The Sun's Rotation Appears Differential plus Other New Views

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ABSTRACT

This is a fresh perspective on the sun that considers its huge spherical size in relation to the finite speed of light. The sun is so extended that it takes light approximately 2.32 seconds to travel from the plane of the solar limb to the plane tangential to the sun at the solar disc's center. The aforementioned information is utilized in this study to support the new viewpoints. Firstly, it is shown that the solar disc is a simultaneous view of successively emitted coaxial spherical circles. Secondly, despite the fact that the sun is gaseous, it is thought to revolve completely as a rigid body at a fixed angular speed, yet an observer on Earth sees it rotate differentially. In a simple mathematical approach, it is found that the sun's rotational speed apparently decreases with latitude. Thirdly, a qualitative examination of how we observe simultaneous whole-surface brightness changes of the sun and sunlike stars indicates that such changes would appear to spread out radially from the center of the solar disc.

1. INTRODUCTION

The speed of light in vacuum, c, is a universal constant of a finite value of 299,792,458 meters per second (approximately 300,000 km/s). Though termed the speed of light, c is actually the speed of all electromagnetic waves (including visible light) in vacuum. According to the special theory of relativity, the speed of light c is the upper limit for the speed at which conventional matter, energy or any signal carrying information can travel through space [1-3]. In our immediate neighborhood and at this upper limit speed, electromagnetic waves appear to propagate instantaneously. But, for considerably long distances and for extended objects, it is important to consider the fact that the speed of light is finite. The American physicist N. James Terrell showed in 1959 the fact that the eye records all the light that is received simultaneously, even though, if the object is extended, this light was emitted at different times from parts of the object at different distances from the observer [4, 5]. The radius R_0 of our mother star, the Sun, has a nominal value of 695,700 kilometers. In light-seconds, the solar radius is equal to 2.32061 [6, 7]. The effect of such an extended solar radius and of the fact that the speed of light is finite on our observation, is discussed in the following sections.

2. THE SOLAR DISC IS A SIMULTANEOUS VIEW OF SUCCESSIVELY EMITTED CO-AXIAL SPHERICAL CIRCLES

The solar disc that appears flat is actually a simultaneous view of the three-dimensional bottom hemisphere of the sun. So, we can think of the solar disc as a set of successively connected spherical circles that are aligned along a co-axis connecting the center of the solar disc to the center of the sphere of the sun. The planes of these spherical circles are, hence, at successively increasing distances from the observer on Earth. The nearest to the observer is the plane tangential to the sphere of the sun at the center of the solar disc, and the farthest is the plane of the solar limb, as shown in **Figure 1**.

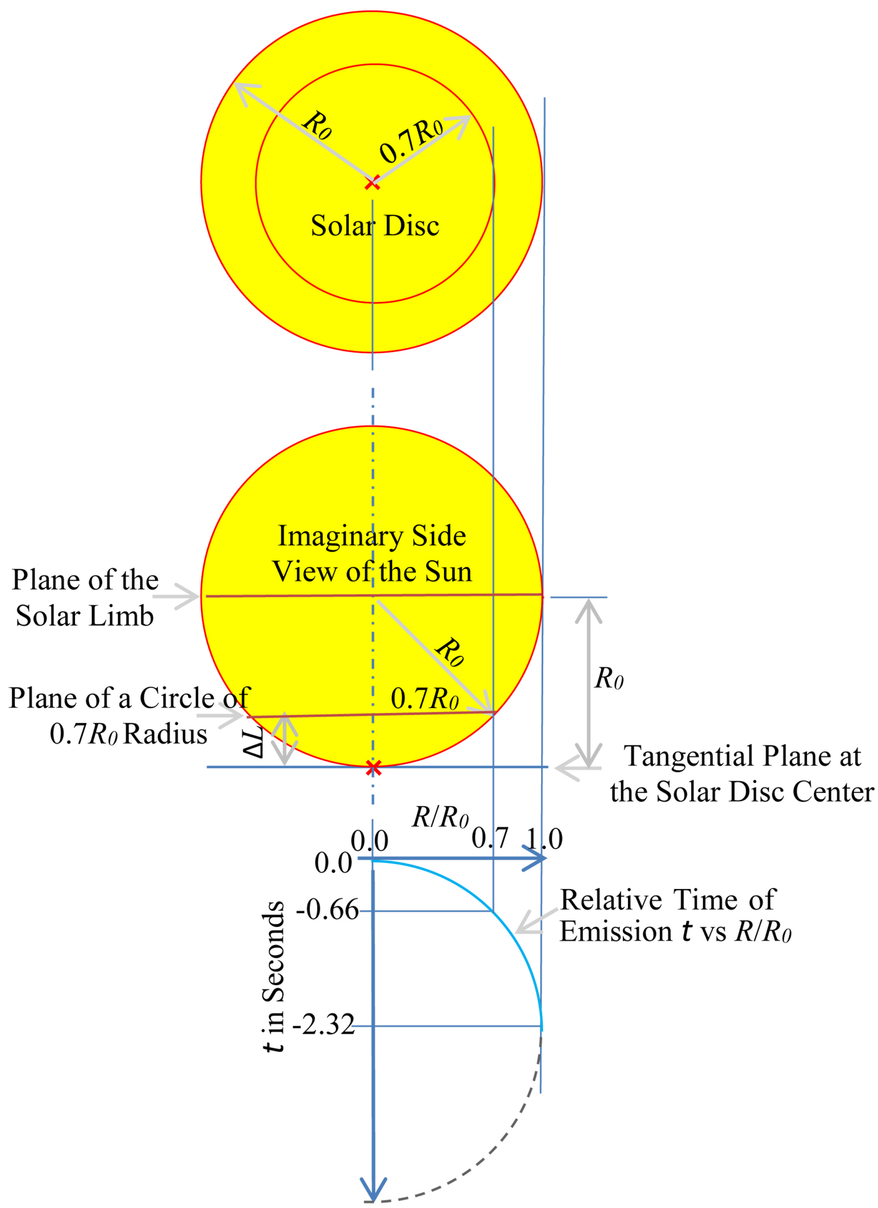

Figure 1. The solar disc as a set of successively emitted co-axial spherical circles.

Relative to the tangential plane, the perpendicular distance ΔL of any subsequent plane is a function of the radius of the spherical circle on that plane. The perpendicular distance ΔL, as shown above in **Figure 1**, increases sinusoidally from zero at the center of the disc to the maximum of one solar radius (2.32 light-seconds) at the solar limb. So, in line with the fact shown by N. James Terrell, as referred to in the introduction, we can say that the observer on Earth sees the whole solar disc simultaneously, even though light was emitted at different times from spherical circles at different distances from the observer. In other words, the circles forming the solar disc are not of the same age. At the constant and finite speed of light, the corresponding period of time taken by light from any circle to reach the tangential plane is thus equal to $\Delta L/c$. It can also be said that $\Delta L/c$ is the time by which light from any given circle was emitted earlier than the light emitted from the center point. As a result, the time of emission t of any circle of radius R/R_0 relative to the disc center is equal to (-) $\Delta L/c$, as illustrated graphically in **Figure 1**. The following simple equations can also be used to calculate the relative time of emission t of any given circle of radius $0 \leq R/R_0 \leq 1$:

$$\Delta L = \left[1 - \sqrt{1 - \left(R/R_0\right)^2}\right] R_0 \tag{1}$$

$$t = -\Delta L/c = -\left[1 - \sqrt{1 - \left(R/R_0\right)^2}\right] R_0/c \tag{2}$$

But, as referred to in the introduction, $R_0/c = 2.32061$ seconds. So, by substituting this value in (2):

$$t = -\Delta L/c = -2.32061\left[1 - \sqrt{1 - \left(R/R_0\right)^2}\right] \tag{3}$$

From (3), for a circle of $0.7R_0$ radius as an example:

$$t = -\Delta L/c = -2.32061\left[1 - \sqrt{1 - 0.7^2}\right] = -0.66 \text{ seconds} \tag{4}$$

3. THE SUN'S ROTATION IS APPARENTLY DIFFERENTIAL

The rotation of the sun is observed to vary with latitude. At the equator, where the latitude angle is equal to zero, the rotation is observed to be the fastest, and it decreases with increasing latitude in either the northern or southern pole direction. Consequently, the period of rotation in the equatorial region is around 24 days, while that in the polar region is 30 to 38 days. The source of this "differential rotation" is an area of current research in astronomy [8-11]. However, since it is an observation via light, it is natural to ask the simple question of whether the Sun's differential rotation is actual or apparent. According to the related literature reviewed, the sun's differential rotation is widely assumed to be actual. The argument behind this assumption is that since the sun is a ball of gas and plasma, it does not have to rotate wholly as a rigid body.

Well, the following alternative explanation of the sun's differential rotation is based on the alternative assumption that the sun, though gaseous, rotates wholly as a rigid body at a constant angular speed. Yet, to the observer on Earth, it appears to rotate differentially. In other words, the rotation of the sun is only apparently differential.

To explain the Sun's differential rotation in light of the above alternative assumption, the distances between the different compared regions of the solar disc must first be known. As shown in the previous section, the solar disc that appears flat is actually the simultaneous view of the three-dimensional bottom hemisphere of the sun. It consists of successively connected spherical co-axial circles that are at different distances from the observer on Earth. So, to compare the rotational speed of different points that are in different regions on the solar disc, let us first find the distance of any given point on the solar disc relative to, say, the plane of the solar limb as a reference. As shown below in **Figure 2**, the perpendicular distance L of any point X from the plane of the solar limb can be obtained from the following simple general equation:

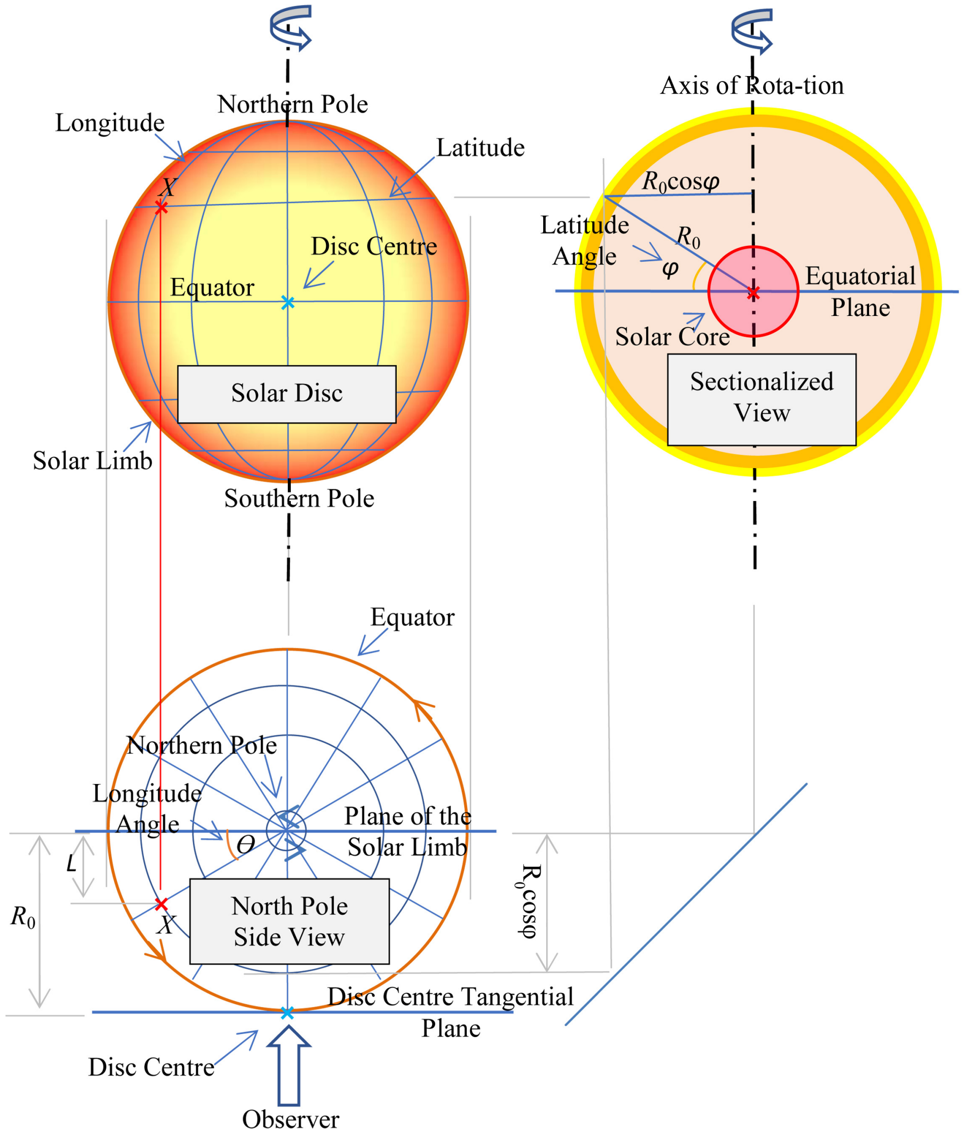

Figure 2. Relative distance of any given point on the solar disc.

$$L = R_0 \cos\varphi \sin\theta \tag{5}$$

where:

- L = The perpendicular distance of point X from the plane of the solar limb.
- R_0 = The solar radius.
- φ = Latitude angle measured in either northern or southern pole direction.
- θ = Longitude angle measured counter clock wise from the left-hand side of the solar limb.

For simplicity of explaining the apparently differential rotation, two extreme points are first considered as shown below in **Figure 3**. Point E is at the equator where the latitude φ is equal to zero and point P is in a polar region that is so close to the observed pole that the latitude φ tends to be 90 degrees. The perpendicular distances L_E and L_P of points E and P respectively from the reference plane of the solar limb, can then be found as follows:

From (5):

$$L_E = R_0 \cos 0^\circ \sin\theta = R_0 \sin\theta \tag{6}$$

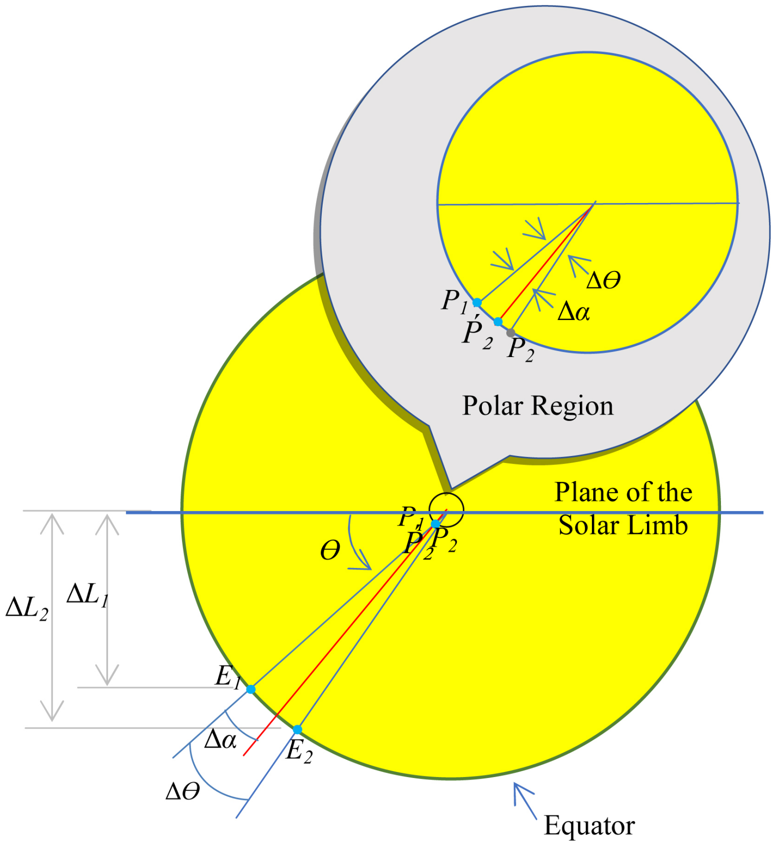

Figure 3. Differential rotation between equatorial and polar regions.

$$L_P = R_0 \cos 90^\circ \sin\theta = 0 \tag{7}$$

Hence from (6) and (7), at any longitudinal angle θ the perpendicular distance ΔL between the planes of the two points is:

$$\Delta L = L_E - L_P = R_0 \sin\theta \tag{8}$$

So, at any point of time t, the observer sees the two points simultaneously, even though the light from point P was emitted earlier than that from point E by the period of time δt, which is the time required for light to travel the perpendicular distance ΔL between the two points at any longitude angle θ, so:

$$\delta t = \Delta L / c \tag{9}$$

From (8) and (9):

$$\delta t = \left(R_0 / c\right) \sin\theta \tag{10}$$

As evident from Equation (10), in this special case of comparing an equatorial point with a polar one, the difference in time of emission δt is a sinusoidal function of the longitude angle $\boldsymbol{\Theta}$. It rises from zero on the left side of the solar limb to a maximum at the center of the solar disc, then falls to zero on the right side of the solar limb. However, as we will see later, the sun's differential rotation decreases with latitude angle.

Now, with the assumption that the sun rotates wholly as a rigid body at a constant angular speed ω, suppose that an observer on Earth started observing the rotation of the points E and P. As shown above in Figure 3, the two points were initially observed to be on the same longitude of an angle θ at the initial positions E_1 and P_1, respectively. The points E and P are assumed to be fixed on the solar surface and hence rotate with the sun. At the end of a very short observational period Δt, the observer will see the equatorial point at, say, position E_2 and will simultaneously see the polar point at $\acute{P}_2$, which was emitted earlier than E_2 by a period δt, being the time required for light to travel the perpendicular distance ΔL_2 between the two points. It must be noted here that, with respect to the observer, the short observational period Δt is the same for observing both points from their initial to their final apparent positions. So, while the equatorial point rotated with the sun for the entire observational period Δt, the polar point rotated for a shorter period of $(\Delta t - \delta t)$ due to light spending a short period δt traveling the perpendicular distance ΔL_2 from the polar point's final apparent position. As the two points are assumed to be rotating with the sun at a constant angular speed ω, the resulting small angular displacement $\Delta \theta$ of the equatorial point during the short observational period Δt will be equal to $\omega(\Delta t)$. This, while the angular displacement of the polar point during the same observational period relative to the observer is equal to $\omega(\Delta t - \delta t)$. As a result, in the range $0 < \theta < 180$, the equatorial point appears to rotate faster than the polar point relative to the observer.

So now, with the alternative assumption that the sun rotates wholly as a rigid body, it has simply been shown that its rotation is only apparently differential. The total apparent differential period of the polar point and of any other point at a latitude angle φ relative to the equator, is shown in the following part of this section.

At the end of the brief observational time Δt, the two final apparent locations are no longer at the same longitude, as illustrated in **Figure 3**. The polar point, at its final apparent position $\acute{P}_2$, is now behind the equatorial point, at its final apparent position E_2. During the short observational period Δt, the instantaneous position of the polar point P_2, which is at the same longitude as E_2, has not yet been seen by the observer. For P_2 to be seen, an additional time period equal to the time it takes light to cross the perpendicular distance between the polar and equatorial points, *i.e.*, equal to δt, must pass. As a result, δt can be considered the sectorial differential period during the observational period Δt. Now, if the observation is continued for a second equal observational period Δt, then the initial position of the equatorial point will be E_2, while that of the polar point will be $\acute{P}_2$, and not P_2, which has not yet been seen by the observer. So, the amount by which the polar point was behind during the first observational period will be added to the next. This simply means that the apparent total differential period of rotation between the equatorial and polar points will be the sum of the small sectorial differential periods observed over a series of short observational periods across the solar disc. As the differential period of rotation is apparent, it applies only to the half of the sun we see. It does not make sense to apply this apparent observation to the other half of the sun that we don't see. Therefore, the total apparent differential period across the solar disc is the total apparent differential period for one complete revolution of the sun.

Now, to find the total differential period, let us divide the bottom semi-circle of the view in **Figure 4** into small, equal sectors, each with a central angle representing the small angular displacement of the equatorial point over a very short observational period Δt. The total differential period ΔT_P between the polar and equatorial points across the solar disc can then be found as follows:

From Equation (10) and **Figure 4**, the total differential period ΔT_P between the polar and equatorial points across the solar disc is as below:

$$\Delta T_P = \left[\left(R_0/c \right) \sum\nolimits_{k=1}^{n} \sin k \left(\Delta\theta \right) \right] \Big/ 86400 \quad (11)$$

where:

- $k = 1, 2, 3, \cdots, n$.
- ΔT_P = The differential period between the polar and equatorial points in days.
- n = The number of sectors across the solar disc.
- 86,400 = 24 × 3600 = The number of seconds per day.
- $\Delta\theta$ = The small sector central angle in radians.

By Lagrange's trigonometric identities [12], Equation (11) can be put in the following form:

$$\Delta T_P = \left(R_0/c \right) \left[\cos\left(\Delta\theta/2 \right) - \cos\left(\left(2n+1 \right) \left(\Delta\theta/2 \right) \right) \right] \Big/ \left[2\sin\left(\Delta\theta/2 \right) \right] \Big/ 86400 \quad (12)$$

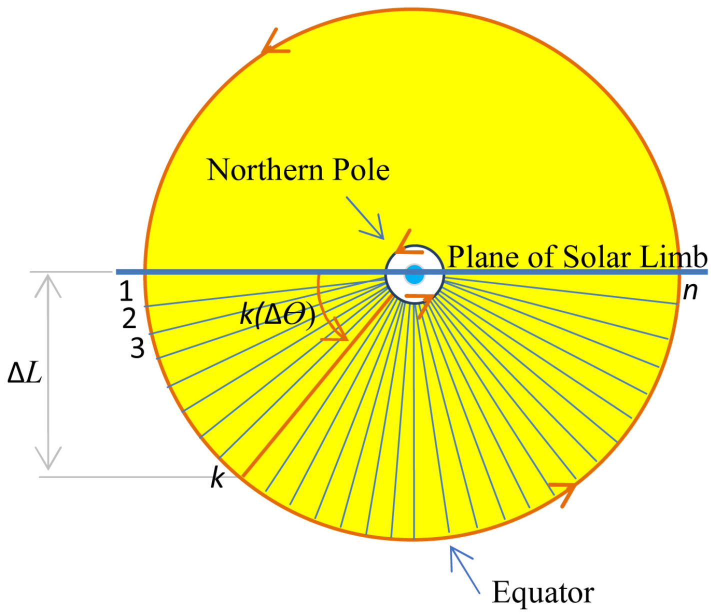

Figure 4. **The sum of the sectorial differential periods of rotaion across the solar disc.**

The sector angle $\Delta\theta$ in radians is given by:

$$\Delta\theta = \pi/n \tag{13}$$

From (12) and (13):

$$\Delta T_P = (R_0/c)\left[\cos(\pi/2n) - \cos(\pi + \pi/2n)\right]/\left[2\sin(\pi/2n)\right]/86400 \tag{14}$$

The number of sectors n is the nearest whole number after dividing half the equatorial period of rotation T_E by the short observational period Δt as follows:

$$n = 86400 T_E/2/\Delta t \tag{15}$$

where:

- T_E = The equatorial period in days.
- Δt = a short observing period in seconds.

From Equation (15) it clear that the number n is very big and hence in Equation (14) the following approximations can be made:

$$\cos(\pi + \pi/2n) \approx \cos\pi = -1 \tag{16}$$

And by the small-angle approximation of trigonometric functions [13]:

$$\cos(\pi/2n) \approx 1 \tag{17}$$

$$\sin(\pi/2n) \approx \pi/2n \tag{18}$$

Substituting (16), (17) and (18) in Equation (14):

$$\Delta T_P = (R_0/c)\left[1 - (-1)\right]/(2\pi/2n)/86400 = (R_0/c)(2n/\pi)/86400 \tag{19}$$

In Equation (15), assuming that Δt is equal to 1 second and substituting in (19);

$$\Delta T_P = (R_0/c) T_E/\pi \tag{20}$$

From (20), the polar point apparent period of rotation T_P relative to the equatorial point is as follows:

$$T_P = T_E + (R_0/c) T_E/\pi \tag{21}$$

Equation (21) is a special case equation for finding the apparent period of a polar point where the latitude angle is 90˚ when given that of the equatorial point, where the latitude angle is zero. Let us now look at the general case equation for calculating the apparent period of rotation T_φ at any latitude angle relative to the equatorial apparent period. Starting from Equation (6), the perpendicular distance L_φ is:

$$L_\varphi = R_0 \cos\varphi \sin\theta \tag{22}$$

Following the same steps as above and keeping n, $\Delta\theta$, and Δt constants as in the special case equation, the polar period relative to T_φ is:

$$T_P = T_\varphi + \left(\left(R_0 \cos\varphi\right)/c\right) T_E / \pi \tag{23}$$

Substituting for T_P from (21) in (23):

$$T_\varphi = T_E + \left(1 - \cos\varphi\right)\left(R_0/c\right) T_E / \pi \tag{24}$$

The general-case Equation (24) can be used to get the apparent differential period at any latitude angle given the equator's period. The relevant periods of rotation at various latitudes are represented in Table 1 for an equatorial period of 24 days. Figure 5 shows a graphic representation of the same outcomes.

Now it can be said with certainty that the rotation of the sun with respect to the observer appears to change with latitude under the alternative hypothesis that it spins wholly as a rigid body at a constant angular speed. Table 1 and Figure 5 demonstrate that the period of rotation increases with latitude. In other words, it appears that as latitude increases, rotational speed decreases.

Table 1. Period of rotation vs latitude angle (Given an equatorial period of 24 days).

Latitude Angle in Degrees	10	20	30	40	50	60	70	80	90
Apparent Period of Rotation in Days	24.3	25.1	26.4	28.1	30.3	32.9	35.7	38.6	41.7

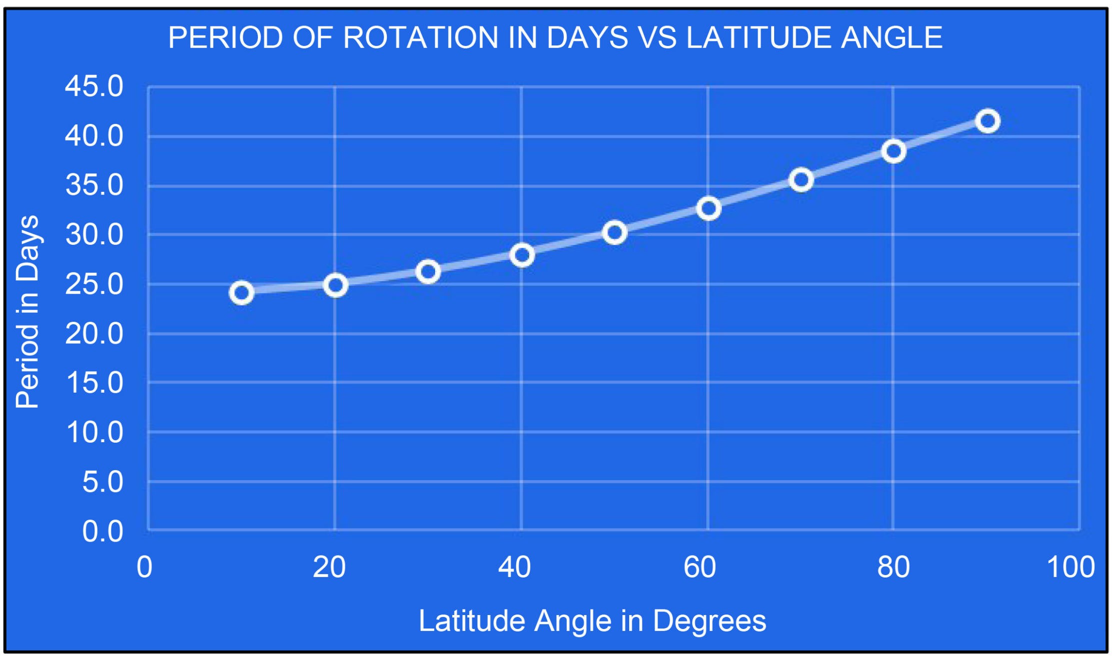

Figure 5. Period of Rotation vs Latitude Angle (Given an equatorial period of 24 days).

4. EFFECT ON OUR VIEW OF A CHANGE IN STAR BRIGHTNESS

Now let's examine how our view of a change in star brightness is affected by the size of the sun or any solar-like star and the finite speed of light. The hypothetical scenarios covered below are merely meant to illustrate this impact; they do not necessarily relate to or provide an explanation for any observed phenomena. The change is assumed to be a simultaneous, whole-surface, noticeable change. In the first scenario, it is assumed that some instantaneous, noticeable, and persisting change in the brightness of the sun took place, as denoted by the two different colours in **Figure 6(a)**. As explained above in Section 2, the circles forming the solar disc are at different distances from the observer. Hence, for an observer on Earth watching the above hypothetical change from just before it starts to the end, it will appear that the change is spreading radially out from the center of the solar disc.

In the second scenario, it is assumed that brightness change is oscillating between two distinctively high and low brightness levels. If the oscillation is slow, then the change will appear to the observer as alternate thick rings expanding radially from the disc center, as shown in **Figure 6(b)**. For rapid oscillation, alternate thin rings will appear to flow radially out from the disc center, as in **Figure 6(c)**.

The third scenario is that of a fading and rebrightening star. Fading is assumed to be so that the corona of a sun-like star can be seen. But with a fading star, the corona will also fade. However, it is known that the corona is the outer atmosphere of the sun or sun-like stars that stretches out many times the solar radius. As shown below in **Figure 6(d)**, when the time line is above the fading star, light from the upper hemisphere of the corona that was emitted when the star was bright will start to be received layer by layer. The gradually decreasing brightness of the corona can possibly overwhelm the gradually fading star.

As illustrated above in **Figure 6(d)**, it will therefore appear as if the star is expanding or exploding while fading out.

5. RESULTS AND DISCUSSION

All the above new views of the sun are based on Terrell's fact that the eye records all the light that is received simultaneously, even though, if the object is extended, this light was emitted at different times from parts of the object at different distances from the observer. The sun, with its 2.32 light-second radius, is an extended object. Light from different parts of the solar disc takes different times to reach the observer on Earth. A simple geometrical and mathematical approach was followed for the quantitative and qualitative explanation of these new views, as follows:

1) A new view of the solar disc: It is well known that the apparently plain solar disc is a view of the bottom hemisphere of the sun. The solar disc is hence a set of successively connected co-axial spherical circles that are at successively increasing distances from the observer on Earth. By the above Terrell's fact, these circles, which are received simultaneously by the observer, must have been emitted at different times. By simple geometry, as shown in Section 2 of this study, given the radius of any circle on the solar disc, the time by which the circle was emitted earlier than the center can be found.

2) The rotation of the sun is only apparently differential: The rotation of the sun is observed to vary with latitude. At the equator, where the latitude angle is equal to zero, the rotation is observed to be the fastest, and it decreases with increasing latitude in either the northern or southern pole direction. As the period is the reciprocal of the angular speed, it can also be said that the period of the sun's rotation increases with latitude. This differential rotation is an area of current research in astronomy. In this study, it is assumed that the sun rotates wholly as a rigid body at a constant speed. Under this alternative assumption, the paper investigates in Section 3 how the rotation appears to an observer on Earth. As shown above, the solar disc is the simultaneous view of the bottom hemisphere of the sun. Hence, in the range $0 < \theta < 180$, the polar regions are more distant from the observer on Earth than the equatorial regions. Now consider one polar and one equatorial points fixed on the surface of the sun. An observer on Earth is observing the rotation of the two points from their initial to their final apparent positions over a short period of time. By the above Terrell's fact, at the end of the short observation period, the observer will simultaneously see

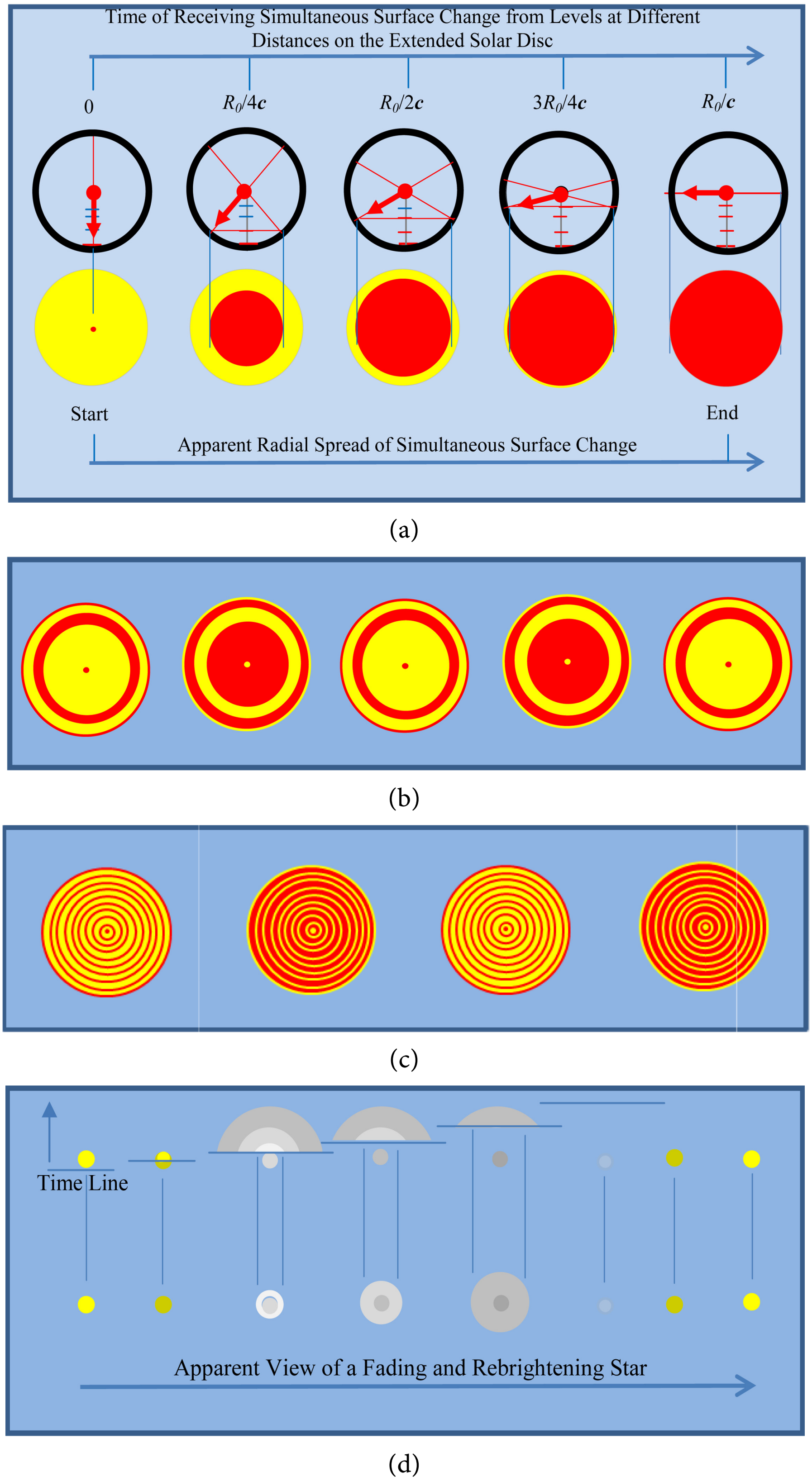

Figure 6. (a) Hypothetical noticeable change in brightness. (b) Slow brightness oscillation. (c) Rapid brightness oscillation. (d) Fading and rebrightening of a star.

the two points at their final apparent positions, even though the polar point was emitted earlier than the equatorial point. In a straightforward mathematical approach, it has been shown in Section 3 that the equatorial point appears to rotate faster than the polar point. The apparent period as a function of the latitude and the equatorial period is given by Equation (24). Assuming that the equatorial period is 24 days, the apparent period at various latitudes is shown in **Table 1** and **Figure 5**, which clearly show that the period of rotation increases with latitude.

3) Effect on our view of a change in star brightness: It can be deduced from Terrell's fact that the light that is simultaneously emitted from parts of an extended object at different distances from an observer will not be simultaneously received by the observer. This light will be received in the order of distances from the shorter to the longer [14]. So when a noticeable, simultaneous, whole surface brightness change of the sun or sun-like stars takes place, it will not be received simultaneously. The change will appear to spread out radially from the center of the solar disc, as shown in Section 4. Another interesting scenario is how we see a fading star. The star appears to be expanding or exploding while fading out. The scenarios in Section 4 are hypothetical, yet they represent a good demonstration of the effect of the size of the star relative to the finite speed of light.

6. CONCLUSION

It is important to keep in mind that the speed of light is finite when making observations regarding extended objects like the sun. The solar disc, which appears flat, is actually a simultaneous picture of the sun's three-dimensional bottom hemisphere, as this study has shown. It is made up of spherical co-axial circles that are connected in a series and are at successively increasing distances from the observer on Earth. For instance, the solar limb's circle is 2.32 light seconds away from the solar disc's center. With the above solar disc view and under the alternative assumption that the sun rotates wholly as a rigid body, it has simply been shown that, with respect to the observer, the sun's rotation appears to vary with latitude. It has also been shown that if a hypothetical noticeable simultaneous change in brightness of the sun or sun like stars took place, it would appear to spread radially from the center of the disc of such a star. Another hypothetical phenomenon is the fading of a sun-like star. It has been illustrated that the star would appear to be expanding or exploding while fading out. Though hypothetical, these scenarios are thought to be important to consider when observing stars.

CONFLICTS OF INTEREST

The author declares no conflicts of interest regarding the publication of this paper.

REFERENCES

1. Speed of Light. https://en.wikipedia.org/wiki/Speed_of_light
2. Special Theory of Relativity Postulates. https://oxscience.com/special-theory-of-relativity/
3. Speed of Light. https://www.britannica.com/science/speed-of-light
4. Skinner, R. (1982) Relativity for Scientists and Engineers. Dover Publications, New York.
5. Terrell Rotation. https://en.wikipedia.org/wiki/Terrell_rotation
6. Solar Radius. https://en.wikipedia.org/wiki/Solar_radius
7. Sun Fact Sheet. https://nssdc.gsfc.nasa.gov/planetary/factsheet/sunfact.html
8. Solar Rotation Varies by Latitude. https://www.nasa.gov/mission_pages/sunearth/science/solar-rotation.html
9. Differential Rotation. https://en.wikipedia.org/wiki/Differential_rotation
10. Does the Sun Rotate? https://www.space.com/does-the-sun-rotate
11. Solar Rotation. https://en.wikipedia.org/wiki/Solar_rotation

12. List of Trigonometric Identities. https://en.wikipedia.org/wiki/List_of_trigonometric_identities
13. Small-Angle Approximation. https://en.wikipedia.org/wiki/Small-angle_approximation
14. Salah Abunaieb (2015) The Universe Is Expanding by Entirely Moving at Some Very High Speed and Not by Flying Away of the Galaxies from Each Other. *International Journal of Advanced Astronomy*, **3**, 78-86. https://doi.org/10.14419/ijaa.v3i2.4922