Cone Rotating in a Fluid

Kern E. Kenyon

4632 North Lane, Del Mar, USA

Correspondence to: Kern E. Kenyon, kernken@aol.com

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ABSTRACT

When a solid cone with smooth side and base rotates about its long axis in a still fluid, theory says that the cone will advance along the direction of the axis, base first and apex last. Bernoulli's law for closed streamline loops is combined with the cross-stream force balance between the centrifugal force and a pressure gradient in order to obtain the result, which is believed to be new. Confirmation of the prediction awaits observational evidence.

1. INTRODUCTION

A solid neutrally buoyant cone with smooth faces rotates at a constant rate about its long axis in a quiet fluid. What is the nature of the forces that act on the cone? It is not surprising that this problem is not discussed in the familiar theoretical hydrodynamics text books.

But there is at least one fluid mechanics text [1] that gives a theory for a solid cylinder rotating in a fluid. Amazingly the perturbed flow next to the cylinder has no friction and there is no friction on the cylinder caused by the fluid either! Right at the surface of the cylinder the flow rotates with the same speed as the cylinder because of friction. Far away from the cylinder the fluid remains quiet and never moves. The fall-off rate of the perturbed flow has been calculated by two different methods to be like $1/r$, where r is the radial distance from the central axis of the cylinder [1, 2]. It is easy to check that in the Navier-Stokes equations in plane polar coordinates this velocity structure has zero friction associated with it.

To my knowledge, there is only one other functional form that the velocity can take in circular geometry whereby the friction term is zero: in the solid body rotation of a fluid inside a hollow vertically rotating cylinder where the velocity varies linearly with r from a maximum at the container's surface to zero at the center. Additionally another case exists for large-scale circular flows where the Coriolis force dominates over the centrifugal force resulting in a velocity shear that is constant and which causes friction to be zero. A tentative application of the model to the hurricane has been put in print recently [3].

One potential significance of the present result for the rotating cone, if extended further, is that a front to back asymmetric but axially symmetric solid may go more easily through a fluid when it rotates about the long axis, since it has already been found that such a non-rotating solid will advance more quickly oriented blunt end first and pointed end last [4]. This is because the forward pointing net reaction force discussed earlier may add to the similarly directed net pressure force described below.

2. METHOD

Extrapolate now from the rotating cylinder to the rotating cone. At the base of the cone the rotational velocity of the fluid stuck to the side is a maximum, and at the apex the rotational velocity is zero. Assume that the fall off rate of the perturbed velocity away from the cone's side, and perpendicular to the axis of rotation, is the same as for the solid cylinder. But the initial speed at the surface now will vary depending on the location on the side surface. Also assume that Bernoulli's law applies to the circular closed streamlines of the flow next to the cone: where the speed is greatest the pressure is least, and vice versa. (Bernoulli's law has been applied to closed streamlines before [5]). As a consequence the pressure will vary over the cone's side surface. What will be the net effect of that variable pressure along the surface?

There is one logical answer to the question, which is a prediction, but for many readers it may be counterintuitive because of what they already think they know: the best way for an asymmetric solid to move through a fluid is to have the pointed end go first and blunt end last (like the mean shape of the waterline of a standard ship at sea, but that common way of thinking turns out to be misguided [4]). However, according to the present approach, a rotating cone should advance with the base leading and the apex trailing, *i.e.* from highest pressure at the apex toward to the lowest pressure on the cone's side near the base. Although this theoretical forecast may seem a likely one, nevertheless it needs to be realized by experiment in the future, assuming no data relevant to the problem exist already.

3. DISCUSSION

Despite the lack of any observational evidence to support the general idea that a rotating cone will advance in the direction of the long axis, base going first, a few qualitative additions to the concept can be made. For example, for a cone with an altitude to base diameter that is relatively small, the horizontal speed should be greater than that for a similar cone with a larger ratio. In other words, a squat cone should move faster than a thin cone for the same base diameter, same rotation speed and same mass. That is because bringing the relatively low and high pressures on the cone's surface closer together will make the pressure gradient larger parallel to the surface. At present it is not possible to estimate the forward progress of the cone from the given parameters.

Another comment is that once the cone begins rotating from a standing start, a certain amount of time will be needed in order to establish the boundary layer flow adjacent to the cone's side. That statement is made by analogy with the better known spin-up time required to get to the steady state when fluid inside a cylinder acquires motion after the cylinder starts rotating. For optimum operation the cone must not advance too quickly in order to allow the boundary layer to be fully developed so that friction vanishes. Consequently a slow rotation rate would probably be better than a faster one. Such details could be worked out observationally.

4. CONCLUSION

If a solid cone, with a smooth side and base, and that is neutrally buoyant in a fluid (water), rotates about its long axis, it is forecast that the cone will move along the direction of the axis base first and apex last. Bernoulli's law applied to closed streamlines is one major part of the theory, and the other piece is the cross-streamline force balance between the centrifugal force and a pressure gradient. From them it is derived that the lowest pressure on the cone's side is near the base and highest pressure is at the apex. Thus a pressure gradient is produced on the cones surface which drives the cone horizontally. Also in the rotary boundary layer attached to the cone's side, friction is computed to be zero. Observations are needed to compare with both concepts which are believed to be new.

CONFLICTS OF INTEREST

The author declares no conflicts of interest regarding the publication of this paper.

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