



# Dynamic Field of the Electron

Isaak Man'kin

Jerusalem, Israel  
Email: igrebnevfam@gmail.com

**How to cite this paper:** Man'kin, I. (2025)  
Dynamic Field of the Electron. *Open Access  
Library Journal*, 12: e13996.  
<https://doi.org/10.4236/oalib.1113996>

**Received:** July 22, 2025  
**Accepted:** September 16, 2025  
**Published:** September 19, 2025

Copyright © 2025 by author(s) and Open  
Access Library Inc.  
This work is licensed under the Creative  
Commons Attribution International  
License (CC BY 4.0).  
<http://creativecommons.org/licenses/by/4.0/>



Open Access

## Abstract

We continue the discussion of the internal structure of elementary particles based on the earlier proposed generalized electrodynamic Maxwell equations. It is assumed that the variety of elementary particles is defined by a complex vector wave function dependent on space-time coordinates. In the general case, the wave function separates into components that are constant in time (the static field) and nonconstant in time (the dynamic field). Previous works have studied the nature of the static field. In particular, it was shown that far from the center of the particle, the electric field is much larger than the magnetic field and asymptotically tends to Coulomb's law. On the other hand, close to the center of the particle, the field changes its nature. In the current work, we study the dynamic field. Due to the difficulty of the problem, we study a simplified spherically symmetric model. Far from the center of the particle, we obtain an analytic solution and show that the wave function decays exponentially. On the basis of this analysis and the accompanying discussion, we propose that inside a particle and within small distances to it, the generalized Maxwell equations describe a unified field that includes gravity. For large objects, gravity is described by Einstein's general theory of relativity, which is based on classical concepts including the energy-momentum tensor, the interval, Riemannian geometry, extremals, etc. For small objects (e.g., elementary particles), the situation seems to be different, and hence, despite numerous efforts, a satisfactory theory for them has not been formed using Einstein's theory. To a certain degree, from our point of view, this resembles the situation that arose upon the comparison of classical and quantum mechanics.

## Subject Areas

Fundamental Physics, Theory of Elementary Particles, Theory of Gravitation, Unified Field

## Keywords

Elementary Particles, Generalized Maxwell Equations of Electromagnetism, Internal Particle Structure, Theory of Gravitation, Unified Field

## 1. Introduction

A systematic theory of elementary particles that agrees with accepted philosophical concepts, from our point of view, currently does not exist. In [1], we attempt to construct such a theory on the basis of the generalized Maxwell equations of electrodynamics.

It follows from quantum mechanics that every particle has an internal energy  $E$  and spin (*i.e.*, its own angular momentum) and from classical relativistic mechanics that  $E = m_0 c^2$  (where  $m_0$  is its rest mass and  $c$  is the speed of light), from which it follows that the particle is not a singular point (as it is usually postulated) but rather has a certain internal structure.

Moreover, considering the fact that during various reactions various types of particles turn into one another, we can naturally come to the conclusion that they exhibit a similar nature. Due to the fact that photons are quanta of the electromagnetic field that take part in almost all reactions, it's natural to conclude that all other particles should be described by equations associated with Maxwell's electrodynamic equations. In our study, the structure of particles is described by a wave vector field  $\psi(\mathbf{r}, t)$ , dependent on the coordinates  $\mathbf{r}$  and time  $t$  [1]:

$$\psi(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) + i\mathbf{H}(\mathbf{r}, t), \quad (1)$$

where  $\mathbf{E}(\mathbf{r}, t)$  and  $\mathbf{H}(\mathbf{r}, t)$  are the electric and magnetic fields, respectively. This way, the wave function is defined by six components that satisfy a self-compatible system of nonlinear equations.

We note that in nonrelativistic quantum mechanics, the scalar Schrodinger's wave function  $\psi(\mathbf{r}, t)$  describes the behavior of point particles in given external fields. In the relativistic case, instead of one scalar wave function, Dirac introduced the analysis of several scalar wave functions  $\psi_v(\mathbf{r}, t)$ , each of which satisfies the relativistic Klein-Gordon equation [2] [3]. The Schrodinger equations, like those of Dirac's, are linear.

In our case, the wave field  $\psi(\mathbf{r}, t)$  defines the internal structure of nonpoint particles according to a self-compatible system of nonlinear equations (the field is defined by an internal source that itself depends on the field as well) [1]:

$$\begin{aligned} \text{rot}\psi &= \frac{i}{c} \frac{\partial \psi}{\partial t} + i\beta \text{grad}\psi^2, \\ \text{div}\psi &= \alpha \psi^2, \end{aligned} \quad (2)$$

where  $\alpha$ ,  $\beta$  are constants.

It was shown that in order for a particle to be stable in time, that is to satisfy the harmonic law  $e^{-i\omega t}$ , it is necessary for there to be a nonconstant wave function  $\tilde{\psi}(\mathbf{r}, t)$  and a constant one  $\psi_0(\mathbf{r})$  (excluding particles that travel at the speed of light, such as photons). The stability condition is defined by

$$\tilde{\psi}^2(\mathbf{r}, t) = 0. \quad (3)$$

Considering the condition (3), the equation for  $\psi_0$  and  $\tilde{\psi}$  separate. However, if  $\psi_0$  does not depend on  $\tilde{\psi}$ , then  $\tilde{\psi}$  becomes a function of  $\psi_0$ .

Earlier works studied the static equation for  $\psi_0(\mathbf{r})$  [1] [4]. In particular, they obtained analytic solutions far from the center of the particles. It was shown that in this case, the electric wave function  $E_0(\mathbf{r})$  is bigger than the magnetic field  $H_0(\mathbf{r})$  and practically satisfies Coulomb's law. Closer to the center of the particle, the structure of  $\psi_0(\mathbf{r})$  becomes more complicated and the electric field does not satisfy the standard Coulomb's law.

The analysis of the dynamic field of the electron (*i.e.*, the field that varies in time) is a more challenging problem. Below, we attempt to solve this problem on the basis of a spherically symmetric model far from the particle's center. From the obtained analytic solution, it is seen that the wave function essentially decays exponentially with distance.

On the basis of this analysis and the accompanying discussion, we propose that inside a particle and within close proximity to it, the generalized Maxwell equations describe a unified field that includes gravity. For large objects, gravity is described by Einstein's general theory of relativity, which is based on classical concepts including the energy-momentum tensor, the interval, Riemannian geometry, extremals, etc. For small objects (e.g., elementary particles), the situation seems to be different, and hence, despite numerous efforts, a satisfactory theory for them has not been formed using Einstein's theory. To a certain degree, from our point of view, this resembles the situation that arose upon the comparison of classical and quantum mechanics.

## 2. Basic Relations

First, we will demonstrate the connection between the wave function  $\psi(\mathbf{r}, t)$ , the electric field  $E(\mathbf{r}, t)$ , and the magnetic field  $H(\mathbf{r}, t)$ . We recall that  $\psi(\mathbf{r}, t)$  is the sum of two terms: a constant term  $\psi_0(\mathbf{r})$  and a term varying in time  $\tilde{\psi}(\mathbf{r})e^{-i\omega t}$  (see [1]):

$$\psi(\mathbf{r}, t) = \psi_0(\mathbf{r}) + \tilde{\psi}(\mathbf{r})e^{-i\omega t}. \quad (4)$$

From (1), we have that

$$\begin{aligned} E(\mathbf{r}, t) + iH(\mathbf{r}, t) &= \psi_0(\mathbf{r}) + \tilde{\psi}(\mathbf{r})e^{-i\omega t}, \\ E(\mathbf{r}, t) &= E_0(\mathbf{r}) + \tilde{E}(\mathbf{r}, t), \\ H(\mathbf{r}, t) &= H_0(\mathbf{r}) + \tilde{H}(\mathbf{r}, t). \end{aligned} \quad (5)$$

Considering that

$$\begin{aligned} \psi_0(\mathbf{r}) &= \psi_{01}(\mathbf{r}) + i\psi_{02}(\mathbf{r}), \\ \tilde{\psi}(\mathbf{r}) &= \tilde{\psi}_1(\mathbf{r}) + i\tilde{\psi}_2(\mathbf{r}). \end{aligned}$$

Equating real and imaginary components in (5), we obtain the following for the constant electric and magnetic fields:

$$E_0(\mathbf{r}) = \psi_{01}(\mathbf{r}), \quad H_0(\mathbf{r}) = \psi_{02}(\mathbf{r}), \quad (6)$$

And for the time-varying fields:

$$\begin{aligned}\tilde{\mathbf{E}}(\mathbf{r}, t) &= \tilde{\psi}_1(\mathbf{r}) \cos(\omega t) + \tilde{\psi}_2(\mathbf{r}) \sin(\omega t), \\ \tilde{\mathbf{H}}(\mathbf{r}, t) &= \tilde{\psi}_2(\mathbf{r}) \cos(\omega t) - \tilde{\psi}_1(\mathbf{r}) \sin(\omega t).\end{aligned}\quad (7)$$

From (7) we see that if the wave function  $\tilde{\psi}$  varies follow a simple harmonic law, then all of the components of the electromagnetic field  $\mathbf{E}(\mathbf{r}, t)$  and  $\mathbf{H}(\mathbf{r}, t)$  vary in time in a more complicated manner and depend on both functions  $\tilde{\psi}_1(\mathbf{r})$  and  $\tilde{\psi}_2(\mathbf{r})$ .

We now shift our focus to the definition of the dynamic field of the electron  $\tilde{\psi}(\mathbf{r})$ , which is defined from the general Equation (2) upon satisfying condition (3), satisfying in particular the Equation [1]:

$$-i\omega\psi_0(\mathbf{r}) \cdot \tilde{\psi}(\mathbf{r}) + \alpha\Delta[\psi_0(\mathbf{r}) \cdot \tilde{\psi}(\mathbf{r})] = 0, \quad (8)$$

where  $a = \beta c / \alpha$  and  $\Delta$  is the Laplacian.

We rewrite (8) into a more compact form:

$$\Delta F(\mathbf{r}) - i\bar{a}F(\mathbf{r}) = 0, \quad (9)$$

where  $F(\mathbf{r}) = \psi_0(\mathbf{r}) \cdot \tilde{\psi}(\mathbf{r})$  — a scalar, and  $\bar{a} = \omega/a$ .

In the spherical coordinate system that we will use,  $F(\mathbf{r})$  takes the form

$$F(\mathbf{r}) = \psi_{0r}(\mathbf{r})\tilde{\psi}_r(\mathbf{r}) + \psi_{0\phi}(\mathbf{r})\tilde{\psi}_\phi(\mathbf{r}) + \psi_{0\theta}(\mathbf{r})\tilde{\psi}_\theta(\mathbf{r}). \quad (10)$$

Defining all of the components of  $\tilde{\psi}(\mathbf{r})$  from the single Equation (9) is clearly not possible. We will solve an approximate problem using the condition that at a sufficient distance from the particle's center, the field  $\psi_0$  essentially satisfies Coulomb's law. As is shown in [1] [4], we can assume that  $\mathbf{E}_0(\mathbf{r}) \rightarrow E_{0r}(\mathbf{r})$ ,  $\mathbf{H}_0(\mathbf{r}) \rightarrow 0$ , and hence assuming (6), we obtain

$$F(\mathbf{r}) \rightarrow F(r) \rightarrow E_{0r}(r)\tilde{\psi}_r(r). \quad (11)$$

In this case, the problem of defining the function  $\tilde{\psi}$  simplifies dramatically and (9) turns into a simpler equation:

$$\Delta_r F(r) - i\bar{a}F(r) = 0, \quad (12)$$

or in the explicit form

$$\frac{d^2 F}{dr^2}(r) + \frac{2}{r} \frac{dF}{dr}(r) - i\bar{a}F(r) = 0. \quad (13)$$

We now seek a solution to the differential Equation (13) in the form

$$F = r^\gamma f(r), \quad (14)$$

where  $\gamma$  is a constant. Plugging (14) into (13), we have that

$$r^\gamma \left[ \frac{d^2 f}{dr^2} - i\bar{a}f \right] + r^{\gamma-1} 2(\gamma+1) \frac{df}{dr} + r^{\gamma-2} [\gamma(\gamma-1) + 2\gamma] f = 0. \quad (15)$$

Supposing that  $r \neq 0$  and equating each term in (15) to zero (*i.e.*, terms with various powers of  $r$ ), we obtain

$$\frac{d^2 f}{dr^2}(r) - i\bar{a}f(r) = 0, \quad (a)$$

$$2(\gamma+1)\frac{df}{dr}=0, \quad (b)$$

$$[\gamma(\gamma-1)+2\gamma]f(r)=0. \quad (c)$$

Assuming that  $f(r)$  and  $df/dr(r)$  are both nonzero, from (b) and (c) we obtain

$$\gamma = -1. \quad (16)$$

The solution to Equation (a) is of the form

$$f = Ae^{\delta r},$$

which plugging into (a) gives

$$\delta^2 - i\bar{a} = 0,$$

$$\delta = \pm\sqrt{\bar{a}}\sqrt{i}.$$

Using both signs for  $\delta$ , we obtain two linearly independent solutions to the differential Equation (10). Considering that  $i = e^{i\pi/2}$ , we have that

$$\sqrt{i} = e^{i\pi/4} = \cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}.$$

From the two possible signs in the above expression for  $\delta$ , we choose the negative sign and hence obtain

$$f = Ae^{-r\sqrt{2\bar{a}}/2}e^{-ir\sqrt{2\bar{a}}/2}. \quad (17)$$

Considering (16), we have that

$$F = \frac{A}{r}f, \quad (18)$$

where  $A$  is a constant. In the end, consider (11) and the fact that  $E_{0r}(r) \cong q/r^2$ , where  $q$  is the charge of the electron, we obtain the following for the wave function  $\tilde{\psi}_r(r)$ :

$$\tilde{\psi}_r(r) = \frac{A}{q}r \exp\left[-\frac{\sqrt{2\bar{a}}}{2}(1+i)r\right]. \quad (19)$$

We point out that, unlike the static field that decays with distance as an inverse square law, the dynamic field decays faster: as an exponential. We emphasize that the solution for the dynamic field of the electron  $\tilde{\psi}(r)$  is obtained from spherical symmetry and is far from the particle's center.

### 3. Some Commentary

We point out that in Equation (19) for the dynamic field of the particle, there are two quantities that would initially seem to have different physical natures: the charge  $q$  and the frequency  $\omega$  (via the constant  $\bar{a}$ ). However

$$\hbar\omega = mc^2,$$

where  $\hbar$  is Planck's constant and  $m$  is the mass of the particle. This way, the dynamic field depends both on the charge and mass. On the other hand, there is

a remarkable resemblance between Coulomb's law and Newton's law of gravitation [5]. Since the mass of a particle defines its gravitational field, don't the above observations serve the purpose of demonstrating that the gravitational field happens to be one aspect of the general field that exhibits an electromagnetic nature, and perhaps is determined by the generalized Maxwell equation [1].

Undoubtedly, these discussions only suggest a new perspective on more general problems which will require further thorough investigation.

### Conflicts of Interest

The author declares no conflicts of interest.

### References

- [1] Man'kin, I. (2022) Elementary Particles' Electrodynamics. *Open Access Library Journal*, **9**, e9129. <https://doi.org/10.4236/oalib.1109129>
- [2] Давыдов, А.С. (1973) Квантовая Механика. Наука.
- [3] Landau, L.D. and Lifshitz, E.M. (1980) Quantum Mechanics. Butterworth-Heinemann, Oxford.
- [4] Man'kin, I. (2023) On the Structure of the Electron. *Open Access Library Journal*, **10**, e10504. <https://doi.org/10.4236/oalib.1110504>
- [5] Landau, L.D. and Lifshitz, E.M. (1980) The Classical Theory of Fields. Butterworth-Heinemann.