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Laplace Decomposition Method for the System of Non Linear PDEs

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Abstract

The Laplace Decomposition Method [1] [2] [3] is applied to a system of non-linear partial differential equations to demonstrate potential applicability to such systems.

Subject Areas

Mathematical Analysis

Keywords

Laplace Decomposition Method, Partial Differential Equations, Laplace Transform

1. Introduction

Differential equations theory is an important Mathematical branch which is used to describe practical problems in physics, chemistry, and biology and so on [4]. It is well known that many phenomena in scientific fields such as reaction-diffusion process, population growth, solid physics, fluid dynamics, Mathematical biology and chemical kinetics, can be modelled by systems of linear or non-linear PDEs. In order to understand and analyze these phenomena well, it needs to know solution of systems of these linear or nonlinear PDEs. So, it is a crucial work to obtain solutions of systems of linear or nonlinear PDEs in the science. With this idea; scientists and mathematicians have developed and searched some methods such as Hirota bilinear method [5], Exp-function method [6], tanh method [7] [8], sine-cosine method [9], Galerkin method [10] and Differential transform method (DTM) [11]. It is more difficult to obtain solutions of nonlinear PDEs than those of linear differential equations. Therefore, it may not always be possible to obtain analytical solutions of these equations [12] [13]. In this case, it is used analytical methods giving series solutions. In these kinds of methods, the solutions are sought in the form of series [1] [2] [3] [14]. Analytical methods are based on finding the other terms of the series from given initial conditions for the problem being considered. At this point, it is encountered the concept of convergence of the series. So, it is necessary to perform convergence analysis of these methods. As this convergence analysis can be carried out theoretically, one can gain information about the convergence of the series solution by looking at the absolute error between the numerical solution and the analytical solution. In some Analytic methods, a very good convergence can be achieved with only a few terms of the series, but more terms can be needed in some problems. That is, if the terms of the series increase, this provides better convergence to the analytical solution.

In this study, I have used LDM to solve a system of nonlinear partial differential equations for two different initial conditions. Later, we compared the obtained results with exact solutions and solution obtained by Method of Differential Quadrature [15]. In this paper, we are not going to explain the LDM. For that, I have referred papers [1] [2] [3] to illustrate this method for a nonlinear system of PDE's.

2. Application

Consider a system of nonlinear partial differential equations on our interest of region given by:

$$u_t = uu_x + vu_y \tag{1}$$

$$v_t = uv_x + vv_y \tag{2}$$

with initial condition

$$u(x, y, 0) = f(x, y) \tag{3}$$

$$v(x, y, 0) = g(x, y) \tag{4}$$

here, we have consider the general form of boundary conditions. Taking Laplace transform of Equations (2.1) and (2.2) with respect to *t*, we get

$$L_t[u_t] = L_t[uu_x + vu_y]$$

$$L_t[v_t] = L_t[uv_x + vv_y]$$

$$su(x, y, s) - u(x, y, 0) = L_t[uu_x + vu_y]$$

$$sv(x, y, s) - v(x, y, 0) = L_t[uv_x + vv_y]$$

$$su(x, y, s) = \frac{1}{s} f(x, y) + \frac{1}{s} L_t[uu_x + vu_y]$$

$$sv(x, y, s) = \frac{1}{s} g(x, y) + \frac{1}{s} L_t[uv_x + vv_y]$$

Taking inverse Laplace transform of above system with respect to "*t*", we get

$$u(x, y, t) = f(x, y) + L_t^{-1}\left[\frac{1}{s} L_t[uu_x + vu_y]\right] \tag{5}$$

$$v(x,y,t) = g(x,y) + L_t^{-1}\left[\frac{1}{s}L_t\left[uv_x + vv_y\right]\right] \tag{6}$$

Let us suppose that,

$$u(x,y,t) = \sum_{n=0}^{\infty} u_n(x,y,t), \quad v(x,y,t) = \sum_{n=0}^{\infty} v_n(x,y,t) \tag{7}$$

be the solution of given system of Equations (2.1), (2.2) in series form. Also we can decompose the nonlinear terms appeared in given system by using adomian polynomials, namely

$$uu_x = \sum_{n=0}^{\infty} A_n, \ vu_y = \sum_{n=0}^{\infty} B_n, \ uv_x = \sum_{n=0}^{\infty} C_n, \ vv_y = \sum_{n=0}^{\infty} D_n \tag{8}$$

where A_n, B_n, C_n and D_n are adomian polynomials [16]. From the Equations (2.5), (2.6), (2.7) and (2.8), we get

$$\sum_{n=0}^{\infty} u_n(x,y,t) = f(x,y) + L_t^{-1}\left[\frac{1}{s}L_t\left[\sum_{n=0}^{\infty} A_n + \sum_{n=0}^{\infty} B_n\right]\right]$$

$$\sum_{n=0}^{\infty} v_n(x,y,t) = g(x,y) + L_t^{-1}\left[\frac{1}{s}L_t\left[\sum_{n=0}^{\infty} C_n + \sum_{n=0}^{\infty} D_n\right]\right]$$

Comparing the both sides of above system of equations, we get the following recursive relations

$$u_0(x,y,t) = f(x,y), \ u_{n+1}(x,y,t) = L_t^{-1}\left[\frac{1}{s}L_t\left[\sum_{n=0}^{\infty} A_n + \sum_{n=0}^{\infty} B_n\right]\right], \ n \geq 0. \tag{9}$$

$$v_0(x,y,t) = g(x,y), \ v_{n+1}(x,y,t) = L_t^{-1}\left[\frac{1}{s}L_t\left[\sum_{n=0}^{\infty} C_n + \sum_{n=0}^{\infty} D_n\right]\right], \ n \geq 0. \tag{10}$$

Note that the solution of (2.1), (2.2) can exhibit a shock phenomenon for finite t; we select f(x, y) and g(x, y) such that the shock occurs for a value of t far from our region of interest. Let

$$f(x,y) = g(x,y) = x + y \tag{11}$$

Therefore from the recursive relation (2.9) and (2.10), we get

$$u_0(x,y,t) = v_0(x,y,t) = x + y$$

then $u_1(x,y,t)$, $v_1(x,y,t)$ can be calculate as

$$\begin{aligned} u_1(x,y,t) &= L_t^{-1}\left[\frac{1}{s}L_t\left[A_0 + B_0\right]\right] \\ &= L_t^{-1}\left[\frac{1}{s}L_t\left[u_0u_{0x} + v_0u_{0y}\right]\right] \\ &= L_t^{-1}\left[\frac{1}{s}L_t\left[(x+y) + (x+y)\right]\right] \\ &= 2t(x+y) \end{aligned}$$

Similarly,

$$v_1(x,y,t) = L_t^{-1}\left[\frac{1}{s}L_t\left[C_0 + D_0\right]\right] = L_t^{-1}\left[\frac{1}{s}L_t\left[u_0v_{0x} + v_0v_{0y}\right]\right] = 2(x+y)t$$

Also, $u_2(x,y,t)$ and $v_2(x,y,t)$ are calculated as

$$\begin{aligned} u_2(x,y,t) &= L_t^{-1}\left[\frac{1}{s}L_t\left[A_1+B_1\right]\right] \\ &= L_t^{-1}\left[\frac{1}{s}L_t\left[\left(u_0u_{1x}+u_1u_{0x}\right)+\left(v_0u_{1y}+v_1u_{0y}\right)\right]\right] \\ &= L_t^{-1}\left[\frac{1}{s}L_t\left[\left(2t(x+y)+2t(x+y)\right)+\left(2t(x+y)+2t(x+y)\right)\right]\right] \\ &= L_t^{-1}\left[\frac{1}{s}L_t\left[8t(x+y)\right]\right] \\ &= 2t^2(x+y) \end{aligned}$$

Similarly,

$$v_2(x,y,t) = 4t^2(x+y)$$

Substitute all the values of $u_0,u_1,u_2,\cdots$ and $v_0,v_1,v_2,\cdots$ in the Equation (2.7), we get

$$u(x,y,t) = (x+y)+2t(x+y)+4t^2(x+y)+\cdots$$

$$v(x,y,t) = (x+y)+2t(x+y)+4t^2(x+y)+\cdots$$

This implies,

$$u(x,y,t) = (x+y)\left[1+2t+4t^2+\cdots\right]$$

$$v(x,y,t) = (x+y)\left[1+2t+4t^2+\cdots\right]$$

$$u(x,y,t) = \frac{x+y}{1-2t}$$

$$v(x,y,t) = \frac{x+y}{1-2t}$$

This is an exact solution of the given system of nonlinear partial differential Equations (2.1) and (2.2). We have verified this through the substitution, which is identical to the solution obtained by R. E. Bellman using the method of differential quadrature [15]. Let we change the initial conditions to

$$f(x,y) = x^2, \quad g(x,y) = y \tag{12}$$

From the recursive relation (2.9), (2.10) and above initial conditions, we get

$$u_0(x,y,t) = x^2, \quad v_0(x,y,t) = y$$

$$u_1(x,y,t) = L_t^{-1}\left[\frac{1}{s}L_t\left[A_0+B_0\right]\right] = 2x^3t$$

Similarly,

$$v_1(x,y,t) = L_t^{-1}\left[\frac{1}{s}L_t\left[C_0+D_0\right]\right] = yt$$

Also, $u_2(x,y,t)$ and $v_2(x,y,t)$ are calculated as

$$u_2(x,y,t) = L_t^{-1}\left[\frac{1}{s}L_t\left[A_1+B_1\right]\right] = 5x^4t$$

Similarly,

$$v_2(x,y,t) = yt^2, \quad u_3(x,y,t) = 14x^5t^3$$

and so on. Substitute all the values of $u_0, u_1, u_2, \cdots$ and $v_0, v_1, v_2, \cdots$ in Equation (2.7), we get

$$u(x,y,t) = x^2\left(1 + 2tx + 5t^2x^2 + 14x^2t^2 + \cdots\right)$$

$$v(x,y,t) = y\left(1 + t + t^2 + \cdots\right) = \frac{y}{1-t}$$

(The shock occurs at $t = \dfrac{1}{4x}$). This is an approximate solution of given system of equations.

3. Conclusion

From the examples above, we can clearly say that we can calculate $u(x,y,t)$ and $v(x,y,t)$ when explicitly solutions exist for given initial functions. More importantly, the methodology [1] [2] [3] does have potential application to the system of nonlinear partial differential equations and clearly in the case of stochastic parameters as well. The given system of equation has a unique solution for the given boundary conditions.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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