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Enhancing Stress Intensity Factor Reduction in Cracks Originating from a Circular Hole in a Rectangular Plate under Uniaxial Stress through Piezoelectric Actuation

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Abstract

Circular holes are commonly employed in engineering designs; however, they often serve as locations where cracks initiate and propagate. This paper explores a novel approach to structural repair by utilizing piezoelectric actuators. The primary focus of this study is to investigate the influence of an adhesively bonded piezoelectric actuator patch placed above a circular hole on the stress intensity factor (SIF) in an aluminium plate. The plate is subjected to uniaxial tensile stress, while the piezoelectric actuator is excited with varying voltage levels. The analysis is conducted using the finite element method (FEM), a powerful numerical technique for simulating complex structures. The study assesses the stress distribution and employs the SIF as an adequate criterion for evaluating the impact of different patch configurations. The results indicate a strong correlation between the applied voltage and the SIF. Whether the SIF increases or decreases depends on the polarization of the piezoelectric actuator. Particularly noteworthy is the finding that rectangular patches in a horizontal orientation significantly reduce the SIF compared to other patch geometries. Moreover, double-sided patches exhibit a pronounced decrease in the SIF compared to single-sided patches. In summary, this research underscores the potential of piezoelectric actuators in mitigating stress intensity in structures with circular hole with crack initiation. It offers valuable insights into the influence of applied voltage, patch geometry, and patch placement on the SIF, thereby contributing to developing effective strategies for enhancing structural integrity.

Keywords

Piezoelectric Actuators, Stress Intensity Factor (SIF), Aluminium Plate, Voltage, Finite Element Method (FEM)

1. Introduction

In the field of mechanical design and structural engineering, it's common to encounter geometric irregularities and sudden cross-sectional changes. These deviations often arise from specific design requirements, such as accommodating features like oil gaps, scores, keyways, or splines. Nearly every machine or structural element exhibits some geometric non-uniformity due to these design considerations. However, these geometric variations give rise to a critical phenomenon known as stress concentration and intensity.

The stress intensity factor (SIF) is a fundamental concept in fracture mechanics, playing a pivotal role in our comprehension and prediction of how cracks and imperfections behave in structural components. This concept is firmly grounded in the theoretical framework of linear elastic fracture mechanics (LEFM), which provides a structured approach for analysing how materials respond near the tips of cracks. LEFM, serving as the cornerstone, operates under specific assumptions. It assumes that materials exhibit linear elastic behaviour (following Hooke's law) and that cracks are significantly smaller than the overall dimensions of the structure. The application of LEFM techniques is frequently observed in lengthy fractures exhibiting small-scale yielding tendencies near the crack tip or the Paris regime [1] [2]. Within these confines, LEFM equips engineers and researchers with a systematic method for evaluating stress and strain distribution in the immediate vicinity of a crack's tip [3]. The practical applications of the stress intensity factor extend widely across engineering and materials science fields. This concept is utilized to assess the structural integrity of components, forecast the fatigue life of materials, and identify critical crack sizes that, when exceeded, may lead to catastrophic failure. As such, the stress intensity factor not only forms the theoretical bedrock of fracture mechanics but also stands as an indispensable tool for real-world assessments of structural reliability and safety [4] [5].

Some of the most often utilized approaches for modelling cracks are the Boundary Element Method (BEM), Finite Difference Method (FDM), Finite Element Method (FEM), and Extended Finite Element Method (XFEM). These methods examine the SIF of a fracture initiation and crack propagation. The FEM is the most widely used computational method for simulating damage and failure under both static and dynamic loadings. It answers various engineering problems, including stress, strain, displacement, and stress intensity factor (SIF). Adaptive remeshing processes, or FEM, have shown to be quite dependable and effective [6]-[12]. The static analysis can be completed precisely by using the FEM to calculate the stress intensity factor at a set of sites on the crack front. Numerous programs, such as ANSYS, FRANC3D, ABAQUS, ZENCRACK, COMSOL, NASTRAN and BEASY, are available today to perform static and fatigue crack growth analyses.

It is unavoidable for fatigue, corrosion, or accidents to cause damages like a crack or notch and delamination in aeronautical, mechanical, and offshore structures during services. Because of the concentration of stress and strain near the damaged areas, these damages will presumably produce structural failures and develop alarmingly [13] [14] [15] [16]. As a result, structural restoration has gained significant interest from academia and business over the past few decades as a valuable and vital research issue. Reducing the concentration of stress or strain at the damaged area of the structure, such as the notch or crack tips and delamination, is a significant goal in the repair design to strengthen the compromised site. The most often utilized technology to extend the life of damaged structures is structural repair using bonded materials [17]. The conventional approach involved mounting or melding extra high-strength patches over the damaged area to enhance the damaged structure's mechanical performance. A 2002 paper by Hart and Boogers detailed a patch repair of cracks in the longeron of an F16 aircraft [18]. Investigated maintenance of aircraft structures with lower cost is one of the prime concerns of regulatory authorities. Carbon fibre-reinforced polymer (CFRP) patches are widely used to repair cracked frames. The demands and application of CFRP compel its price to increase soon [19].

The customizable mechanical properties of innovative materials have made them useful in structural repair applications, given the limits of traditional approaches. A class of common intelligent materials known as piezoelectric materials are substances that exhibit the electro-mechanical coupling effect, meaning that when an external load is applied to a piezoelectric material, an electric charge is produced. A mechanical deformation is generated when an electric field is applied to the piezoelectric material. Piezoelectric materials possess active electro-mechanical properties, which have led to their investigation and successful application in structural restoration for various damages. Thus, strategies to boost the generated repair force and enhance the intelligent materials' adaptability for multiple structures and working environments would be the focus of future work on structural repair utilizing smart materials. More research should be done on the electro-mechanical characteristics and design of the piezoelectric stack in order to enhance the force produced by the smart structure. However, research on the piezoelectric fiber composite material would be crucial to improving the versatility of using smart materials as actuators and sensors in various engineering constructions [20].

An alternative approach to structural restoration is to use Shape Memory Alloys (SMAs) as the actuator rather than piezoelectric ceramics because SMAs can produce greater force and withstand higher deformations. The primary issue with employing SMAs as actuators for structural restoration is that their reaction time is typically slower when subjected to active control than that of piezoelectric materials, which react electromechanically more quickly. Reducing the SMAs' response time is yet another fascinating and valuable research area in this area [21] [22] [23].

The customizable mechanical properties of smart materials have made them useful in structural repair applications, given the limits of traditional approaches. A class first presented research work on fixing a crack in a beam under transverse static loading. To ensure the mitigation of stress surrounding the crack, a piezoelectric patch bonded to the beam was subjected to a voltage that was derived from a computational model of a simply supported beam by Wang *et al.* [24]. Touogam's [25] study contributes to the understanding of the structural behaviour of the double-layer steel plate concrete shaft and provides valuable insights for optimising the design of sandwiched structures. A piezoelectric patch-bonded vibrating delaminated beam structure was designed for repair by Wu and Wang [26]. Using piezoelectric patches, UK Rao [27] studied how to control vibration in a simply supported beam. Jones and Callinan's [28] study offers insights on positioning composite patches to reduce crack tip stress intensity, limit fiber stress in the patch, and optimize adhesive bond shear stress. Piezoelectric patches fix cracks in aircraft panels, Shah *et al.* [29] explored the patches surrounding a hole in an isotropic plate. Gopi Krishna [30] investigates the potential for reducing the stress concentration factor (SCF) using various piezoelectric geometric patches. The study by Peter [31] investigates the use of piezoelectric actuators for active mending on the parallel side of a plate's central crack.

Conclusively, our study is a groundbreaking attempt to tackle a particular and widespread problem in engineering works. Our study posits a fresh and significant contribution to the field of structural repair, given its concentrated investigation of circular holes as hotspots for crack instigation and propagation, as well as its comprehensive consideration of patch configurations and their effect on SIF. The knowledge obtained from this study could be used to rethink methods for improving structural integrity when cracks are starting and circular holes are present, providing a more specialised and efficient way to handle engineering difficulties.

2. Problem Description

The objective of this work is to investigate and determine the ideal geometric design for both single- and double-sided piezoelectric patches to optimize the stress intensity of a circular hole with a 20˚ V-notch on both sides of a plate, as shown in **Figure 1**, that is being subjected to a uniaxial tensile load. In the numerical study, a 200 × 100 mm rectangular aluminium plate with a 1 mm thickness and a 25 mm diameter central hole with a 20˚ V-notch have a 20 mm crack length extending from the plate's centre. The plate is subjected to a pressure force of 1 MPa in the y direction. It is considered and assumed that piezoelectric patches of Geometry Circle (diameter 45 mm), Square (45 × 45 mm), and Rectangle in both vertical (45 × 90 mm) and horizontal (90 × 45 mm) orientations, are perfectly bonded with the host plate through an adhesive of thickness 0.0125 mm. The patches have a 0.5 mm thickness. Under the assumption of flawless bonding between the plate and patch, piezoelectric patches that are polarized in the thickness direction are taken into consideration for examination. Table 1 and Table 2 list the items taken into consideration for the analysis along with their characteristics.

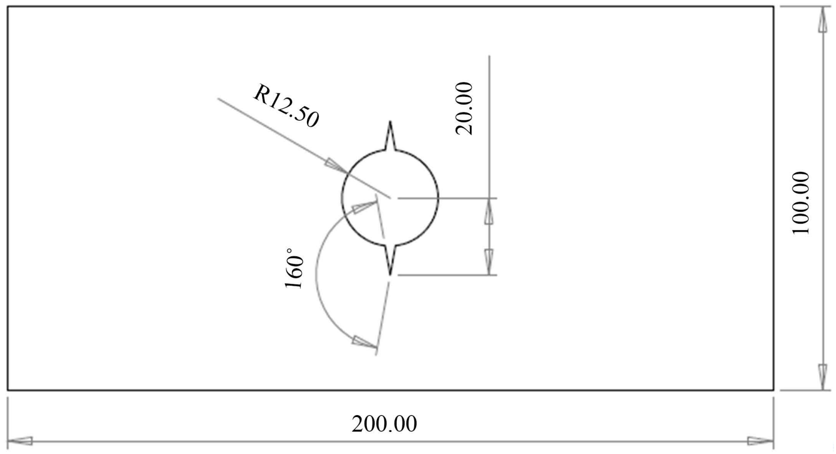


Figure 1. Host plate (All dimensions are in mm).

Table 1. Isotropic material properties.

Parameters	Symbols	Aluminium Alloy	Adhesive
Density [kg/m^3]	ρ	2770	1000
Poisson's ratio [1]	ν	0.33	0.3
Young's modulus [GPa]	E	71	5.09

Table 2. Materials properties of PYT-5H With respect to Z-direction polarization.

Parameters	Matrix
Elastic Constants [GPa] (C_{ij})	$\begin{bmatrix} 126 & 79.5 & 84.1 & 0 & 0 & 0 \\ 79.5 & 126 & 84.1 & 0 & 0 & 0 \\ 84.1 & 84.1 & 117 & 0 & 0 & 0 \\ 0 & 0 & 0 & 23.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 23.25 & 0 \\ 0 & 0 & 0 & 0 & 0 & 23.25 \end{bmatrix}$
Piezoelectric Coefficients [C/m^2] (e_{ij})	$\begin{bmatrix} 0 & 0 & -6.5 \\ 0 & 0 & -6.5 \\ 0 & 0 & 23.3 \\ 0 & 0 & 0 \\ 0 & 17 & 0 \\ 17 & 0 & 0 \end{bmatrix}$
Permittivity constants [C/(V-m)] (ε_{ij})	$\begin{bmatrix} 1.505 & 0 & 0 \\ 0 & 1.505 & 0 \\ 0 & 0 & 1.302 \end{bmatrix} \times 10^{-8}$

3. Finite Element Modelling and Analysis

The most important numerical method for solving challenging engineering issues with some degree of accuracy is Finite Element Method (FEM). Software for finite element analysis, ANSYS Workbench 2023/R3, is utilized for this. Based on a 20-node brick SOLID 226 finite element, a piezoelectric FEM formulation is created for simulation [32]. This formulation links the impacts of associated physics within the element matrices or load vectors, which contain all relevant terms needed for the connected physics solution. SolidWorks 2022 is used to construct the 3D model for analysis, and simulation software is used to import the geometry. An ANSYS Workbench 2023/R3 is used to import a Piezo and MEMS ACT Extension for static fracture analysis to do piezoelectric simulation [33].

In this work, the stress intensity factor under linear elastic fracture simulation is investigated using a static structure. The Extended Finite Elements Method (XFEM) has been used extensively in fracture modelling to calculate interior cracks. XFEM, which was added to the Ansys toolkit, removes the requirement for re-meshing fracture tip regions. Rather, it delineates an expanded finite element enrichment region encircling a fracture point and in areas where it is conceivable that the fracture tip may proliferate. The centre of the element is divided from the unique volume elements in the enrichment zone by XFEM. This method, rather than meshing, it breaks existing cells to produce a finer mesh [34] [35].

Bonding the piezoelectric actuator patch on one side and another side of the host plate with adhesive material is subjected to 1 MPa of load at one end by fixing on another end. On the circumference of the circular notch and the V-notch fracture, fine meshing is done with the tetrahedron patch conforming method. The V-notch crack is selected as a pre-meshed crack object. It uses nodal-named selection to analyse crack fronts. For all the piezoelectric actuator patch geometries. For all the piezoelectric actuator patch geometries, an applied voltage load of −30 to +30 with an interval of 5 volts is applied for simulation. **Figure 2** shows the flowchart for the crack simulation processes used by Ansys Workbench.

4. Results

The stress intensity factor (SIF) for the plate without a piezoelectric patch must first be established in order to validate the finite element model. Based on the analytical result [36], the mode-one stress intensity factor is 9.635 Mpa·mm$^{0.5}$. **Figure 3** displays the contour map derived from the FEM study for the SIF in the plate. According to the picture, the maximum stress intensity factor (SIF) is 9.647 Mpa·mm$^{0.5}$. This is in close agreement with the analytical answer.

4.1. Single Sided Piezoelectric Patch

The voltage load is applied to the bottom face of a single-sided piezoelectric

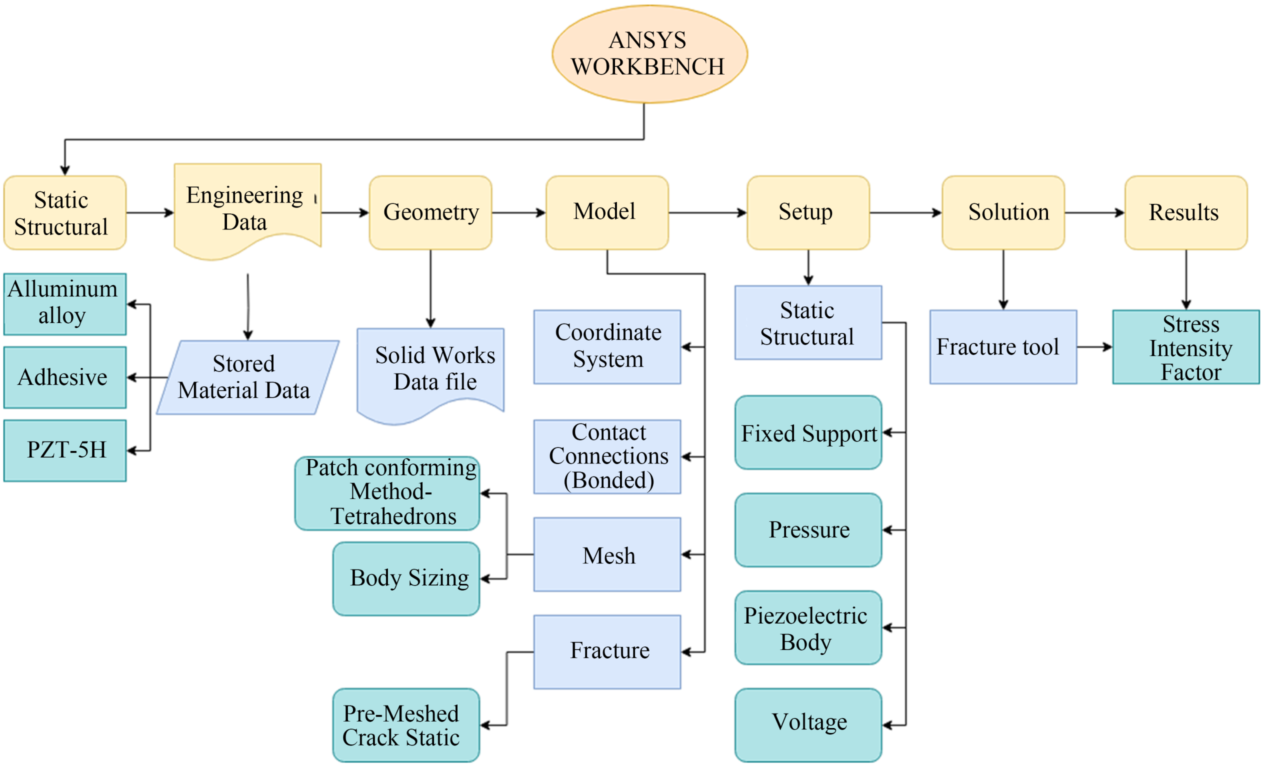


Figure 2. Flowchart for the ANSYS simulation procedures.

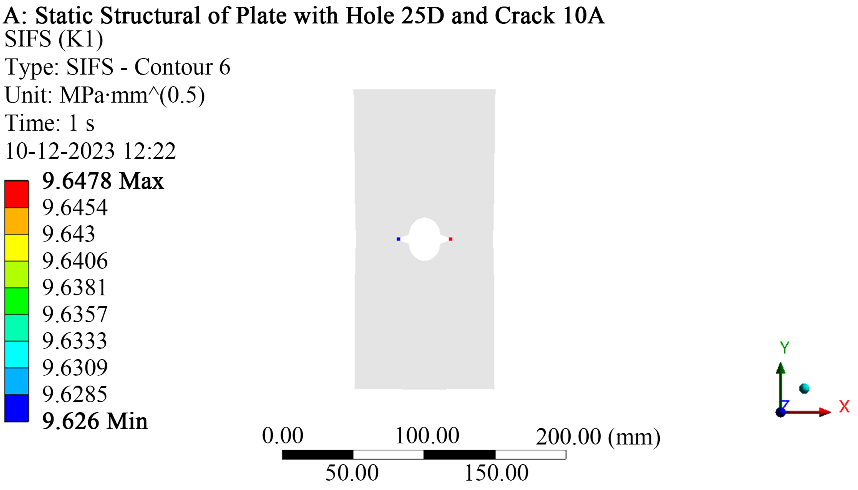
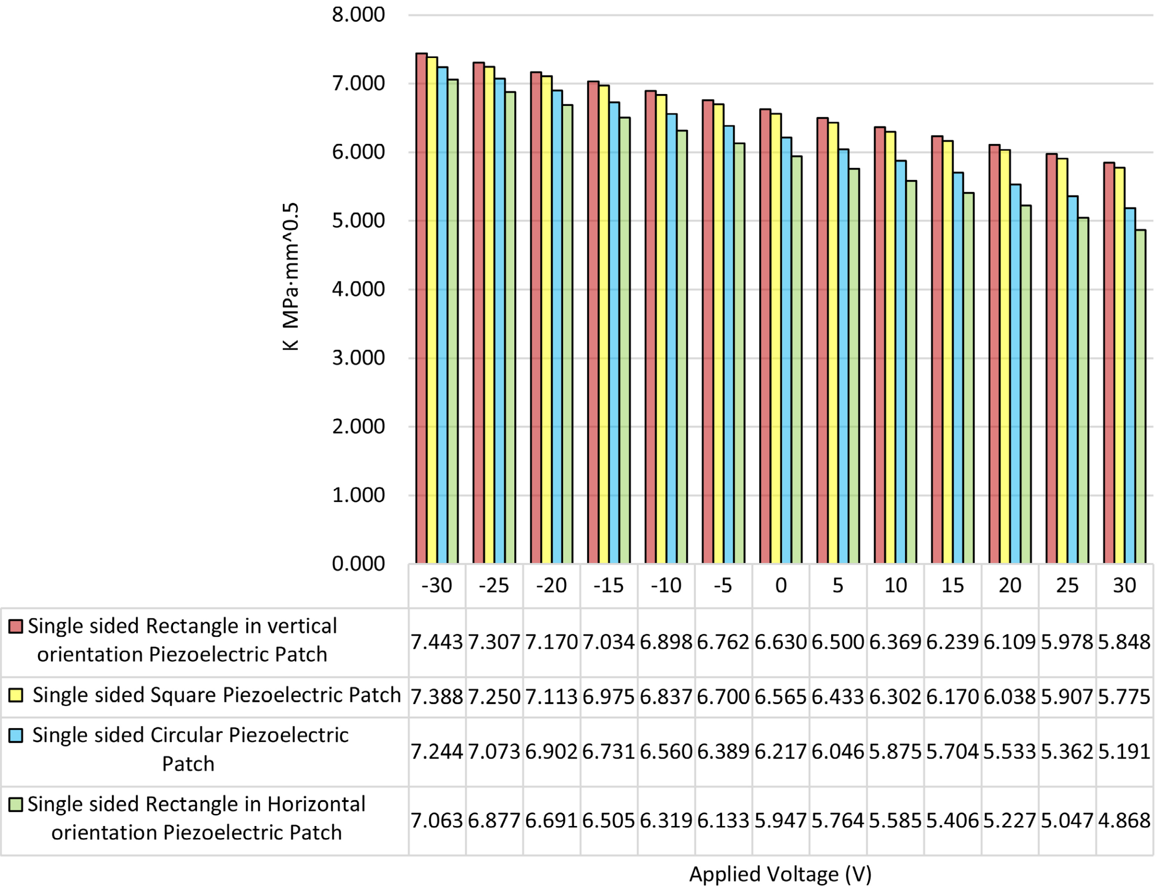


Figure 3. SIF in the plate without piezoelectric patch.

patch on a host plate. A closed circle loop is produced by the patch's opposing side, ideally zero. The rectangle in the vertical orientation patch shows 6.63 Mpa·mm$^{0.5}$ of SIF when no voltage, or zero voltage, is applied. However, the square, circle and rectangle in the horizontal orientation show 6.56 Mpa·mm$^{0.5}$, 6.21 Mpa·mm$^{0.5}$, and 5.94 Mpa·mm$^{0.5}$, respectively. In contrast, the SIF valves with single-sided piezoelectric patches of all considered shapes and no voltage load produced three to four units of SIF less values when compared to the fractures without piezoelectric patches.

The top face of the patch becomes positive when a negative voltage load is applied to its bottom face. by providing voltages of −5, −10, −15, −20, −25, and −30 to the bottom faces of each shape model under consideration. As illustrated in **Figure 4**, the SIF is rising for the following patches: 6.76 Mpa·mm$^{0.5}$ to 7.44 Mpa·mm$^{0.5}$ for a rectangle vertical patch, 6.7 Mpa·mm$^{0.5}$ to 7.38 Mpa·mm$^{0.5}$ for a square patch, 6.38 Mpa·mm$^{0.5}$ to 7.24 Mpa·mm$^{0.5}$ for a circular patch, and 6.13 Mpa·mm$^{0.5}$ to 7.06 Mpa·mm$^{0.5}$ for a rectangular horizontal patch. The findings show that stress is created in the piezoelectric patch and the SIF magnitude increases when negative voltage is applied to the bottom face and positive voltage is applied to the top face.

The patch's top face turns negative when a positive voltage load is applied to its bottom face by providing voltages to the bottom faces of each form model under consideration 5, 10, 15, 20, 25, and 30. As seen in **Figure 4**, the SIF drops for rectangular vertical patches are 6.5 Mpa·mm$^{0.5}$ to 5.84 Mpa·mm$^{0.5}$, square patches are 6.43 Mpa·mm$^{0.5}$ to 5.77 Mpa·mm$^{0.5}$, circular patches are 6.04 Mpa·mm$^{0.5}$ to 5.19 Mpa·mm$^{0.5}$, and rectangular horizontal patches are 5.76 Mpa·mm$^{0.5}$ to 4.86 Mpa·mm$^{0.5}$. The findings show that when a positive voltage is applied to the bottom face, and a negative voltage to the top face, the piezoelectric patch is compressed, which lowers the SIF magnitude.



	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	30
🟥 Single sided Rectangle in vertical orientation Piezoelectric Patch	7.443	7.307	7.170	7.034	6.898	6.762	6.630	6.500	6.369	6.239	6.109	5.978	5.848
🟨 Single sided Square Piezoelectric Patch	7.388	7.250	7.113	6.975	6.837	6.700	6.565	6.433	6.302	6.170	6.038	5.907	5.775
🟦 Single sided Circular Piezoelectric Patch	7.244	7.073	6.902	6.731	6.560	6.389	6.217	6.046	5.875	5.704	5.533	5.362	5.191
🟩 Single sided Rectangle in Horizontal orientation Piezoelectric Patch	7.063	6.877	6.691	6.505	6.319	6.133	5.947	5.764	5.585	5.406	5.227	5.047	4.868

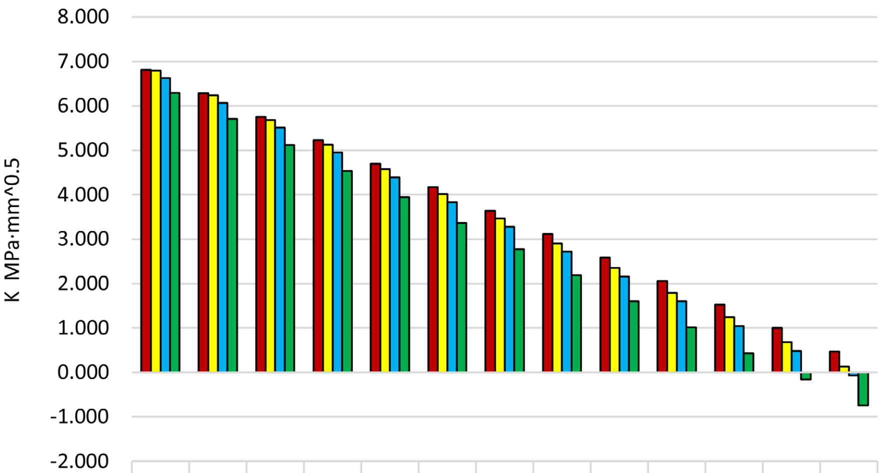
Figure 4. SIF in the plate with single sided piezoelectric patch.

4.2. Double Piezoelectric Patch

Two-sided piezoelectric actuator patch glued to the host plate adhesive for improved SIF values. Each front patch's bottom face and the back patch's top face get an equal amount of voltage. On every shape patch under consideration, there is initially no voltage *i.e.*, zero voltage. Following are the findings: Rectangles of the following sizes: vertical: 3.6 Mpa·mm$^{0.5}$; square: 3.46 Mpa·mm$^{0.5}$; circular: 3.27 Mpa·mm$^{0.5}$; and horizontal: 2.77 Mpa·mm$^{0.5}$.

After applying a voltage of −30 V, FEA analysis yielded SIF 6.81 Mpa·mm$^{0.5}$ for the double-sided rectangle in the vertical orientation patch. Until the load is at +30 V, the analysis is performed for each increment of 5 V of voltage. SIF drops with each set of stresses applied until it eventually reaches 0.47 Mpa·mm$^{0.5}$. When the square patch was applied, the SIF decreased similarly for a voltage load ranging from −30 to +30. For the double-sided circular patch, the SIF was 1.04 Mpa·mm$^{0.5}$ at 20 V, 0.48 Mpa·mm$^{0.5}$ at 25 V, and −0.07 Mpa·mm$^{0.5}$ at 30 V before decreasing again. The SIF reaches −0.16 Mpa·mm$^{0.5}$ and −0.74 Mpa·mm$^{0.5}$, respectively, for a double-sided Rectangle in a horizontal patch on analyses of 25 and 30 V, as represented in **Figure 5**. The V-notch crack is undergoing compressive stress, as shown by the stress intensity factor's negative value.

Out of all the double-sided piezoelectric actuator patch analyses that were taken into consideration, the rectangle in horizontal mode performs better in mitigating SIF than the square, circle, and rectangle in vertical mode.



	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	30
🟥 Double sided Rectangle in vertical orientation Piezoelectric Patch	6.815	6.286	5.757	5.229	4.700	4.171	3.643	3.114	2.585	2.057	1.528	0.999	0.471
🟨 Double sided Square Piezoelectric Patch	6.795	6.239	5.684	5.128	4.572	4.017	3.461	2.906	2.350	1.794	1.239	0.683	0.128
🟦 Double sided Circular Piezoelectric Patch	6.627	6.068	5.510	4.952	4.393	3.835	3.276	2.718	2.160	1.601	1.043	0.485	-0.07
🟩 Double sided Rectangle in Horizontal orientation Piezoelectric Patch	6.292	5.705	5.119	4.532	3.946	3.359	2.773	2.186	1.600	1.013	0.427	-0.16	-0.74

Applied Voltage (V)

Figure 5. SIF in the plate with double sided piezoelectric patch.

5. Discussion

The stress intensity factor of a V-notch fracture and a circular notch in a rectangular plate subjected to uniaxial tension and the impact of the piezoelectric patch's form at various applied voltages are examined. Based on the simulation, the following findings are made to discuss.

As the piezoelectric patch is polarized in the thickness direction for the need of a compressive strain in the length direction of the patch, to achieve this compressive strain, the piezoelectric patch bottom face, *i.e.*, the face that is adhesively attached to the host plate, should be charged with a positive voltage. The other face of the path automatically becomes a negative charge. If the bottom face of the patch is given a positive voltage and the top face is given a negative voltage, this results in tensile strain. Because piezoelectric material is a quartz crystal material, when electrons flow from positive to negative, the piezoelectric material will compress. When the flow is inverted, the material will excite decompression. In the case of a double-sided piezoelectric patch, *i.e.*, two patches are parallelly sandwiched to the host plate physically, but in terms of the electric circuit method of representation, piezoelectric patch-hotplate-piezoelectric patch in series connection, when the second patch end face is given positive voltage and the front face of the first patch is considered negative voltage as mode 2, it leads to a compressive strain of the two piezoelectric patches, and the inverse *i.e.*, mode 1 results in tensile strain, as shown in **Figure 6**.

The SIF values are decreasing towards negative magnitudes, starting at 25 volts in a double-sided rectangle of a horizontally oriented piezoelectric patch. Negative stress intensity variables frequently indicate that a compressive stress field is acting on the fracture.

Negative solutions are, however, erroneous in a physical sense; therefore, it should be aware of that. It is expected that the faces of a realistic fracture located in a stress field with partial or full compression will be partially or entirely closed, making interpenetration impossible.

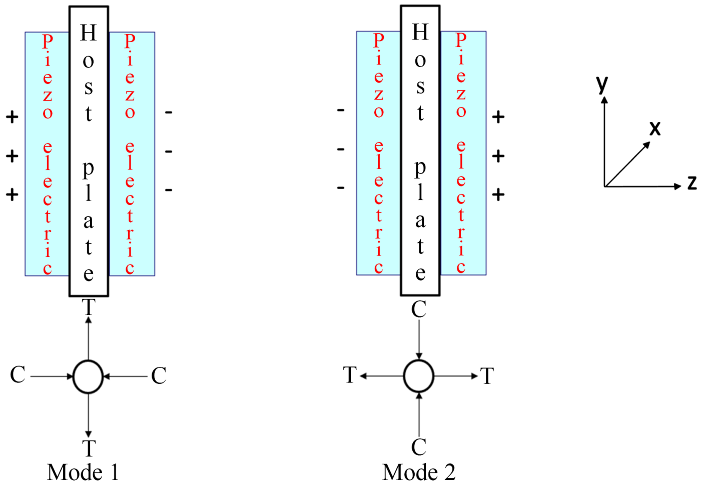


Figure 6. Piezoelectric patch in series with direction Z-direction Polarisation.

6. Conclusions

Research has provided critical new understandings regarding the influence of piezoelectric patch shape on stress intensity variables. A comprehensive knowledge of the connection between patch form and stress reduction has been made possible by the methodical investigation of single-sided and double-sided piezoelectric patches, which include a variety of configurations such as squares, circles, and rectangles in horizontal orientation in addition to rectangles in vertical orientation.

Most importantly, the arrangement of geometries according to stress intensity variables demonstrated the unique benefit of the rectangle in horizontal orientation, which showed a significant decrease in stress intensity compared to its equivalents. This demonstrates how important it is to consider patch geometry while maximising stress-reduction tactics.

Findings are further complicated by the observed decrease in plate size for plates with single-sided piezoelectric patches, which is explained by the model's asymmetry and the patch material's intrinsic behaviour. Despite this, our research highlights how well these patches reduce stress intensity factors across various geometries.

Notably, the rectangle positioned horizontally stood out as an anomaly, exhibiting a low-stress intensity factor. This anomaly calls for more research into the particular dynamics of this geometry and the underlying processes that give rise to its peculiar behaviour.

Research has practical ramifications for designing and implementing piezoelectric patches in real-world applications, especially in fractured structures that are affected by uniaxial force. The selection and positioning of piezoelectric patches for optimal stress mitigation can be guided by understanding how various geometries influence stress intensity variables.

In conclusion, this work contributes to understanding how stress intensity variables, applied voltages, and piezoelectric patch geometry interact. The intricate understandings advance the discipline of structural engineering and offer new avenues for applying piezoelectric materials for stress reduction in various contexts, including structural restoration.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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