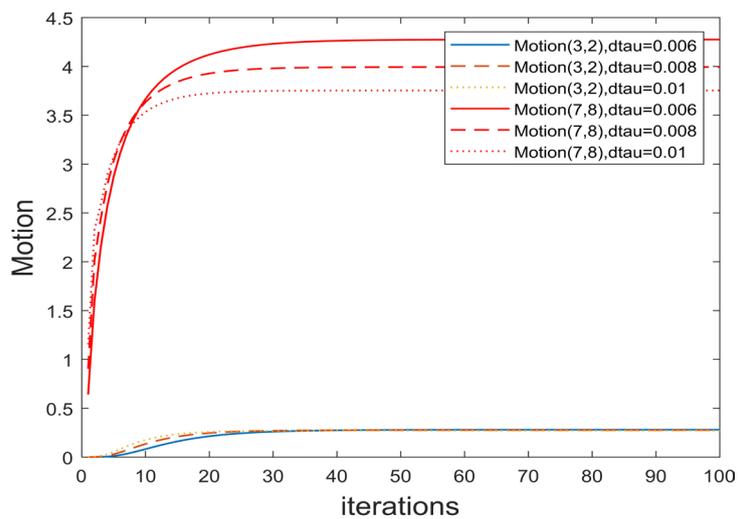


Figure 11. The effect of the time-step on the motion.

7) In motion equation, when  $r = 1$ , and the time-step increases, the faster steady state will be reached.

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## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

## References

- [1] Mohammed, A.-K. and Mohamed, S. (2010) Finite Difference Solutions of MFM Square Duct Flow with Heat Transfer Using Matlab Program. In: Leite, E.P., Ed., *Matlab-Modelling, Programming and Simulations*, Sciyo, Rijeka, 365-388.
- [2] Singha, K. (2009) Analytically Solution to the Problem of MHD Free Convection of an Electrically Conducting Fluid between Two Heated Parallel Plates in the Presence of an Induced Magnetic Field. *International Journal to Applied Mathematics and Computation*, **1**, 183-193.
- [3] Cookey, C. and Omubo-Pepple, V. (2010) On Steady Hydro Genetic Flow of a Radiating Viscous Fluid through a Horizontal Channel in a Porous Media. *American Journal of Scientific and Industrial Research*, **1**, 203-208.
- [4] Dalia, S. and Ioan, P. (2011) A Study of Entropy Generation Minimization in an Inclined Channel. *WSEAS Transactions on Heat and Mass Transfer*, **6**, 31-40.
- [5] Sudershan, S., Reddy, R. and Reddy, J. (2012) Radiation and Chemical Reaction Effects on Free Convection MHD Flow through a Porous Medium Bounded by Vertical Surface, Pelugia Reserch Library. *Advances in Applied Science Research*, **3**, 1603-1610.
- [6] Mukhopadhyay, S. and Mandal, I. (2015) Magneto Hydrodynamic (MHD) Mixed Convection Slip Flow and Heat Transfer over a Vertical Porous Plate. *Engineering Science and Technology an International Journal*, **18**, 98-105.  
<https://doi.org/10.1016/j.jestch.2014.10.001>
- [7] Kumari, K. and Goyal, M. (2017) Heat and Mass Transfer Flow of MHD Viscous Dissipative Fluid in a Channel with a Stretching and Porous Plate. *An International Journal of Applied Mathematics and Information Sciences Letters*, **5**, 81-87.  
<https://doi.org/10.18576/amisl/050301>
- [8] Brewster, M. (1972) *Thermal Radiative Transfer Properties*. John Wiley and Sons, New York.
- [9] Bhattacharyya, K. (2013) MHD Stagnation-Point Flow of Casson Fluid and Heat Transfer over a Stretching Sheet with Thermal Radiation. *Journal of Thermodynamics*, **2013**, Article ID: 169974. <https://doi.org/10.1155/2013/169674>
- [10] Eugeni, E.M. (2017) *Programming with MATLAB for Scientists: A Beginner's Introduction*. CRC Press, Taylor & Francis Group, Boca Raton, London, New York.