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The Movement of Orbits and Their Effect on the Encoding of Letters in Partition Theory II

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Abstract

This paper is complementary to the work of Shareef and Mahmood in 2019, on the effect of the movement of orbits within each English letter was prepared using the partition theory. The difference from the research referred to here is that we will adopt a word from any number of English letters and study this movement on the 2^{nd} and 3^{rd} orbits and study the difference here in the new case about what is present only with one letter of the English language letters which was discussed in the Part I.

Subject Areas

Mathematics, Partition Theory

Keywords

Partition Theory, Encoding, e-Abacus Diagram

1. Introduction

Let *r* be a non-negative integer. A partition $\mu=(\mu_1,\mu_2,\cdots,\mu_n)$ of *r* is a sequence of non-negative integers such that $|\mu|=\sum_{j=1}^{n}\mu_j=r$ and for $\forall$ $j\geq 1$, $\mu_j\geq\mu_{j+1}$. Fix μ is a partition of *r*, choosing an integer b greater than or equal to the number of parts of μ and defining $\beta_i=\mu_i+b-i, 1\leq i\leq b$. The set $\{\beta_1,\beta_2,\cdots,\beta_b\}$ is said to be the set of β-number for μ, see [1] [2].

Let *e* be a positive integer number greater than or equal to 2, we can represent β-number by a diagram called *e*-Abacus diagram: (Table 1).

Where every β will be represented by a bead (●) and else that by (-) which takes its location in Table 1.

2. Orbits

A formula was adopted in [3] for the format of the orbits for any English letter,

Table 1. e-Abacus diagram.

Run. 1	Run. 2	···	Run. e
0	1	···	$e-1$
e	$e+1$	···	$2e-1$
$2e$	$2e+1$	···	$3e-1$
:	:	:	:

$$\begin{array}{ccccc} a_{22} & a_{23} & a_{24} & \cdots & a_{28} & a_{29} \\ & a_{32} & & & a_{39} & \\ a_{42} & a_{43} & a_{44} & \cdots & a_{48} & a_{49} \end{array}$$

Figure 1. When the word of 2 letters.

$$\begin{array}{cccccc} a_{22} & a_{23} & a_{24} & \cdots & a_{2(13)} & a_{2(14)} \\ & a_{32} & & & a_{3(14)} & \\ a_{42} & a_{43} & a_{44} & \cdots & a_{4(13)} & a_{4(14)} \end{array}$$

Figure 2. When the word of 3 letters.

we have three orbit according to 3.1; *only the case of* 2*-orbit is discussed there because it is the one that has the most influence in that research and the rest.* Now, if we have a word of 2, 3 or more letters. Will we try encoding on each letter separately or we will use the encoding on each word? And because the process of calculating the partition is based on all the word, see [4] [5], we had to find a mechanism to encode the word according to the movement of the orbits.

2.1. Behavior of Each Orbit to Any Word

Obviously there are three orbits:

1) Orbit: It is the external orbit which will remain the same without any change so that we can read the partition of the word before the change, see [6].

2) Orbit: It is the middle orbit which takes the following location.

3) Orbit: It is the last orbit and his movement will be explained in section 3 of this paper.

2.2. $w_2 = 1$ for Any Word

Depending on the number of letters in each word, specifically **Figure 1** or **Figure 2**, the movement will be according to the following:

$$a_{22} \to a_{23} \to \cdots \to a_{28} \to a_{29} \to a_{39} \to a_{49} \to a_{48} \\ \to \cdots \to a_{43} \to a_{42} \to a_{32} \to a_{22}$$

or

$$a_{22} \to a_{23} \to \cdots \to a_{2(13)} \to a_{2(14)} \to a_{3(14)} \to a_{4(14)} \to a_{4(13)}$$
$$\to \cdots \to a_{43} \to a_{42} \to a_{32} \to a_{22}$$

Thus we can make the following rules:

Rule 2.2.1: When choosing a partition for any word has $\hbar$ letters where $e=5$ and the value of β_i was equal to the location $a_{\alpha\lambda}$ within $[0;1;0]$ will be:

$$\begin{cases} a_{\alpha(\lambda\mp1)} & \text{if } \alpha=2\,(\text{or } 4) \wedge \lambda=2,3,\cdots,(5\hbar-2)\,(\text{or } 3,4,\cdots,5\hbar-1) \text{ respectively,} \\ a_{(\alpha\mp1)\lambda} & \text{if } \alpha=2,3\ (\text{or } 3,4) \wedge \lambda=5\hbar-1\,(\text{or } 2) \text{ respectively,} \end{cases}$$

Proof: Since $w_2=1$ the transition process will be in two ways and each of them has two opposite directions. The first way (with traditional direction) will be taken $a_{2\lambda} \to a_{2(\lambda+1)}$ when $2 \le \lambda \le 5\hbar-2$, as for the opposite direction of the same way, it is $a_{4\lambda} \to a_{2(\lambda-1)}$ when $3 \le \lambda \le 5\hbar-1$. Now, we come to the second way (the traditional direction), it will be $a_{\alpha(5\hbar-1)} \to a_{(\alpha+1)(5\hbar-1)}$ when $2 \le \alpha \le 3$, and for last direction then we have $a_{\alpha 2} \to a_{(\alpha-1)2}$ when $3 \le \alpha \le 4$.

For example (Figure 3),

The Word Way

$$=\left(42,40,37,36,35,32,30,27^2,26,25,24^3,23^6,22,21^2,18^2,15^2,12,6^3\right)$$

Will be (Figure 4):

$$\text{WAY}^{[0;1;0]}=\left(42,40,37,36,35,33,32,30,27^2,26^2,24^2,23^6,22,21^2,19^2,16^2,12,6^3\right)$$

Rule 2.2.2: When choosing a partition for any word has $\hbar$ letters where $e=5$ and the value of β_i was equal to the location $a_{\alpha\lambda}$ within $[0;2;0]$ will be:

$$\begin{cases} a_{\alpha(\lambda\mp2)} & \text{if } \alpha=2\,(\text{or } 4) \wedge \lambda=2,3,\cdots,5\hbar-3\,(\text{or } 4,5,\cdots,5\hbar-1)\ \textit{resp.,} \\ a_{(\alpha+1)(\lambda\mp1)} & \text{if } \alpha=2\,(\text{or } 3) \wedge \lambda=5\hbar-2\,(\text{or } 5\hbar-1)\ \textit{resp.,} \\ a_{(\alpha-1)(\lambda\pm1)} & \text{if } \alpha=4\,(\text{or } 3) \wedge \lambda=3\,(\text{or } 2)\ \textit{resp.,} \\ a_{(\alpha\mp1)\lambda} & \text{if } \alpha=2\,(\text{or } 4) \wedge \lambda=5\hbar-1\,(\text{or } 2)\ \textit{resp..} \end{cases}$$

Proof: By using the same method of rule (2.2.1) we have 4 ways A,B,C,D (with two opposite directions) as the following:

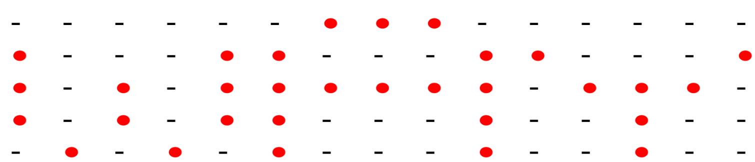

Figure 3. The word WAY.

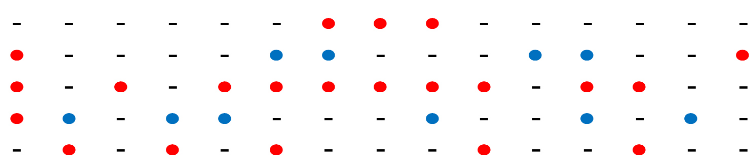

Figure 4. The word WAY when $w_2=1$.

If $(5\hbar-1)$ is odd, then:

$$A: a_{22} \to a_{24} \to \cdots \to a_{2(5\hbar-4)} \to a_{2(5\hbar-2)} \to a_{3(5\hbar-1)} \to a_{4(5\hbar-2)} \to \cdots \to a_{42} \to a_{22},$$

$$B: a_{23} \to a_{25} \to \cdots \to a_{2(5\hbar-3)} \to a_{2(5\hbar-1)} \to a_{4(5\hbar-1)} \to a_{4(5\hbar-3)} \to \cdots \to a_{42} \to a_{22}.$$

If $(5\hbar-1)$ is even, then:

$$C: a_{22} \to a_{24} \to \cdots \to a_{2(5\hbar-3)} \to a_{2(5\hbar-1)} \to a_{4(5\hbar-1)} \to a_{4(5\hbar-3)} \to \cdots \to a_{42} \to a_{22},$$

$$D: a_{23} \to a_{25} \to \ldots \to a_{2(5\hbar-3)} \to a_{2(5\hbar-1)} \to a_{4(5\hbar-1)} \to a_{4(5\hbar-3)} \to \cdots \to a_{42} \to a_{22}.$$

Then all the relationships above are achieved.

For example the word (Figure 5).

$$\text{WAY}^{[0;2;0]} = \left(42,40,37,36,35,32,30,27^2,26^2,24^2,23^6,22,21,19^2,16^2,12,6^3\right)$$

Rule 2.2.3: When choosing a partition for any word has $\hbar$ letters where $e=5$ and the value of β_i was equal to the location $a_{\alpha\lambda}$ within $[0;w_2;0]$ will be:

(I) If $w_2 = 3$ then

$$\begin{cases} a_{\alpha(\lambda\mp3)} & \text{if } \alpha=2\ (\text{or } 4) \wedge \lambda = 2,3,\cdots,5\hbar-4\ (\text{or } 5,6,\cdots,5\hbar-1)\ \textit{resp.}, \\ a_{(\alpha+1)(\lambda\mp2)} & \text{if } \alpha=2\ (\text{or } 3) \wedge \lambda = 5\hbar-3\ (\text{or } 5\hbar-1)\ \textit{resp.}, \\ a_{(\alpha-1)(\lambda\pm2)} & \text{if } \alpha=4\ (\text{or } 3) \wedge \lambda = 4\ (\text{or } 2)\ \textit{resp.}, \\ a_{(\alpha\mp2)(\lambda\mp1)} & \text{if } \alpha=2\ (\text{or } 4) \wedge \lambda = 5\hbar-2\ (\text{or } 3)\ \textit{resp.}, \\ a_{(\alpha\mp2)(\lambda\pm1)} & \text{if } \alpha=2\ (\text{or } 4) \wedge \lambda = 5\hbar-1\ (\text{or } 2)\ \textit{resp.}. \end{cases}$$

(II) If $w_2 = 4$ then

$$\begin{cases} a_{\alpha(\lambda\mp4)} & \text{if } \alpha=2\ (\text{or } 4) \wedge \lambda = 2,3,\cdots,5\hbar-5\ (\text{or } 6,7,\cdots,5\hbar-1)\ \textit{resp.}, \\ a_{(\alpha+1)(\lambda\mp3)} & \text{if } \alpha=2\ (\text{or } 3) \wedge \lambda = 5\hbar-4\ (\text{or } 5\hbar-1)\ \textit{resp.}, \\ a_{(\alpha-1)(\lambda\pm3)} & \text{if } \alpha=4\ (\text{or } 3) \wedge \lambda = 5\ (\text{or } 2)\ \textit{resp.}, \\ a_{(\alpha\mp2)(\lambda\mp2)} & \text{if } \alpha=2\ (\text{or } 4) \wedge \lambda = 5\hbar-3\ (\text{or } 4)\ \textit{resp.}, \\ a_{(\alpha\mp2)\lambda} & \text{if } \alpha=2\ (\text{or } 4) \wedge \lambda = 5\hbar-2\ (\text{or } 3)\ \textit{resp.}, \\ a_{(\alpha\mp2)(\lambda\pm2)} & \text{if } \alpha=2\ (\text{or } 4) \wedge \lambda = 5\hbar-1\ (\text{or } 2)\ \textit{resp.}. \end{cases}$$

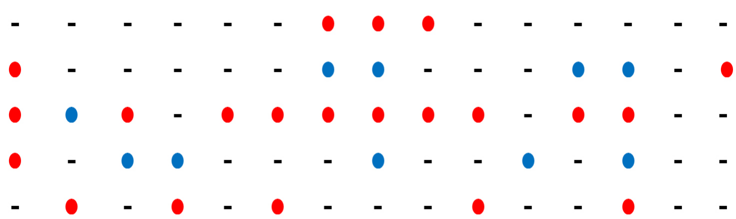

Figure 5. The word WAY when $w_2 = 2$.

(III) If $w_2 = 5$ then:

$$\begin{cases} a_{\alpha(\lambda\mp5)} & \text{if } \alpha=2\,(\text{or }4)\wedge\lambda=2,3,\cdots,5\hbar-6\,(\text{or }7,8,\cdots,5\hbar-1)\ \textit{resp.},\\ a_{(\alpha+1)(\lambda\mp4)} & \text{if } \alpha=2\,(\text{or }3)\wedge\lambda=5\hbar-5\,(\text{or }5\hbar-1)\ \textit{resp.},\\ a_{(\alpha-1)(\lambda\pm4)} & \text{if } \alpha=4\,(\text{or }3)\wedge\lambda=6\,(\text{or }2)\ \textit{resp.},\\ a_{(\alpha\mp2)(\lambda\mp3)} & \text{if } \alpha=2\,(\text{or }4)\wedge\lambda=5\hbar-4\,(\text{or }5)\ \textit{resp.},\\ a_{(\alpha\mp2)(\lambda\mp1)} & \text{if } \alpha=2\,(\text{or }4)\wedge\lambda=5\hbar-3\,(\text{or }4)\ \textit{resp.},\\ a_{(\alpha\mp2)(\lambda\pm1)} & \text{if } \alpha=2\,(\text{or }4)\wedge\lambda=5\hbar-2\,(\text{or }3)\ \textit{resp.},\\ a_{(\alpha\mp2)(\lambda\pm3)} & \text{if } \alpha=2\,(\text{or }4)\wedge\lambda=5\hbar-1\,(\text{or }2)\ \textit{resp.}. \end{cases}$$

(IV) If $w_2 = 6$ then we have:

$$\begin{cases} a_{\alpha(\lambda\mp6)} & \text{if } \alpha=2\,(\text{or }4)\wedge\lambda=2,3,\cdots,5\hbar-7\,(\text{or }8,9,\cdots,5\hbar-1)\ \textit{resp.},\\ a_{(\alpha+1)(\lambda\mp5)} & \text{if } \alpha=2\,(\text{or }3)\wedge\lambda=5\hbar-6\,(\text{or }5\hbar-1)\ \textit{resp.},\\ a_{(\alpha-1)(\lambda\pm5)} & \text{if } \alpha=4\,(\text{or }3)\wedge\lambda=7\,(\text{or }2)\ \textit{resp.},\\ a_{(\alpha\mp2)(\lambda\mp4)} & \text{if } \alpha=2\,(\text{or }4)\wedge\lambda=5\hbar-5\,(\text{or }6)\ \textit{resp.},\\ a_{(\alpha\mp2)(\lambda\mp2)} & \text{if } \alpha=2\,(\text{or }4)\wedge\lambda=5\hbar-4\,(\text{or }5)\ \textit{resp.},\\ a_{(\alpha\mp2)\lambda} & \text{if } \alpha=2\,(\text{or }4)\wedge\lambda=5\hbar-3\,(\text{or }4)\ \textit{resp.},\\ a_{(\alpha\mp2)(\lambda\pm2)} & \text{if } \alpha=2\,(\text{or }4)\wedge\lambda=5\hbar-2\,(\text{or }3)\ \textit{resp.},\\ a_{(\alpha\mp2)(\lambda\pm4)} & \text{if } \alpha=2\,(\text{or }4)\wedge\lambda=5\hbar-1\,(\text{or }2)\ \textit{resp.}. \end{cases}$$

Finally,

(V) If $w_2 = 7$ then we have:

$$\begin{cases} a_{\alpha(\lambda\mp7)} & \text{if } \alpha=2\,(\text{or }4)\wedge\lambda=2,3,\cdots,5\hbar-8\,(\text{or }9,10,\cdots,5\hbar-1)\ \textit{resp.},\\ a_{(\alpha+1)(\lambda\mp6)} & \text{if } \alpha=2\,(\text{or }3)\wedge\lambda=5\hbar-7\,(\text{or }5\hbar-1)\ \textit{resp.},\\ a_{(\alpha-1)(\lambda\pm6)} & \text{if } \alpha=4\,(\text{or }3)\wedge\lambda=8\,(\text{or }2)\ \textit{resp.},\\ a_{(\alpha\mp2)(\lambda\mp5)} & \text{if } \alpha=2\,(\text{or }4)\wedge\lambda=5\hbar-6\,(\text{or }7)\ \textit{resp.},\\ a_{(\alpha\mp2)(\lambda\mp3)} & \text{if } \alpha=2\,(\text{or }4)\wedge\lambda=5\hbar-5\,(\text{or }6)\ \textit{resp.},\\ a_{(\alpha\mp2)(\lambda\mp1)} & \text{if } \alpha=2\,(\text{or }4)\wedge\lambda=5\hbar-4\,(\text{or }5)\ \textit{resp.},\\ a_{(\alpha\mp2)(\lambda\pm1)} & \text{if } \alpha=2\,(\text{or }4)\wedge\lambda=5\hbar-3\,(\text{or }4)\ \textit{resp.},\\ a_{(\alpha\mp2)(\lambda\pm3)} & \text{if } \alpha=2\,(\text{or }4)\wedge\lambda=5\hbar-2\,(\text{or }3)\ \textit{resp.},\\ a_{(\alpha\mp2)(\lambda\pm5)} & \text{if } \alpha=2\,(\text{or }4)\wedge\lambda=5\hbar-1\,(\text{or }2)\ \textit{resp.}. \end{cases}$$

For example the word (**Figure 6**).

$$\mathrm{WAY}^{[0;4;0]}=\left(43,41,38,37,36,31,27,24^2,23^3,22^6,21^5,18^2,13,12,6^3\right)$$

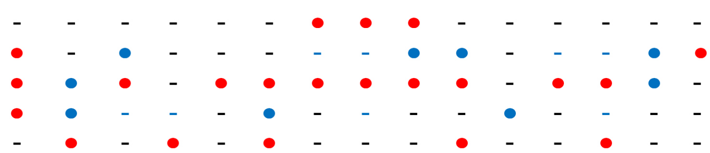

Figure 6. The word WAY when $w_2 = 4$.

$$a_{33}\ a_{34}\ a_{35}\ a_{36}a_{37}a_{38}$$

Figure 7. The positions in orbit 3 for any word have 2 letters.

$$a_{33}\ a_{34}\ a_{35}\ \ldots\ a_{3(11)}a_{3(12)}a_{3(13)}$$

Figure 8. The positions in orbit 3 for any word have 3 letters.

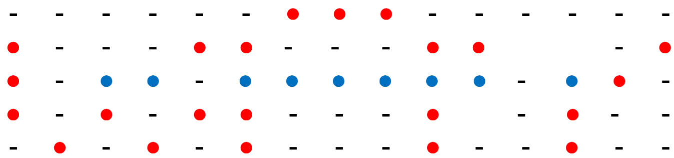

Figure 9. The word WAY when $w_3 = 1$.

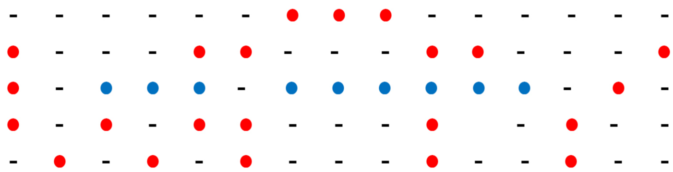

Figure 10. The word WAY when $w_3 = 2$.

3. The Movement of w_3

By the results of [3], mentioned that w_3 per letter has no effect as only one position, but in the case of a word consisting of more than one letter, its impact is very important. On this basis in the case of a word that contains only two letters, then (Figure 7, Figure 8).

<u>**Rule 3.1:**</u> When choosing a partition for any word have ($\hbar > 1$) letter where e = 5 and the value of β_i was equal to the location $a_{\alpha\lambda}$ within $[0;0;w_3]$ will be:

$$\begin{cases} a_{\alpha(\lambda+w_3)} & \text{if } \alpha = 3 \wedge \lambda = 2,3,\cdots,\left(5\hbar-\left(w_3+2\right)\right) \\ a_{3t} & \text{if } \alpha = 3,\ \lambda = \left(5\hbar-\left(w_3+1\right)\right),\left(5\hbar-\left(w_3\right)\right),\cdots,5\hbar-2 \\ & \quad \wedge\, t = 3,4,\cdots,\left(w_3+2\right), \text{ respectively.} \end{cases}$$

For example, the word (Figure 9, Figure 10).

$$\text{WAY}^{[0;0;1]} = \left(42,40,37,36,35,32,30,27,25,24^2,23^6,22^3,21,18^2,15^2,12,6^3\right)$$

and

$$\text{WAY}^{[0;0;2]} = \left(42,40,37,36,35,32,30,27^2,26,25,24,23^6,22^3,21,18^2,15^2,12,6^3\right).$$

4. Results and Discussion

1) This encoding made the first encoding of English letters more difficult in terms of finding the origin of the word.

2) A regular shape was used from e-Abacus diagram and we can think of using an irregular shape in the future.

3) It is quite possible to merge both w_2 and w_3 at the same time by merging the previous relationships with each other.

5. Conclusions

1) The above technique can be used on letters of other languages that do not use the same letters.

2) This technique can be used in tiling, were the colors and shapes vary.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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