

Universes Being Invisible on Earth outside the Portals Are Visible in Portals

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ABSTRACT

The authors of the existing version of the special theory of relativity had to use the principle of light speed non-exceedance to explain relativistic formulas obtained therein within the space of real numbers. The principle implied existence of only our visible universe and absence of physical content in imaginary numbers. However, since the principle is just a postulate, *i.e.* an unproven assumption, it has always carried little credibility. The paper refutes the principle by the experimentally proven principle of physical reality of imaginary numbers. As follows from the principle of physical reality of imaginary numbers, relativistic formulas of the existing version of the STR are wrong and incorrectly explained, and conclusions drawn from them are misguided. In other words, this version is incorrect¹ and, thus, the STR was not actually created in the 20th century. Moreover, it could not be created in the 20th century, as its creation required experimental data obtained in the 21st century. The paper provides an alternative version of the STR containing relativistic formulas corrected given experimental data obtained in the 21st century. These formulas imply that instead of the Monoverse, whose existence is supposed in the existing version of the STR, there is a Multiverse, whose universes are mutually invisible (and the invisibility is explainable) and therefore it has been referred to as hidden. The paper explains that existence of invisible universes of the hidden Multiverse gives rise to the phenomenon of dark matter and dark energy that is actually a sort of optical effect (however, not electromagnetic, but gravitational), a shadow, rather than some physical substance². It also explains that existence of other invisible universes outside the hidden Multiverse gives rise to the phenomenon of dark space. Invisible universes are claimed to really exist, which can be experimentally proved by astronomical observations in portals, where universes not visible outside

¹No undertaking may be considered complete, unless it is properly done. No one would drink coffee, if salt is put thereto instead of sugar.

²And, therefore, it cannot be detected at the Large Hadron Collider.

portals become partially visible. Therefore, constellations observed in the starry sky inside the portals are different from those observed in our visible universe. The alternative version of the special theory of relativity can also successfully solve other issues of astrophysics. In particular, it can explain where antimatter is located and why it does not annihilate with matter, as well as where tachyons are located and why they don't violate the principle of causality, etc.

1. INTRODUCTION

The special theory of relativity (STR) [1-3] is considered to be the greatest achievement of physics of 20th century. The theory asserts that according to the principle of light speed non-exceedance our visible universe is the one and only. However, such assertion has always been reasonably mistrusted³, since it follows from the above-mentioned postulated principle that is actually an unproven assumption. Moreover, the trigger for appearance of this principle in the STR was quite obvious and had nothing to do with any serious theoretical research. The authors of the STR [4-7] needed the principle only because they did not have correct explanation for relativistic formulas (the main result of their theory) taking imaginary values at hyper-light speeds within the framework of real number physics. And in order to avoid non-recognition of their theory, they would like to divert attention from this circumstance, one way or another.

It seemed as if the use of the principle of light speed non-exceedance allowed them to do this. Nevertheless, the authors of the existing version of the STR were so discouraged by this circumstance that further STR development ceased for almost a century⁴. And thus, incomplete and imperfect STR in its existing version came into use as a generally acknowledged theory.

2. UNSUCCESSFUL OPERA EXPERIMENT

In 1958, Pavel Alekseyevich Cherenkov, Igor Evgenievich Tamm and Ilya Mikhailovich Frank received the Nobel Prize for the discovery and interpretation of Cherenkov radiation [8], emitted when charged particles move through a transparent medium at a speed greater than the speed of light in that medium. Apparently, the discovery refuted the principle of light speed non-exceedance and, therefore, the existing version of the STR. However, over some time the principle was resurrected by revision stating that it implied movement of a physical body only in a vacuum.

Nevertheless, doubts about the truth of the principle of light speed non-exceedance have remained even in its revised version. In this regard, there was apparent necessity for an experiment that would be able to confirm or refute the principle of light speed non-exceedance in a vacuum, and thereby prove or refute the principle of physical reality of imaginary numbers. The OPERA experiment, aimed to detect superluminal neutrinos, would seem to be appropriate. On September 23, 2011 the OPERA collaboration published [9] a sensational report on registration of superluminal neutrinos contradicting the STR. However, on March 15, 2012 the ICARUS collaboration published [10] a report on refutation of the OPERA experiment, but still the ICARUS proved nothing. That has created illusion of irrefutability of the STR.

³The STR was criticized by Oliver Heaviside, Nikola Tesla, Nobel Prize winner Albert Abraham Michelson, Nobel Prize winner Friedrich Wilhelm Ostwald, Nobel Prize winner Joseph John Thomson, Nobel Prize winner Svante August Arrhenius, Nobel Prize winner Philipp Eduard Anton von Le-nard, Nobel Prize winner Alvar Gullstrand, Nobel Prize winner Wilhelm Carl Werner Otto Fritz Franz Wien, Nobel Prize winner Walther Hermann Nernst, Nobel Prize winner Ernest Rutherford, 1st Baron Rutherford of Nelson, Nobel Prize winner Johannes Stark, Nobel Prize winner Frederick Soddy, Nobel Prize winner Percy Williams Bridgman, Nobel Prize winner Edwin Mattison McMillan, Nobel Prize winner Hideki Yukawa, Nobel Prize winner Hannes Olof Gösta Alfvén and many other outstanding scientists.

⁴At that time, political and other considerations far from science began to influence the outcome of scientific discussions. Thus, decisions on banning criticism of the theory of relativity were made three times in the Soviet Union: in 1934, by the resolution of the Central Committee of the All-Union Communist Party (Bolsheviks) on the discussion of relativism; in 1942, by the resolution of the Presidium of the Academy of Sciences of the Soviet Union on the theory of relativity; and in 1964, by the closed decree of the Presidium of the Academy of Sciences of the Soviet Union that forbade all scientific councils, journals and departments to accept, consider, discuss and publish works criticizing the theory of Albert Einstein. In other countries, for example in Germany, the attitude to the STR was also ambiguous.

3. ALTERNATIVE EXPERIMENTAL PROOFS OF PHYSICAL REALITY OF IMAGINARY NUMBERS

Creation of such illusion was presumably the true goal of the OPERA and ICARUS experiments. That is, the OPERA and ICARUS experiments were probably just an advertising campaign, which, in the absence of scientific arguments, was supposed to prolong existence of the incorrect STR version. And such an assumption seems quite plausible, since it is confirmed by facts. Results of experimental studies of special processes in linear electric circuits [11-16] that successfully proved physical reality of concrete imaginary numbers⁵ and made OPERA experiment unnecessary were actually published in 2008-2010, *i.e.*, prior to publication of OPERA experiment results. And along with subsequent experimental studies of such processes [17-26] they have never been refuted, unlike the OPERA experiment.

Publications [11-26] presented even three experimental proofs of physical reality of imaginary numbers:

- A proof using oscillatory transient processes [15-17, 18, 25, 26]. It implied that there would be no tsunami, pianos and church bells would not sound and even children's swing wouldn't sway after being pushed by parents, if the principle of light speed non-exceedance were true;
- A proof using oscillatory resonant processes [11-14, 16, 18, 19, 25, 26]. It implied that there would be neither television, nor radiolocation, nor GPS trackers, nor mobile phones, etc, if the principle of light speed non-exceedance were true;
- A proof using Ohm's law in the interpretation of Steinmetz [20-26], proposed in 1893 and known to all educated people. It implied that the principle of light speed non-exceedance could be refuted even before creation of the STR.

The proofs presented are quite simple and can be verified and confirmed in any radio engineering laboratory, and therefore are irrefutable. To make sure thereof, let's demonstrate one of them, for example, the last proof, since it is the simplest and clearest.

Recall that in accordance with the symbolic method of calculating electric circuits proposed by Steinmetz, which is now used by all electric and radio engineers, resistance is measured by real numbers, while capacitive and inductive reactance is measured by bipolar imaginary numbers. Consequently, impedance of electric circuits containing resistors, capacitors and inductors is measured by complex numbers. Thus, if, in accordance with the principle of light speed non-exceedance, imaginary capacitive and inductive reactance were actually imaginary, *i.e.* physically non-existent, then electric circuit impedance would always be determined only by resistors and therefore, never depend on frequency. However, all engineers have been aware for more than a hundred years that impedance of LCR circuit changes, as frequency of voltage applied thereto varies. This is due to change in amount of current flowing through the LCR circuit. This unequivocally proves physical reality of imaginary reactance of capacitors and inductors, and thereby physical reality of any imaginary numbers.

Besides, if, according to the principle of light speed non-exceedance, imaginary reactance of capacitors and inductors were actually imaginary, then there would be no resonance in electric LCR circuits and electric filters could hardly be created. As a result, neither electric engineering, nor radio engineering, nor other exact sciences would exist. However, they do. And for more than a century, millions of engineers have used Ohm's law in the interpretation of Steinmetz in their daily practical activities, thereby proving physical reality of imaginary numbers, which physicists of 21st century still consider unproved on the basis of the unsuccessful OPERA experiment. Thus, using the principle of light speed non-exceedance postulated in the STR, physicists assert impossibility of existence of television, telecommunications, radar and radio navigation, impossibility of sounding of musical instruments and church bells, and deny existence of tsunami and even children's hanging swings.

Therefore, it is high time for physics to recognize the fallacy of relativistic formulas developed in the existing version of the STR in the 20th century and the principle of light speed non-exceedance used to explain them.

⁵*i.e.* provided with references to units used to measure parameters of corresponding physical objects and processes.

4. FALLACY OF THE EXISTING VERSION OF THE STR

Now, having proved the principle of physical reality of imaginary numbers, we have a great opportunity to explain the meaning of relativistic formulas of the STR both in the range $0 \leq v < c$ and $c \leq v$. And since people think in visual images, we use the graphs (see **Figures 1(a)-(c)**) of relativistic formulas to make the explanation more clear

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \quad (1)$$

$$\Delta t = \Delta t_0 \sqrt{1 - \left(\frac{v}{c}\right)^2} \quad (2)$$

$$l = l_0 \sqrt{1 - \left(\frac{v}{c}\right)^2} \quad (3)$$

where m_0 is the rest mass of a physical body;
 m is the relativistic mass of a moving physical body;
 Δt_0 is the rest time of a physical body;
 Δt is the relativistic time of a moving physical body;
 l_0 is the rest longitudinal length of a physical body;

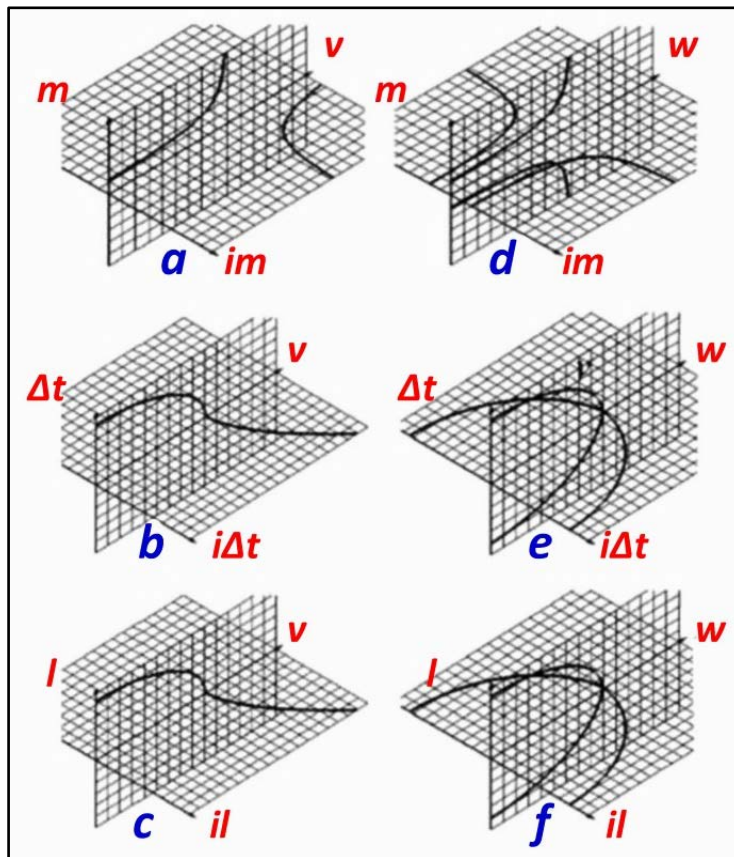


Figure 1. Graphs of functions (1)-(3) corresponding to the existing version of the STR, and (4)-(6) corresponding to its alternative version.

l is the relativistic longitudinal length of a moving physical body;
 v is the velocity of a moving physical body;
 c is the speed of light.

As can be seen, fragments corresponding to the intervals $0 \leq v < c$ and $c \leq v$ are expressed in different forms on all these graphs. But according to the principle of physical reality of imaginary numbers, both intervals should still correspond to really existing universes, albeit different: the interval $0 \leq v < c$ should correspond to our visible universe, and the interval $c \leq v$ to some other invisible universe that lies beyond the horizon of events. However, an invisible universe corresponding to the interval $c \leq v$ is inexplicable.

Hence, logical conclusion is that the formulas (1)-(3) are false. Consequently, the ***existing version of the STR is also false. So, we have to admit that the STR was not actually created in the 20th century. Moreover, it could not be created at that time, since experimental data required for its creation were obtained only in the 21st century.***

And such a result is even expected in creation of the STR. Physical truth can be guessed by no postulates, if its search is not corrected by appropriate experiments⁶. That's why the Nobel Prize winner Stephen Weinberg was right in his statement concerning postulate-based theories: "*Scientific theories cannot be deduced by purely mathematical reasoning*".

We would add that any postulate in physics should sooner or later be replaced by either confirming or refuting experiment. Theories containing postulates are rather hypotheses.

5. TRUTH OF THE ALTERNATIVE VERSION OF THE STR FIGURE 1

Graphs shown in **Figures 1(d)-(f)** should correspond to the explainable relativistic formulas of the STR [27] and the following formulas

$$m = \frac{m_0 i^q}{\sqrt{1 - \left(\frac{v}{c} - q\right)^2}} = \frac{m_0 i^q}{\sqrt{1 - \left(\frac{w}{c}\right)^2}} \quad (4)$$

$$\Delta t = \Delta t_0 i^q \sqrt{1 - \left(\frac{v}{c} - q\right)^2} = \Delta t_0 i^q \sqrt{1 - \left(\frac{w}{c}\right)^2} \quad (5)$$

$$l = l_0 i^q \sqrt{1 - \left(\frac{v}{c} - q\right)^2} = l_0 i^q \sqrt{1 - \left(\frac{w}{c}\right)^2} \quad (6)$$

where $q = \left\lfloor \frac{v}{c} \right\rfloor$ is the "floor" function (**Figure 2(a)**) of argument $\frac{v}{c}$.

$w = v - qc$ is the local velocity (**Figure 2(b)**) for each universe, which can take values only in the range $0 \leq w < c$.

v is the velocity measured from our visible universe.

The parameter q in the formulas (4)-(6) means the fourth spatial dimension, whose integer values in the Multiverse correspond to the coordinates of mutually invisible universes [28, 29] included therein. Therefore, such a Multiverse is referred to as hidden. In the hidden Multiverse, different q values correspond to the following universes:

- the quantity $q = 0$ corresponds to our visible universe (as $i^0 = 1$ and, therefore, $0 \leq v < c$ for it), which is referred to as tardyon due to the name of subatomic particles moving at sub-light speed;
- the quantity $q = 1$ corresponds to an adjacent invisible universe (as $i^1 = i$, and, therefore, $c \leq v < 2c$ for it, that is, it is located beyond the event horizon), which is referred to as tachyon due to the name of subatomic particles moving at hyper-light speeds;

⁶Therefore, for example, the WMAP and Planck experiments (see below) turned out to be unguessed.

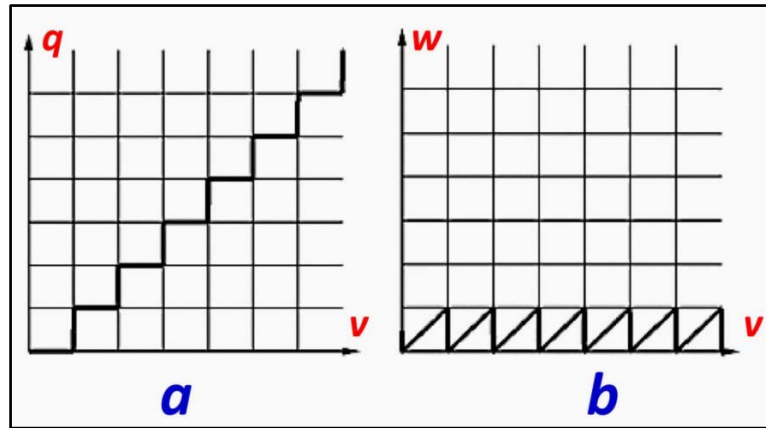


Figure 2. Graphs of functions $q(v)$ and $w(v)$.

- the quantity $q=2$ corresponds to an invisible tardyon antiverse (as $i^2 = -1$, and, therefore, $2c \leq v < 3c$ for it, that is, it is also located beyond the event horizon), since according to the relativistic formulas (4)-(6) all its physical quantities take on values opposite in sign to those of the same quantities in a tardyon universe;
- the quantity $q=3$ corresponds to an invisible tachyon antiverse (as $i^3 = -i$, and, therefore $3c \leq v < 4c$ for it, that is, it is also located beyond the event horizon), since according to the relativistic formulas (4)-(6) all its physical quantities take on values opposite in sign to those of the same quantities in a tachyon universe;
- the quantity $q=4$ corresponds to another invisible tardyon universe (as $i^4 = 1$, and, therefore, for it, that is, it is also located beyond the event horizon), since according to the relativistic formulas (4)-(6) all its physical quantities take on the same values as those of the quantities in a tachyon universe, for which $4c \leq v < 5c$;
- the quantity $q=5$ corresponds to another invisible tachyon universe (as $i^5 = i$, and, therefore, $5c \leq v < 6c$ for it, that is, it is also located beyond the event horizon), since according to the relativistic formulas (4)-(6) all its physical quantities there take on the same values as those of the quantities in a tardyon universe, for which $q=1$, and so on.

Moreover, the universes of the hidden Multiverse have actually no fixed location in space. They continuously drift, touching and even slightly penetrating into each other. And adjacent invisible universes exchange their physical content through numerous transitional zones, called portals or stargates [30-33], generated at penetration points. As a result, over billions of years of existence, mass/energy of these universes has significantly averaged.

6. EXPLANATION OF THE PHENOMENON OF DARK MATTER AND DARK ENERGY

However, new ideas are worth something in science only if they allow solving new issues. In astrophysics, these are, for example, issues concerning explanation of the phenomenon of dark matter and dark energy, existence of antimatter and tachyons, experimental confirmation of existence of invisible universes, etc.

As it follows from the above, the idea of uniqueness of our visible universe, postulated⁷ in the existing version of the STR, has failed a test.

Therefore, we shall check the idea of multiplicity of physical worlds, alternative to the idea of Monoverse, for compliance with the criterion of fruitfulness. Let's start with the phenomenon of dark matter, discovered by Jan Hendrik Oort [34] and Fritz Zwicky [35] in 1932-33, and the phenomenon of dark energy, discovered by Saul Perlmutter [36], Brian Schmidt [37] and Adam Riess [38] in 1998-1999. All of

⁷Since it follows from the postulated principle of light speed non-exceedance.

them were awarded the Nobel Prize for their discoveries. Stressing the importance of the discoveries, the Nobel Prize laureate Adam Riess wrote: “*Humanity is on the verge of a new physics of the Universe. Whether we want it or not, we will have to accept it*”. Such a new physics is actually concerned below.

The phenomena of dark matter and dark energy [39-42] are referred to as such for their obscurity. It is not clear why they are invisible in any range of electromagnetic oscillations and therefore can only be detected indirectly by their gravitational manifestations. It is even more incomprehensible why no molecules, atoms and subatomic particles have so far been found in dark matter and dark energy. This is completely inconsistent with modern fundamental concepts of physical chemistry as to the essence of matter. Many other things also remain an enigma. Therefore, explanation of the phenomenon of dark matter and dark energy is the most important issue of modern physics.

Professor Michio Kaku commented on this issue as follows: “*Of course, a whole bunch of Nobel Prizes is waiting for scientists who can reveal the secrets of the “dark energy” and “dark matter”*”.

Albert Einstein explained very clearly the reason for obscurity of the phenomenon of dark matter and dark energy in the existing version of the STR: “*Insanity: doing the same thing over and over again and expecting different results*”. Sir Isaac Newton was of the same opinion: “*No great discovery was ever made without a bold guess*”.

That is, astrophysicists created all these obscurities by wrong formulation of the problem. They sought an explanation of this phenomenon, which would certainly correspond to the existing version of the STR, stating that we live in the Monoverse. They thought that any alternative explanations for this phenomenon were of no interest. After all, even a few attempts to study possible structures of the Multiverse [43-49] have still been commented on as fundamentally unverifiable.

However, the phenomenon of dark matter and dark energy becomes quite explicable [50-58] as soon as problem formulation is changed and explanation is sought within the hidden Multiverse:

- dark matter and dark energy are not real physical entities that can be found either in the microcosm or macrocosm. They are rather a certain image (gravitational, rather than optical and still less electromagnetic) of invisible universes of the hidden Multiverse, a sort of a shadow;
- dark matter is generated by parallel universes of the hidden Multiverse adjacent to our universe;
- dark energy is generated by other parallel universes of the hidden Multiverse, more distant from us;
- images corresponding to dark matter and dark energy have⁸ no chemical composition. This alone suggests and proves existence of the Multiverse, rather than Monoverse;
- dark matter and dark energy are invisible because universes of the hidden Multiverse generating them are invisible.

Such an explanation meets the criterion of Occam’s razor and therefore is quite plausible.

7. ANALYSIS OF EXPERIMENTAL DATA OBTAINED BY WMAP AND PLANCK SPACECRAFT

Extremely valuable additional information on the structure of the hidden Multiverse can be discovered in analysis of data obtained by WMAP [59] and Planck [60] spacecraft. Thus, according to Planck data, total mass/energy of the entire Multiverse contains 4.9% of ordinary baryonic matter (according to WMAP—4.6%), 26.8% of dark matter (according to WMAP—22.4%) and 68.3% of dark energy (according to WMAP—73%). Therefore, we can conclude that:

- the entire Multiverse contains $(100\%)/(4.9\%) = 20.4$ parallel universes according to Planck data (and $(100\%)/(4.6\%) = 21.8$ parallel universes according to WMAP data);
- dark matter is generated by $(26.8\%)/(4.9\%) = 5.5$ parallel universes according to Planck data (and $(22.4\%)/(4.6\%) = 4.9$ parallel universes according to WMAP data);
- dark energy is generated by $(68.3\%)/(4.9\%) = 13.9$ parallel universes according to Planck data (and $(73.0\%)/(4.6\%) = 15.9$ parallel universes according to WMAP data).

And it’s quite obvious that such experimental results could be guessed by no postulates in the 20th century.

⁸Try to find at least one molecule in any shadow.

8. TRUTH OF THE ALTERNATIVE VERSION OF THE STR (CONTINUED)

However ... the results obtained do not correspond to the relativistic formulas (4)-(6), since, according to WMAP and Planck data, our tardyon universe has five-six adjacent universes, rather than two. Consequently, the hidden Multiverse has actually three extra dimensions, rather than one.

Therefore, relativistic formulas of the STR (4)-(6) should be once more corrected as follows

$$m = \frac{m_0 (i_1)^{q-q_0} (i_2)^{r-r_0} (i_3)^{s-s_0}}{\sqrt{1 - \left[\frac{v}{c} - (q+r+s-q_0-r_0-s_0) \right]^2}} = \frac{m_0 (i_1)^{q-q_0} (i_2)^{r-r_0} (i_3)^{s-s_0}}{\sqrt{1 - \left(\frac{w}{c} \right)^2}} \quad (7)$$

$$\Delta t = \Delta t_0 (i_1)^{q-q_0} (i_2)^{r-r_0} (i_3)^{s-s_0} \sqrt{1 - \left[\frac{v}{c} - (q+r+s-q_0-r_0-s_0) \right]^2} \quad (8)$$

$$l = l_0 (i_1)^{q-q_0} (i_2)^{r-r_0} (i_3)^{s-s_0} \sqrt{1 - \left[\frac{v}{c} - (q+r+s-q_0-r_0-s_0) \right]^2} \quad (9)$$

$$= l_0 (i_1)^{q-q_0} (i_2)^{r-r_0} (i_3)^{s-s_0} \sqrt{1 - \left(\frac{w}{c} \right)^2}$$

where q, r, s are the extra dimensions;

q_0, r_0, s_0 are the coordinates of our visible universe in the hidden Multiverse;

v is the velocity measured relative to our visible tardyon universe;

c is the speed of light;

$w = v - (q+r+s-q_0-r_0-s_0)c$ is local velocity of the universe corresponding to the coordinates i_1, i_2, i_3 that can take values only in the range $0 \leq w \leq c$;

i_1, i_2, i_3 are the imaginary units in hypercomplex numbers [61], called quaternions, that are interconnected by the following relations

$$i_1^2 = i_2^2 = i_3^2 = -1 \quad (10)$$

$$i_1 i_2 i_3 = i_2 i_3 i_1 = i_3 i_1 i_2 = -1 \quad (11)$$

$$i_1 i_3 i_2 = i_2 i_1 i_3 = i_3 i_2 i_1 = 1 \quad (12)$$

Consequently, structure of six-dimensional space (see **Figure 3**) of the hidden Multiverse [62] is described by the formula $f_{q,r,s}(x, y, z) + i_1 q + i_2 r + i_3 s$, where

- the term $i_1 q + i_2 r + i_3 s$ is the coordinates of the corresponding invisible universe;
- and the term $f_{q,r,s}(x, y, z)$ determines distribution of physical content in this universe.

Professor Lisa Randall suggested: “*We can be living in a three-dimensional space sinkhole in a higher-dimensional universe*”. And her suggestion actually proved true.

Therefore, the probable structure of the hidden Multiverse would seem to be as that shown in **Figure 4**. It has the following features:

- a screw structure containing three turns, eight universes in each;
- each turn contains one tardyon universe, one tardyon antiverse, three tachyon universes and three tachyon antiverses;
- the screw structure has a beginning and an end connected to the universes of other Multiverses through corresponding portals;
- other Multiverses along with our hidden Multiverse form the Hyperverse;
- invisible universes of the hidden Multiverse that are adjacent to our visible universe inexplicably evoke the phenomenon of dark matter;
- other invisible universes of the hidden Multiverse inexplicably evoke the phenomenon of dark energy;

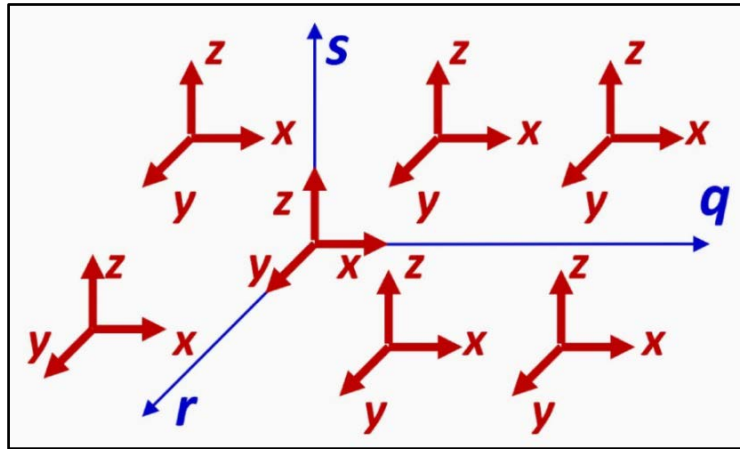


Figure 3. Six-dimensional space of the hidden multiverse.

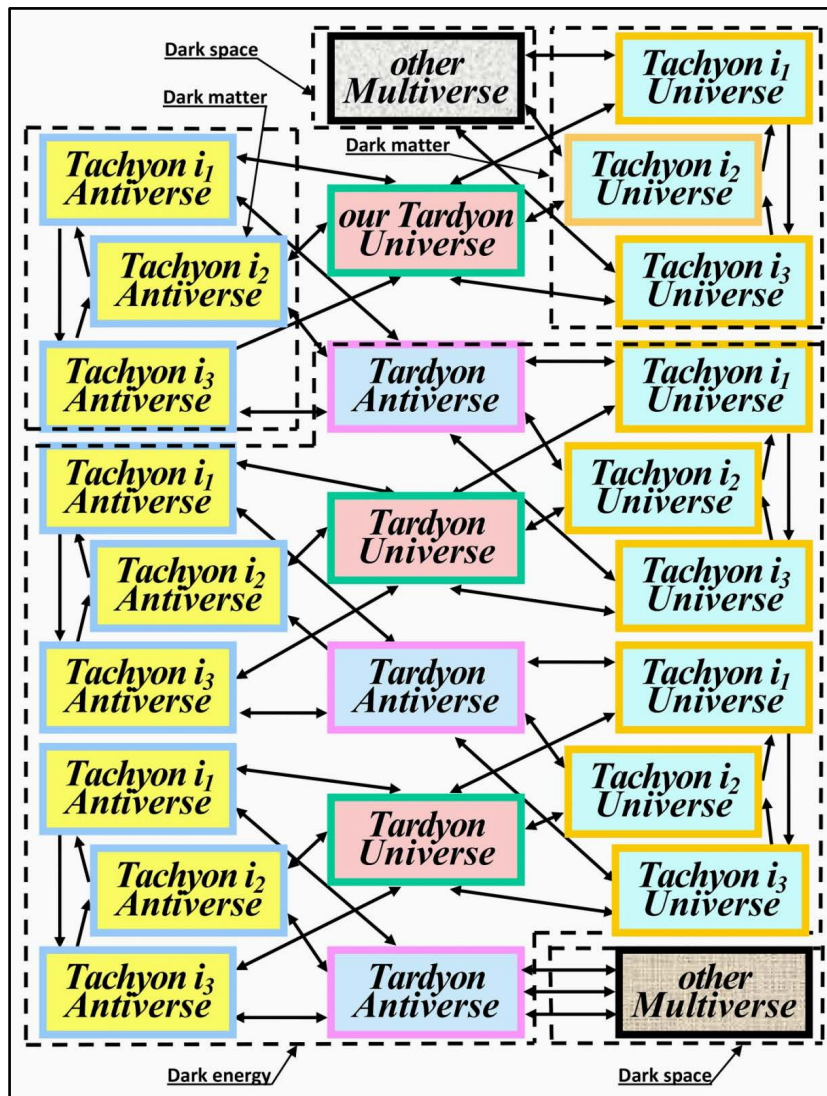


Figure 4. Probable quaternion structure of the hidden Multiverse in six-dimensional space.

- invisible universes outside the hidden Multiverse inexplicably evoke the phenomenon of dark space;
- universes of the hidden Multiverse are interconnected by bidirectional portals corresponding to the formula (10) and by unidirectional portals corresponding to the formulas (11) and (12).

This structure of the hidden Multiverse contains twenty-four universes. Therefore, it does not correspond to the results of WMAP and Planck data analysis.

Figure 5 shows possible structure of the hidden Multiverse corresponding to the results of WMAP and Planck data analysis and containing twenty-two parallel universes. If compared with the structure given in Figure 4, it lacks two invisible universes of the hidden Multiverse. Instead, adjacent invisible universes are connected through portals to invisible universes of two (or one) other Multiverses outside the hidden Multiverse. Besides, our hidden Multiverse can also be connected through portals with these other Multiverses in a different way.

Another possible structure of the hidden Multiverse containing twenty-one parallel universes is depicted in Figure 6. As can be seen, this structure lacks three invisible parallel universes. Instead, adjacent invisible parallel universes are connected through their portals to invisible parallel universes of other three (or two or one) Multiverses. At the same time, our hidden Multiverse can be connected through portals with these other Multiverses in a different way.

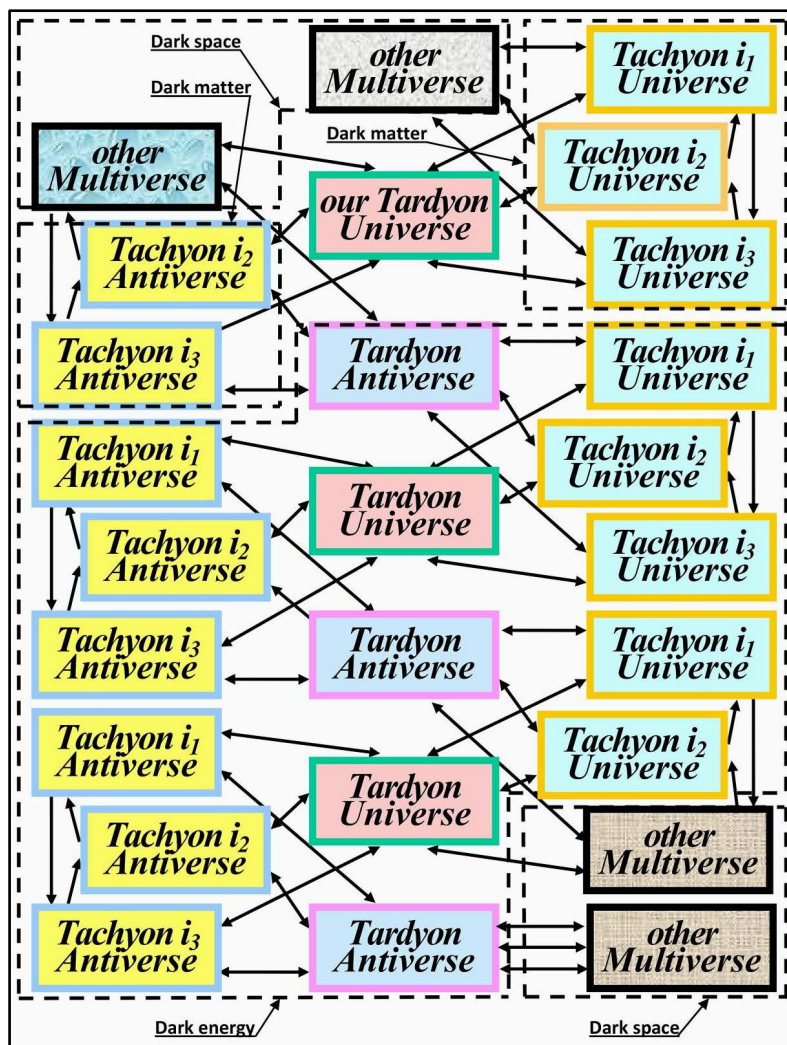


Figure 5. Possible quaternion structure of the hidden multiverse in six-dimensional space.

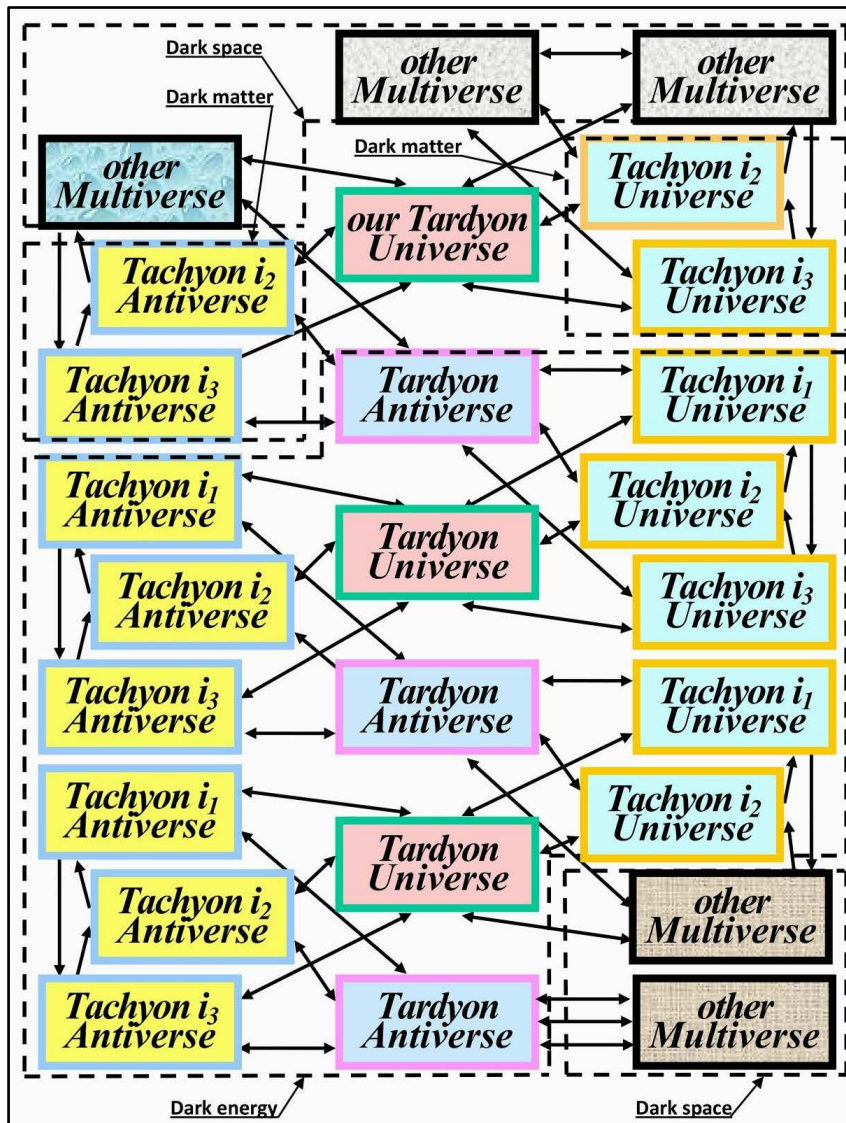


Figure 6. Another possible quaternion structure of the hidden multiverse in six-dimensional space.

Finally, one more possible structure of the hidden Multiverse containing twenty parallel universes is depicted in Figure 7. This structure lacks four invisible parallel universes. Instead, adjacent invisible parallel universes are connected through portals to invisible parallel universes of other four (or three, or two, or one) Multiverses. At the same time, our hidden Multiverse can be connected through portals with these other Multiverses in a different way.

9. ANTIMATTER AND TACHYONS

Thus, an attempt to explain the phenomenon of dark matter and dark energy by existence of invisible universes can't be considered unsuccessful. Now let's see whether the same approach is useful in relation to the problem of explaining the existence of antimatter and tachyons [57], which until recently have been as inexplicable as dark matter and dark energy. Their explanation was hindered by such insurmountable contradictions, as the necessity of placing matter and antimatter in the Monoverse so that they did not annihilate and necessity of placing tachyons in the Monoverse so that they did not violate the principle of

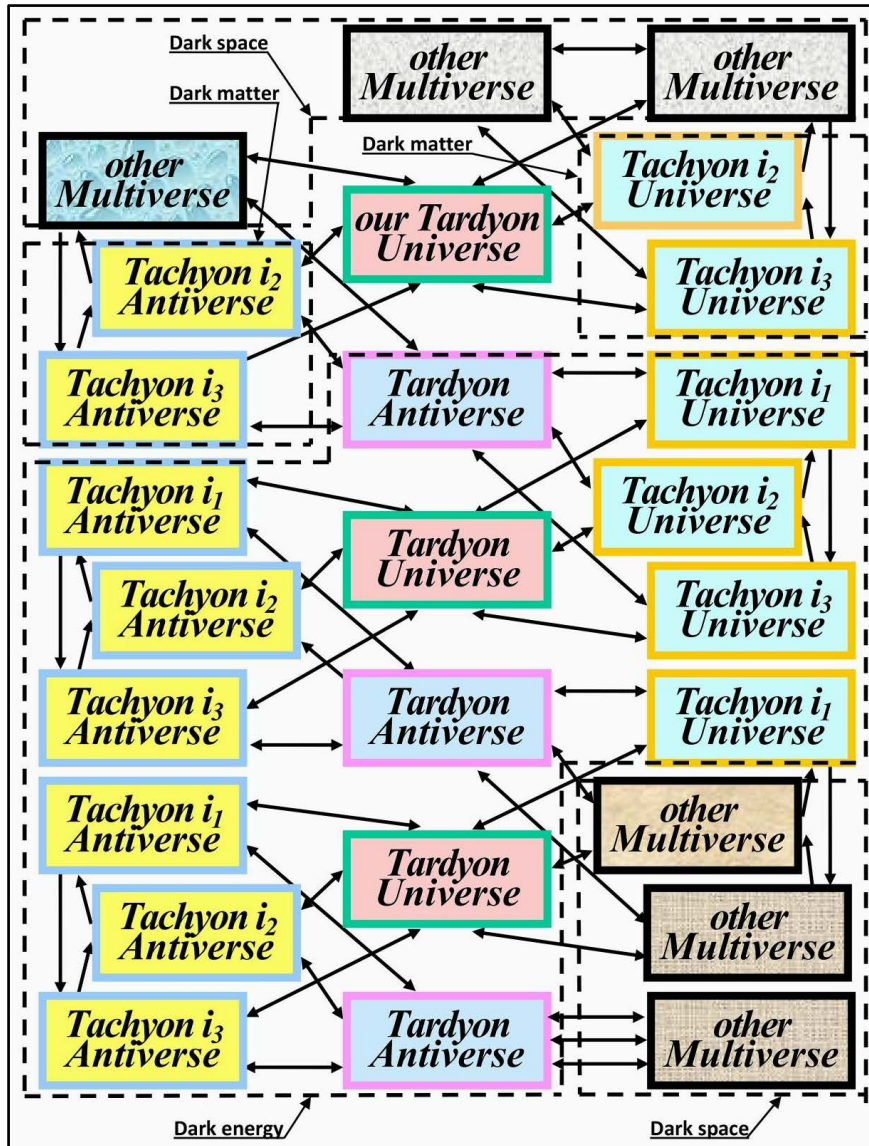


Figure 7. One more possible quaternion structure of the hidden multiverse in six-dimensional space.

causality. As a result, the problems of explanation seemed completely unsolvable. And they are really unsolvable in the context of Monoverse corresponding to the existing version of the STR.

But why, then, taking into account these circumstances, has there been no attempt to abandon the principle of light speed non-exceedance and the Monoverse hypothesis following therefrom? Then it would have long ago become clear that relativistic formulas and the STR need correction.

One way or another, this has not been done. Therefore, we do it now. Explanation of these astrophysical problems appears to be extremely simple with a different approach. It has already been received. The explanation lies in the names of invisible universes of the hidden Multiverse. Antimatter in the hidden Multiverse exists in antiverses, which are as many as universes. Matter and antimatter do not annihilate in the hidden Multiverse, because its tardyon and tachyon universes and antiverses alternate in the above manner, *i.e.* are placed in different dimensions. Tachyons do not violate the principle of causality for the same simple reason: they are in tachyon rather than in tardyon universes and antiverses, *i.e.*, also in different dimensions.

10. DISCOVERY OF THE PHENOMENON OF DARK SPACE

Noteworthy is the fact that some invisible universes evoking the phenomenon of dark matter and dark energy⁹ shown in the structure diagrams of the hidden Multiverse (Figures 5-7), corresponding to WMAP and Planck data, are connected to invisible universes located outside the hidden Multiverse and evoking the phenomenon of dark space¹⁰ (Figure 8(a)).

There is a possibility that our visible tardyon universe is also connected to them (see Figure 8(b)). In that case it would become possible to study invisible universes evoking the phenomena of not only dark matter and dark energy, but also dark space [63-65]. However, which of these two structures actually exists is the question that can be answered only after astronomical research of invisible universes in portals on Earth described below. It should then become clear how many adjacent invisible universes our visible universe has and would be possible to study them somehow.

11. HOW TO SEE CONSTELLATIONS IN THE STARRY SKY OF INVISIBLE UNIVERSES

It is plain to see that the above corrected version of the STR providing very clear explanation is based on the concept of real physical existence of invisible universes. However, despite its logical impeccability, it cannot be considered complete, until existence of invisible universes is proved experimentally.

So how physical reality of invisible universes can be proved? An experimentally supported answer to this question can actually be received only from appropriate astronomical observations. This appears to be simple. Presumably, even amateur astronomers can do this. They only need to know places, where invisible universes are seen. Herewith, in order to understand that the universes we see are not our universe, but other universes that are invisible outside the portals, constellations in their starry sky should evidently differ from those in our visible universe. And it will not be difficult to verify this, since stars in different universes are most likely placed in extremely different ways¹¹ and, therefore, constellations of invisible universes and our visible universe are completely different.

To see the other constellations, we shall do the same thing as we do at home. To see what is going on in other invisible rooms from the room we are in, we need to come in other rooms or at least to come to the doors of these other rooms and look inside. Portals [30-33] are analogues of those doors. At least some of numerous so-called anomalous zones [66] are portals on Earth.

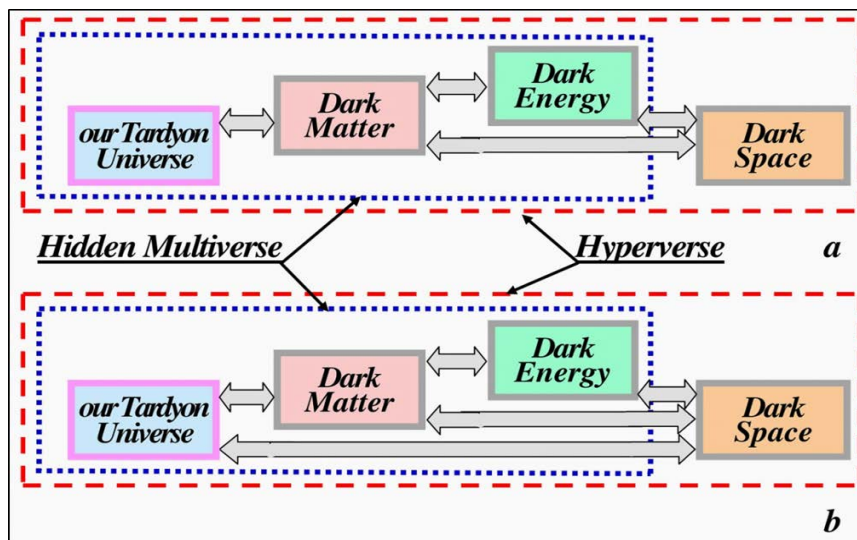


Figure 8. Possible structures of the hyperverses.

⁹For simplicity denoted as “dark matter” and “dark energy” in Figure 8.

¹⁰For simplicity denoted as “dark space” in Figure 8.

¹¹Like contents of different rooms in our home.

Therefore, to see other constellations in the sky of other universes, which, one might recall, are invisible, we have to move on foot or by car to any of these adjacent invisible universes¹² through portals. Then the adjacent universe would become visible to us, whereas our visible universe would become invisible. Such a journey through portals to adjacent invisible universes may be difficult and even dangerous¹³ to implement, as one can get lost in portals and fail to find the way back. However, since, when moving through the portals, image of the starry sky of our universe is gradually replaced by images of the starry sky of adjacent universes, we wouldn't need to move too far into the portals. Even in this case, constellations in the starry sky of portals would noticeably differ from those on Earth outside the portals. Therefore, changes in constellation¹⁴ configuration, *i.e.* relative position of stars in the sky, registered in the portals as we enter into them, would be the most indisputable evidence of existence of invisible universes.

Such an experiment would be simpler and more sensational than a similar experiment conducted by Sir Arthur Stanley Eddington in 1919 [67].

12. INVISIBLE WORLD

First, However, imaginary numbers are used not only in the STR. They are used in all other exact sciences. And it remains to be understood what physical sense they have in these sciences. Anyway, this circumstance proves that in addition to the visible world we know, there is also an invisible and unknown world.

Moreover, in the Euler's formula $\exp(ix) = \cos(x) + i\sin(x)$ describing oscillation processes, the real term $\cos(x)$ of any physical nature (mechanical oscillations of a pendulum, acoustic oscillations generated by speech, sea waves, radio waves, etc.) is inextricably linked with the imaginary term $i\sin(x)$, physical sense of which is incomprehensible. Nevertheless, it is clear that this circumstance proves not just existence of the invisible world, but also its interrelation with our visible world.

13. THE PRINCIPLE OF RELATIVISM

What is the contribution of Albert Einstein to the creation of new physics, physics of the future? Although much has been written about Albert Einstein, nothing has been written about the main thing – his role in creating new physics. This is because nobody needs new physics today. An old physics denying physical reality of imaginary numbers is now in demand, since the principle of light speed non-exceedance is still valid in spite of the above irrefutable arguments against the existing version of the STR. New physics, in contrast to the existing physics of real numbers, is the physics of imaginary numbers. This is the physics of invisible world. And it does exist, as the principle of physical reality of imaginary numbers has already been conclusively proved. It unambiguously refutes the principle of light speed non-exceedance. The existing version of the STR studied according to all textbooks of physics would collapse like a house of cards without the principle of light speed non-exceedance.

But Albert Einstein asserted: “*There is no single idea, which I would be sure that it will stand the test of time*”. He understood that the postulate of light speed non-exceedance was short-lived and the STR version created in the 20th century was just a temporary intermediate option that would inevitably be corrected. Therefore, the phrase at the beginning of this article containing words “*relativistic formulas of the existing version of the STR are wrong and incorrectly explained, and conclusions drawn from them are misguided*” by no means detracts from the merits of Albert Einstein, who himself understood this very well: “*Anyone who has never made a mistake has never tried anything new*”.

Albert Einstein created new things, even ahead of his time. Having no experimental data he needed in the 20th century, he had to replace them with postulates. He took the first step toward creating the alter-

¹²Or at least to the portals connecting them.

¹³One should use (not yet existing) portal navigation devices similar to marine compass, in order not to get lost in portals, while moving therein. Such devices could use, for example, cellular telecommunication networks to determine relative attenuation of radio signals in accordance with the relativistic formulas (7)-(9) when moving away from portal entrance towards its exit. This would make it possible to determine correct direction of motion.

¹⁴It is even useful to capture these changes on video while a vehicle with a telescope placed thereon is moving.

native version of the STR by creating the existing incorrect version of the STR, although the formulas (1)-(3) turned out to be false. However, the correct formulas (4)-(6) and the alternative correct version of the STR would not been created soon, if the formulas have not been given in all physics textbooks and the existing incorrect version of the STR has still been not studied according to these textbooks.

Albert Einstein introduced the principle of relativism into physics, according to which some physical quantities may, under certain conditions, depend on other physical quantities. In the STR this is velocity dependence. Other similar dependencies will inevitably be revealed in the future. And in the future, not only quantitative changes, as in the existing version of the STR, but also qualitative changes, as already in the alternative version of the STR, will be taken into account. For example, when water is heated to 99°C it only remains hot, and upon reaching 100°C, it turns into steam. Therefore, in the STR, when the velocity in the relativistic formulas (1)-(3) and (4)-(6) varies in the range $0 \leq v < c$, there are only quantitative changes in physical quantities. However, in the alternative version of the STR, also above-mentioned qualitative changes take place at velocities $c \leq v$. Consequently, the new physics is much more complicated than both classical physics and the existing version of the STR. However, it still remains to be created.

14. CONCLUSIONS

The article shows that the existing version of the special theory of relativity (STR) is incomplete. Its further development after Albert Einstein ceased due to the fact that the creators of the STR could not prove physical reality and explain physical sense of concrete imaginary numbers. Therefore, relativistic formulas obtained therein have been found to be incorrect and misexplained using the wrong principle of light speed non-exceedance. The relativistic formulas have entailed wrong conclusions consisting in existence of only our visible universe and absence of any real physical content in imaginary numbers. Ultimately, the *existing version of the STR turned out to be wrong*.

The principle of light speed non-exceedance is refuted in the article by the experimentally proven principle of physical reality of imaginary numbers that makes the unsuccessful OPERA experiment unnecessary. Thus, the STR hypothesis on the uniqueness of our visible universe is also refuted. Our visible universe along with about twenty other mutually invisible universes is proved to form the hidden Multiverse.

The hypothesis of the hidden Multiverse makes it possible to explain the phenomenon of dark matter and dark energy. It is shown that dark matter and dark energy are a certain image (gravitational, rather than optical and still less electromagnetic) of the invisible universes, a sort of a shadow, rather than any physical substances. It is explained that the phenomenon of dark space is due to existence of other invisible universes of the Hyperverses.

It is also explained that *existence of invisible universes can be proved experimentally*. This requires astronomical observations in the portals to register constellations in their starry sky that have never been noticed in the starry sky outside the portals. Moreover, constellations may be different in different portals. Therefore, they belong to different invisible universes.

Such experimental proof of existence of invisible universes, as well as experimentally proven principle of physical reality of imaginary numbers and the data obtained by the WMAP and Planck spacecraft *manifest the truth of the alternative version of the STR*.

Finally, in addition to the visible world, we know the corrected alternative version of the STR proves the existence of invisible and, therefore, considerably unknown world that has yet to be studied by science of the future.

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CONFLICTS OF INTEREST

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