

Depth Analysis of Divergence in Teaching

Wenbing Wu, Rihua Liu

Nanchang Normal University, Nanchang, China

Correspondence to: Wenbing Wu, wwbysq@fjnu.edu.cn

Keywords: Divergence, Field Theory, Quantitative Method

Received: November 5, 2021

Accepted: December 21, 2021

Published: December 24, 2021

Copyright © 2021 by author(s) and Scientific Research Publishing Inc.

This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).

<http://creativecommons.org/licenses/by/4.0/>



Open Access

ABSTRACT

The concept of divergence is the basis of electromagnetic field theory, and has been a special difficulty in mathematical theory. In this paper, the connotation of divergence itself is truly displayed by quantitative method for the first time. Through these methods, the concepts of divergence can be grasped more easily. The explanation of divergence is helpful to strengthen the understanding of the concept and has a certain reference significance.

1. INTRODUCTION

In the theory of electromagnetic field, in order to simplify the operation, some operator symbols are introduced. They have become indispensable tools in the field theory analysis. The Hamiltonian operator and Laplacian operator are widely used. Hamiltonian operator, the mathematical symbol is ∇ . Divergence is a quantity that describes the extent to which air converges from the surrounding area to or from a certain point. The divergence of 3-D space represents the change rate of unit volume of any gas block in unit time. The volume expansion of an air mass is called divergence, and the volume contraction of an air mass is called convergence. Divergence, as an important mathematical tool, has been widely used in various aspects of scientific research, such as interaction of population by temperature treatment [1], variational Inequalities [2]. Due to the abstract characteristics of divergence, both the lecturer and the learner will find it difficult in the process of teaching and learning the concept. The existing literature has also carried out a lot of discussion on this problem, but all of them are based on the further analysis of the existing theory [3, 4]. More relevant literature is based on the calculation of gradient and divergence [5-7]. This paper proposes a concrete understanding method for the concept of divergence.

2. QUANTITATIVE UNDERSTANDING OF DIVERGENCE

2.1. Flux and Divergence

First of all, the theoretical knowledge related to divergence must be listed.

1) Gauss formula:

Let a space closed region Ω be surrounded by a piecewise smooth closed surface Σ , and the functions $P(x, y, z)$, $Q(x, y, z)$ and $R(x, y, z)$ have first order continuous partial derivatives on Ω , then there is a

formula

$$\iiint_{\Omega} \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right) dv = \oiint_{\Sigma} P dydz + Q dzdx + R dxdy \quad (1)$$

2) Definition of Flux:

Given a vector field

$$\mathbf{A}(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k} \quad (2)$$

The integral of the directed surface Σ in the field is:

$$\Phi = \iint_{\Sigma} \mathbf{A} \cdot d\mathbf{S} = \iint_{\Sigma} \mathbf{A} \cdot \mathbf{n}^0 dS = \iint_{\Sigma} P dydz + Q dzdx + R dxdy \quad (3)$$

It's called the flux of the vector field $\mathbf{A}(x, y, z)$ through the directed surface Σ .

3) Definition of Divergence:

Let the vector field $\mathbf{A}(x, y, z)$, set a closed surface of the point M in the field, and the area enclosed

by Σ is V . When V shrinks to the point M , if the limit $\lim_{V \rightarrow M} \frac{\oiint_{\Sigma} \mathbf{A} \cdot d\mathbf{S}}{V}$ exists, the limit value is called the divergence at the point M , denoted as $\text{div } \mathbf{A}$.

Where

$$\text{div } \mathbf{A} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \quad (4)$$

Here introduce Hamilton operator:

$$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k} \quad (5)$$

then:

$$\text{div } \mathbf{A} = \nabla \cdot \mathbf{A} \quad (6)$$

Gauss formula can be expressed as:

$$\int_V \nabla \cdot \mathbf{A} dv = \oiint_{\Sigma} \mathbf{A} \cdot d\mathbf{s} \quad (7)$$

From the Gauss flux theorem:

$$\oiint_{\Sigma} \mathbf{D} \cdot d\mathbf{s} = \sum q \quad (8)$$

Then the following equation can be obtained:

$$\nabla \cdot \mathbf{D} = \rho \quad (9)$$

2.2. Explanation of Divergence

According to the explanation of divergence in general textbooks, divergence is a scalar, which can be understood as the flux passing through the closed surface of unit volume, that is, the change rate of flux to volume at a given point in the field. Divergence represents the strength of the source, so in the passive area where the source does not exist, the divergence of each point should be equal to zero.

To imagine a charged sphere divergence through spatial imagination in [Figure 1](#). Suppose that the charged sphere is wrapped with a spherical surface, then the surface will surround all the power lines emitted by the sphere. Assuming that the number of lines is 10,000, and the volume of this surface is equal to 1, then the volume density of power lines is 10,000; if the surface is expanded to 10 times of the original,

according to Gauss theorem Formula (1), the power lines surrounded by the surface will not change and is always equal to 10,000, then the volume density of power lines is 1000; therefore, the volume of this surface changes from 1 to 10, in this process, the volume density of power lines changes from 10,000 to 1000, and the change ratio of this density with the volume increasing by 10 times is $(10,000 - 1000)/10,000 = 90\%$; then suppose that the volume of this surface changes from 10 to 100, and then through the same analysis, the proportion of power lines density changing with volume increasing 10 times is $(1000 - 100)/1000 = 90\%$ too.

It is particularly emphasized here that the volume density of power lines obtained by the above method is the value calculated when the unit volume element is at a certain surface position of the spherical surface, such as points A, B and C in **Figure 2**; for point D, a spherical surface must be drawn at this point and then calculated.

According to the above analysis results, it can be considered that the electric field divergence, that is, the change rate of power lines to volume at any point in the electric field, refers to the change of the volume density of power lines at this point. It is proportional to the distance between the given point and the charged sphere (the volume of the corresponding spherical surface), and this ratio is fixed at any point outside the charged sphere.

For example, the number of light emitted by the sun in **Figure 3** is fixed. With the distance from the sun, the number of light in the unit space outside the sun will gradually decrease. The concept of divergence perfectly expresses our feelings that we can only understand but difficult to express.

According to the textbook explanation, divergence is the change rate of power lines volume density. According to the previous analysis results, the true meaning of this sentence can be gotten: electric field divergence is the change rate of power lines to volume at any point in the electric field, which is proportional to the distance between the given point and the charged sphere. This ratio is fixed at any point outside the charged sphere, that is, the number of power lines in a unit volume increases or decreases in proportion to the distance between the volume element and the charged sphere.

Through the above analysis, we can get the essential connotation of the divergence symbol itself, that is, the divergence represents the change rate of the number of power lines contained in one unit volume in the field, and this number will change with the distance of the unit volume from the charged sphere.

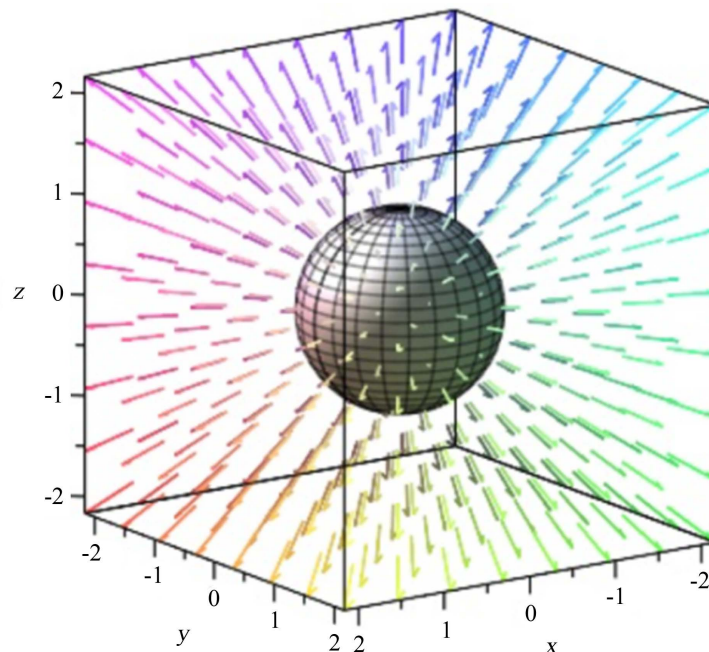


Figure 1. Charged sphere.

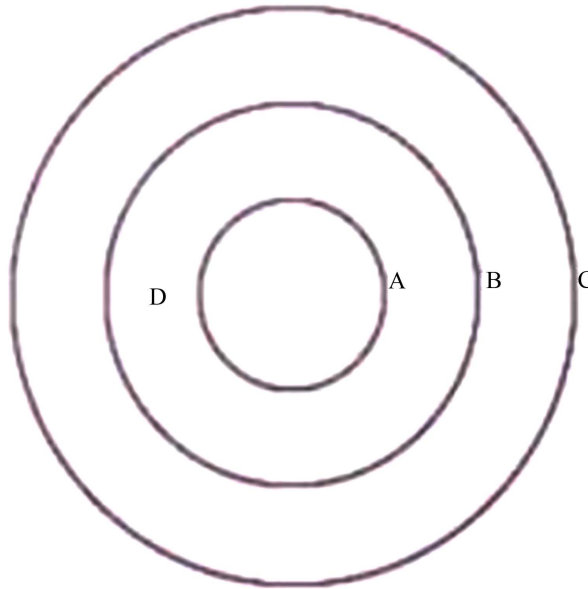


Figure 2. Divergence calculation.

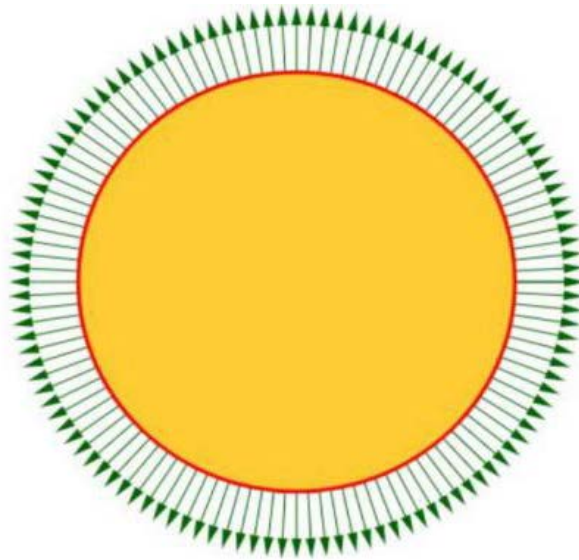


Figure 3. Light emitted by the sun.

3. CONCLUSION

Through quantitative calculation, the connotation of divergence is deeply analyzed: Firstly, through the divergence picture of a charged sphere, the change ratio of flux with the volume changing is calculated, the result shows that this ratio is fixed at any point outside the charged sphere. Through the above visual display of the concept of gradient, the incomprehensible concept of divergence becomes concrete, so the concept of divergence becomes easily grasped for the students.

FUNDING

This paper is supported by Research Foundation of the Nanchang Normal University for Doctors (NSBSJJ2018014).

CONFLICTS OF INTEREST

The authors declare no conflicts of interest regarding the publication of this paper.

REFERENCES

1. Boche, H. and Pohl, V. (2018) Calculating the Hilbert Transform on Spaces with Energy Concentration: Convergence and Divergence Regions. *IEEE Transactions on Information Theory*, **65**, 586-603. <https://doi.org/10.1109/TIT.2018.2859328>
2. Wang, Y., Li, Q.Z., Zhou, F.L., *et al.* (2019) A New Method with Hilbert Transform and Slip-SVD-Based Noise-Suppression Algorithm for Noisy Power Quality Monitoring. *IEEE Transactions on Instrumentation and Measurement*, **68**, 987-1001.
3. Cui, X., Li, C., Geng, Y., *et al.* (2019) Secondary Envelope Extraction Based on Multiple Hilbert Transforms for Laser Self-Mixing Micro-Vibration Measurement. *Applied Optics*, **58**, 9392-9397. <https://doi.org/10.1364/AO.58.009392>
4. Castillo, J., Mocquet, A. and Saracco, G. (2018) Wavelet Transform: A Tool for the Interpretation of Upper Mantle Converted Phases at High Frequency. *Geophysical Research Letters*, **28**, 4327-4330. <https://doi.org/10.1029/2001GL013214>
5. Mor-Yossef, Y. (2019) A Stable, Positivity-Preserving Scheme for Cross-Diffusion Source Term in RANS Turbulence Models: Application to $k-\omega$ Turbulence Models. *Computers & Fluids*, **191**, Article ID: 104234. <https://doi.org/10.1016/j.compfluid.2019.104234>
6. Sun, S., Jin, C., Si, J., *et al.* (2021) Single-Cell Analysis of Ploidy and Transcriptome Reveals Functional and Spatial Divergency in Murine Megakaryopoiesis. *Blood*, **138**, 1211-1224. <https://doi.org/10.1182/blood.2021010697>
7. Wang, G. and She, Y. (2007) Topological Description of Divergency and Consistency of Two-Valued Propositional Theories. *Acta Mathematica Sinica*, **50**, 841-850.