Modeling and Numerical Simulation of Heat Transfers in a Metallic Pressure Cooker Isolated with Kapok Wool

Drissa Ouedraogo¹, Serge Wendsida Igo², Gaël Lassina Sawadogo¹, Abdoulaye Compaoare², Belkacem Zeghmati³, Xavier Chesneau³

¹Laboratoire d’Energies Thermiques Renouvelables (LETRE), Université Joseph KI-ZERBO, Ouagadougou, Burkina Faso
²Département Energie, Institut de Recherche en Sciences Appliquées et Technologies (IRSAT/CNRST), Ouagadougou, Burkina Faso
³Laboratoire de Mathématiques et Physique (LAMPS), Université de Perpignan Via Domitia, Perpignan, France

Email: *ouedraogodri2016@gmail.com

Abstract

In this work, a numerical study of heat transfers in a metallic pressure cooker isolated with kapok wool was carried out. This equipment works like a thermos, allowing finishing cooking meals only thanks to the heat stored at the beginning of cooking, which generates energy savings. Cooked meals are also kept hot for long hours. In our previous work, we have highlighted the performances of the pressure cooker when making common dishes in Burkina Faso. Also, the parameters (thickness and density) of the insulating matrix allowing having such performances as well as the influence of the climatic conditions on the pressure cooker operation were analyzed in detail in this present work. The numerical methodology is based on the nodal method and the transfer equations obtained by making an energy balance on each node have been discretized using an implicit scheme with finite differences and resolved by the Gauss algorithm. Numerical results validated experimentally show that the thickness of the kapok wool as well as its density play an important role in the pressure cooker operation. In addition, equipment performances are very little influenced by the weather conditions of the city of Ouagadougou (Burkina Faso).

Keywords

Pressure Cooker, Kapok Wool, Heat Transfers, Modeling, Nodal Method

1. Introduction

Thermal insulation is one of the most used solutions to save energy in different...
sectors (building, catering, industry, etc.). In many sectors, natural insulation is more and more preferred over synthetic insulation because of the requirements related to sustainable development. Natural insulators can be vegetable, animal or mineral origin. Among these natural insulators that are found in abundance in tropical countries like Burkina Faso, is kapok wool. Numerous studies have demonstrated the ability of this fiber to compete with synthetic insulation.

Experimental studies conducted by Voumbo et al. [1] on kapok wool, using the plate method, show very interesting thermophysical properties. Indeed, they have shown that the thermal conductivity of kapok wool varies between 0.03 and 0.04 W/m·K for a density between 5 and 40 kg/m³ and an average thermal diffusivity of $17.1 \times 10^{-7}$ m²/s.

J. C. Damfeu et al. [2] determined by an experimental measurement using the radial flux method the thermo-physical properties at low density of the following natural fibers: fibers kapok; peanut shell fibers; rattan fiber and coconut fiber. The results presented show that the thermal conductivity of kapok wool ($\lambda = 0.045$ W/m·K) is in agreement with that reported by Voumbo et al. [1].

A. Wereme et al. [3] compared the storage capacity of ice in a container insulated with kapok wool and in the shell of a refrigerator insulated with polyurethane. They found that the insulated container with kapok wool could keep the ice for 6 days while the shell of the refrigerator could do it in 7 days, which shows that kapok wool can compete with polyurethane.

In the same way aming Wang, [4] experimentally analyzed the heat transfer in the kapok using a sensor (HIH-3610). It appears that the use of kapok as a thermal insulating material in the fabric helps minimize heat loss from the human body. The influence of temperature and wind speed on heat transfer through the kapok was analyzed and compared to that of cotton. It has been shown that with kapok the heat losses of the human body are lower than those obtained by cotton.

In the building sector, a study focused on a comparative analysis of the thermal performance of a non-insulated house with that of a house whose roof is insulated with kapok fiber shows that kapok fiber is suitable for insulating the roof in residential buildings in a hot climate [5].

Other work has shown that the combination of kapok wool and other materials can lead to very interesting materials in terms of thermal insulation. Adulkareem S et al. [6] show through an experimental study that composite materials resulting from the combination of kapok wool and sugarcane bagasse or coconut fiber at percentages of 50% lead to very good insulating materials, have shown that adding a small amount of kapok wool to the plaster leads to an improvement in the thermophysical properties of the composite material [1].

Because of its very interesting thermophysical properties, we have recently used kapok wool in the thermal insulation of a pressure cooker intended for cooking and meals conservation. The experimental study by Drissa Ouedraogo et al. [7] which followed showed that the device makes it possible to achieve
enormous energy savings (between 30% and 75%) in the cooking of local dishes, and its constant time is about 60 hours. However, even if these results are mainly due to the thermophysical properties of kapok wool, the influence of other parameters such as the thickness and density of the insulating matrix on the performance of the device was not elucidated during this study. However, these parameters are essential when reproducing or resizing the device. In addition, the behavior of the pressure cooker in a typical climate of Burkina Faso, throughout the year is not known, which could limit its application in a given period. Consequently, the objective of this work is to carry out a numerical simulation in order on the one hand to understand the influence of the parameters (thickness, density) of the insulating matrix on the operation of the device and on the other hand to know the influence of the climate on his behavior during the year.

2. Pressure Cooker Modeling

2.1. Description of the Pressure Cooker

The pressure cooker has a parallelepiped shape \((L \times W \times H = 65 \text{ cm} \times 60 \text{ cm} \times 50 \text{ cm})\) box made of 2 mm thick steel sheet which encloses the insulation system consisting of kapok wool. The thickness of kapok wool varies from 1 to 20 cm while its density varies from 10 to 50 kg/m\(^3\). The kapok wool is protected from the outer wall by a 15 mm thick wooden sheet. The pot containing hot water is placed in the middle of the pressure cooker and surrounded by kapok wool (Figure 1).

![Figure 1. Description of the pressure cooker.](image-url)
2.2. Mathematical Formulation

2.2.1. Simplifying Hypotheses

We adopt the following simplifying hypotheses:
- The hot water temperature inside the pressure cooker is uniform;
- The thermo-physical properties of air and materials are constant;
- The external temperature is taken equal to the ambient temperature;
- Heat exchanges through the cover wall are negligible;
- Materials are assimilated to gray bodies;
- The celestial vault is considered to be a black body;
- The atmospheric diffuse radiation is isotropic.

According to the above hypotheses, the model of heat transfers in the pressure cooker is represented in the Figure 2.

2.2.2. Heat Transfers Equations in the Pressure Cooker

In general, modeling a thermal system using the nodal method amounts to setting up a network of thermal capacities ($C_j = \rho V_j C_{pm}$), heat sources and thermal coefficients. Heat transfers are represented by a thermal resistance network. By applying Kirchhoff's law [8], to the network of thermal resistances schematizing the different modes of heat transfer, we give the general heat balance equation by expression (1). This equation is based on nodal analysis by Boyer et al., [9] and Thomas Nganyaa et al. [10].

$$m_i C_{pi} \frac{\partial T_i}{\partial t} = \sum_{j \neq i} G_{ij} S(T_j - T_i) + Q_i$$

With:

**Figure 2.** Pressure cooker heat transfer model.
$j$ index of the medium for which $T_j$ is a potential connected to the potential $T_i$, $Q$, heat source (W), $G_{ij}$, heat transfer coefficient between the media $i$ and $j$ (W/m²K) according to the heat transfer mode $x$ (conduction, convection, radiation) and $S$, surface of the section considered (m²). The thermal heat transfer coefficients $G_{ij}$ are defined by the formal expression of the energy flow $\Phi_{ij}$ exchanged between two adjacent nodes $i$ and $j$

$$\Phi_{ij} = G_{ij} (T_i - T_j)$$ (2)

**Figure 3** show the heat transfer model.

![Figure 3. Pressure cooker heat transfer model.](image)

The heat transfer equations resulting from the nodal model of the pressure cooker are summarized below:

- At the sheet steel
  - The external face of the pressure cooker
    $$\frac{m_i C_p}{S} \frac{\partial T_{fi}}{\partial t} = h_{s,amb} (T_{amb} - T_{fi}) + h_{s,ciel} (T_{ciel} - T_{fi}) + \frac{\lambda_f}{e_f} (T_f - T_{fi})$$ (3)
  - The inner side of the pressure cooker
    $$\frac{m_i C_p}{S} \frac{\partial T_{ci}}{\partial t} = \frac{\lambda_f}{e_f} (T_f - T_{fi}) + \frac{\lambda_{c,i}}{e_{c,i}} (T_{ci} - T_{ci})$$ (4)
  - At the wood sheet level
    $$\frac{m_i C_k}{S} \frac{\partial T_{ki}}{\partial t} = \frac{\lambda_k}{e_{k}} (T_{fi} - T_{ki}) + h_{int} (T_{ki} - T_{int})$$ (5)
  - At the kapok level
    $$\frac{m_i C_k}{S} \frac{\partial T_{ki}}{\partial t} = \frac{\lambda_k}{e_{k}} (T_{ki} - T_{ki}) + h_{int} (T_{ki} - T_{int})$$ (6)
  - Within the pot
    $$\frac{m_i C_w}{S} \frac{\partial T_w}{\partial t} = \sum h_{int} (T_w - T_{int})$$ (7)

### 2.2.3. Expressions of Heat Transfer Coefficients

The heat exchanges in the pressure cooker take place according to the three
modes (convective, radiative and by conduction).

The convective exchange coefficient between the outer wall of the pressure cooker and the ambient air depends on the action of the wind speed. This coefficient is deduced from the relation of Mac Adams [11][12].

\[ h_{cv} = 5.67 + 3.86V \]  

- The heat transfer coefficients by natural convection between the interior air of the pressure cooker enclosure and the walls are determined thanks to the correlations deduced in the case of a vertical wall [13][14].

\[ N_u = 0.68 + 0.67 R_u^{1/4} \left[ 1 + \left( \frac{0.492}{P_f} \right)^{9/16} \right]^{4/9}; 10^{-5} \leq R_u \leq 10^{12} \]  

- The heat transfer coefficient between the underside of the horizontal wall of the pot and the ambient air is deduced from the relation of Incroprera & De Witt [15].

\[ N_u = 0.27 R_u^{1/4}, 10^3 \leq R_u \leq 10^{10} \]  

- The coefficient of radiative transfer between the external wall of the pressure cooker and the celestial vault is determined by the Expression (11):

\[ h_{r-p-ciel} = \frac{\sigma}{\varepsilon_p + F_{ciel}} \left( T_i^2 + T_p^2 \right) \left( T_i + T_{ciel} \right) \]  

\[ F_{ciel} = \frac{3 \pi + 2b}{2 \pi (3 + 2b)} \]  

\[ b \] represents a parameter which is a function of the anisotropy of the sky. For an isotropic sky (b = 0) the radiative from factor is equal to 0.5. In the literature, there are several correlations for determining the temperature of the sky. We selected the one proposed by Swinbank [18].

\[ T_{sky} = 0.05527 T_{amb}^{1.5} \]  

### 2.3. Discretization of Equations

The Equations (3)-(7) are discretized by an implicit method with finite differences. This method is based on a Taylor series development which transforms partial differential equations into a system of algebraic equations. So, the discretized equations are given by the Expressions (13)-(22).

- At the sheet steel level
  - The external face of the pressure cooker

\[ \frac{m_f C_{ef} T_i^{t+\Delta t} - T_f^{t}}{\Delta t} = h_{ef} \left( T_i^{t+\Delta t} - T_{ef}^{t+\Delta t} \right) + \frac{\lambda_f}{\varepsilon_f} \left( T_f^{t+\Delta t} - T_{ef}^{t} \right) + h_{r-p-ciel} \left( T_{ef}^{t+\Delta t} - T_f^{t} \right) \]
\[
\left(\frac{m\lambda}{C_p} + h_{eq} + h_{p,eq}\right)\frac{T^t_{fj} + \frac{\lambda_f}{e_f} T_{ci}^t - h_{p,eq} T_{ci}^t - h_{eq} T_{amb}^t}{S} = m\lambda T^t_{fj}
\]

- The inner side of the pressure cooker

\[
\frac{m\lambda}{C_p} \times \frac{T_{ci}^{t+\Delta t} - T_{ci}^t}{\Delta t} = \frac{\lambda_c}{e_c} \left(T_{ci}^{t+\Delta t} - T_{ci}^t\right) + \frac{\lambda_k}{e_k} \left(T_{ki}^{t+\Delta t} - T_{ki}^t\right)
\]

- At the wood sheet

\[
\frac{m\lambda}{C_p} \times \frac{T_{ci}^{t+\Delta t} - T_{ci}^t}{\Delta t} = \frac{\lambda_c}{e_c} \left(T_{ci}^{t+\Delta t} - T_{ci}^t\right) + \frac{\lambda_k}{e_k} \left(T_{ki}^{t+\Delta t} - T_{ki}^t\right)
\]

- At the kapok wool level

\[
\frac{m\lambda}{C_p} \times \frac{T_{ki}^{t+\Delta t} - T_{ki}^t}{\Delta t} = \frac{\lambda_c}{e_c} \left(T_{ci}^{t+\Delta t} - T_{ci}^t\right) + \frac{\lambda_k}{e_k} \left(T_{ki}^{t+\Delta t} - T_{ki}^t\right)
\]

- Within the pot

\[
\frac{m\lambda}{C_p} \times \frac{T_0^{t+\Delta t} - T_0^t}{\Delta t} = h_{b,\text{int}} \left(T_0^{t+\Delta t} - T_{ki}^t\right) + h_{b,\text{int}} \left(T_0^{t+\Delta t} - T_{ki}^t\right) + h_{b,\text{int}} \left(T_0^{t+\Delta t} - T_{ki}^t\right)
\]

The algebraic Equations (14)-(23) can be written in the form of a system of equations as follows

\[
A T^{t+\Delta t} = T^t
\]

- Initial conditions

At \(t = 0\), \(T_0 = 90^\circ C\).

\[
T_f = T_c = T_k = T_{amb}
\]

2.4. Solving the System of Equations

At \(t_0 + \Delta t\), we assigned an arbitrary value to the temperatures on components of the pressure cooker. The heat transfer coefficients (convective, radiative) are thus calculated. The resolution of the system of algebraic Equations (3)-(7) leads to new values of the temperature of the components of the pres-
sure cooker which are compared with the arbitrary value. If the difference between these two temperatures is greater than the desired precision, the values of the calculated temperatures replace the arbitrary value and the procedure described below is repeated until the convergence is obtained. The convergence was obtained when the following criterion was satisfied:

\[
\frac{T^{n+\Delta t} - T^n}{T^{n+\Delta t} - T^{n-\Delta t}} \leq 10^{-3}
\]

(25)

3. Results and Discussion

3.1. Model Validation

Using the methodology described in Section 2.4, we obtain the numerical curve of hot water cooling under the conditions: Hot water initial temperature: 90°C, ambient temperature: 25°C, kapok wool density: 50 kg/m³, kapok wool thickness: 15 cm. This numerical curve is then compared to the hot water cooling curve obtained experimentally under the same conditions (Figure 4).

![Figure 4](image.png)

We note a good qualitative agreement between these two results. The discrepancies do not exceed 5% and are due to the various empirical correlations used for the calculation of the heat transfer coefficients.

3.2. Influence of Kapok Wool Thickness and Density on the Pressure Cooker Operation

Figure 5 shows the influence of the thickness \( e_k \) (for 1 to 20 cm) of kapok wool on the cooling curve of the hot water contained in the pressure cooker. The initial temperature inside the pot is taken at 90°C and the outside temperature is 25°C.
We note that the temporal evolution of the water temperature decreases rapidly to tend asymptotically towards a constant value for small thicknesses of kapok wool (1, 2, 3, 4 and 5 cm). This leads to rapid cooling of hot water for low values of the thickness of the insulation. For large thicknesses of kapok wool, particularly from 15 cm, we note that the cooling curve flows to straight lines, which indicates a low cooling of the hot water over time. This result is explained by the fact that at low thickness, the thermal resistance of the insulating material is low, which increases transfers to the outside environment.

Figure 6 shows the influence of the density (for 5 to 50 kg/m$^3$) of kapok wool on the cooling curve of the hot water contained in the pressure cooker. The initial temperature inside the pot is taken at 90°C and the outside temperature is 25°C.
As in the previous case, it can be seen that the increase of the kapok wool density leads to poor cooling of the hot water over time. On the other hand, for low density value, a very rapid cooling of the hot water is observed. This result is due to the fact that at low density, the thermal diffusivity of the insulating material is high, which increases transfers to the outside environment.

3.3. Simulation of the Pressure Cooker Operation with Meteorological Data from the City of Ouagadougou

3.3.1. Monthly Change in Ambient Air Temperature

Figure 7 shows the monthly change in ambient air temperature following a seasonal cycle at Ouagadougou.

![Ambient temperature graph](image)

Figure 7. Ambient temperature of the typical year [19].

We note that the highest temperatures are observed in March, April and May. The maximum temperature was recorded during the month of April and reached the value of 40°C. The lowest temperatures are observed during the months of December, January and August. January is the coldest month with a maximum and minimum temperature of 32°C and 16°C respectively.

3.3.2. Influence of the Hottest and Coldest Months on the Operation of the Pressure Cooker

Figure 8 shows the influence of the hottest and coldest months on the operation of the pressure cooker. The initial temperature inside the pot is taken at 90°C.

We notice that there is a similar change in the cooling curve for the two months. The difference between the two curves is more pronounced for the high times because of the temperature gradient between the two months. However, the transfers are more important for the month of January which present lower temperatures. The GAP in final temperatures between the two months is approximately 4°C, which is relatively small compared to the initial temperature of the hot water.
Figure 8. Influence of the hottest and coldest mounts on the cooling curve of the hot water.

3.3.3. Influence of the Hottest Month on the Wall of the Pressure Cooker

Figure 9 shows the influence of the hottest month on the wall of the pressure cooker. The initial temperature inside the pot is taken at 90°C.

We note that the temperature of the wall follows that of the ambient air which reaches a maximum between 12 h and 15 h because of the sunshine. Despite the fluctuations in the wall temperature, it can be seen that the cooling curve of the hot water is not disturbed, which shows the effectiveness of the insulation.
3.3.4. Influence of the 12 Typical Days of the Year on the Operation of the Pressure Cooker

Figure 10 shows the influence of the 12 typical day of year on the operation of the pressure cooker. The initial temperature inside the pot is taken at 90°C.

The observation in Figure 10 shows that the evolution over time of the cooling curve for the typical 12 days is similar. We also note that the temperature gradient between the different cooling curves is very low from one month to the next. These results show that the use of the pressure cooker is not conditioned by climatic by climatic variations during the year.

4. Conclusions

A metallic pressure cooker insulated with kapok wool has been numerically investigated in the present work. The numerical methodology is based on the nodal method and the transfer equations obtained by making an energy balance on each node have been discretized using an implicit scheme with finite differences and resolved by the Gauss algorithm. The main results are summarized as follows:

- The pressure cooker performances are linked to the good thermophysical properties of kapok wool,
- The increase of the thickness and density of kapok wool leads to optimal operation of the pressure cooker,
- The pressure cooker operation is very little influenced by the climatic conditions of the city of Ouagadougou. Consequently, it can be used at any time of the year.

Its permanent use will considerably reduce energy consumption in the catering sector, as we have shown in our previous work.
Acknowledgements

The authors express their deep gratitude to the Embassy of France in Burkina Faso, the National Research Fund for Innovation and Development (FONRiD) and the International Science Program (ISP) of UPPSALA University for their financial support which allowed the realization of this work.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References


des échanges radiatifs avec le ciel. Thèse de doctorat d’ingénieur Physicien EPF.


Nomenclatures

$A$: Heat transfer coefficient matrix
$C_{wt}$: Mass heat capacity of water (J/kg·K)
$C_{cm}$: Centimeter
$C_{p}$: Specific heat of the material (J/kg·K)
$C_{pfs}$: Specific heat of sheet steel (J/kg·K)
$C_{pwc}$: Specific heat of wood sheet (J/kg·K)
$C_{pkk}$: Specific heat of kapok wool (J/kg·K)
$C_{eq}$: Specific heat (J/kg·K)
$C_{eqp}$: Equivalent heat capacity of the pot container (liquid, solid) (J/kg·K)
$e_{kk}$: Thickness of kapok wool (m)
$h_{int}$: Overall internal exchange coefficient of the source (W/m²·K)
$h_{ij}$: Heat transfer coefficient between media $i$ and $j$ depending on the heat transfer mode (convection, radiation) (W/m²·K)
$h_{r-p-ciel}$: Coefficient of heat transfer by radiation) (W/m²·K)
$h_{s-c}$: Heat transfer coefficient by natural convection with the outside environment (W/m²·K)
$H$: Pressure cooker height (m)
$L$: Pressure cooker length (m)
$m_{w}$: Mass of water (kg)
$m_{kk}$: Masse of kapok (kg)
$m_{fs}$: Masse of sheet steel (kg)
$m_{wc}$: Masse of wood sheet (kg)
$N_{u}$: Nusselt number
$L$: Pressure cooker length (m)
$Q$: Heat source (W)
$R_p$: Raleigh number
$S$: Surface of the section considered (m²)
$T_i$: Temperature at node $i$ (K)
$T_j$: Temperature at node $j$ (K)
$T_{fs}$: Temperature at the outer node of the iron sheet (K)
$T_{fsi}$: Temperature at the internal node of the iron sheet (K)
$T_{wc}$: Temperature at the internal node at the plywood (K)
$T_{kk}$: Temperature at the internal node at the kapok level (K)
$T_{pc}$: Internal pressure cooker temperature (K)
$T_{sk}$: Sky temperature (K)
$T_{amb}$: Ambient temperature (K)
$T_{fs1}$: Temperature of the outer left side of the pressure cooker at the level of the steel sheet
$T_{fs2}$: Temperature of the inner left side of the pressure cooker at the level of the steel sheet
$T_{fs3}$: Temperature of the external front face of the pressure cooker at the level of the steel sheet
\(T_{f1}\): Temperature of the inner front face of the pressure cooker at the level of the steel sheet

\(T_{f5}\): Temperature of the outside right side of the pressure cooker at the level of the steel sheet

\(T_{f6}\): Temperature of the inner right side of the pressure cooker at the level of the steel sheet

\(T_{f7}\): Temperature of the outer rear face of the pressure cooker at the level of the steel sheet

\(T_{f8}\): Temperature of the inner rear side of the pressure cooker at the level of the steel sheet

\(T_{c1}\): Temperature of the left side of the pressure cooker at the level of the plywood

\(T_{c2}\): Temperature of the front of the pressure cooker at the level of the plywood

\(T_{c3}\): Temperature of the right side of the pressure cooker at the level of the plywood

\(T_{c4}\): Temperature of the back of the pressure cooker at the level of the plywood

\(T_{k1}\): Temperature of the left side of the pressure cooker at the level of the kapok wool

\(T_{k2}\): Temperature of the front of the pressure cooker at the level of the kapok wool

\(T_{k3}\): Temperature of the right side of the pressure cooker at the level of the kapok wool

\(T_{k5}\): Temperature of the back of the pressure cooker at the level of the kapok wool

\(\Delta t\): Time step (s)

\(T_{t+\Delta t}\): Temperature vector at time \(t + \Delta t\)

\(T_t\): Temperature vector at time \(t\)

\(\rho\): Specific mass heat (kg/m\(^3\))

\(\lambda_f\): Conductivity of sheet steel (W/m-K)

\(\lambda_c\): Conductivity of wood sheet (W/m-K)

\(\lambda_k\): Conductivity of kapok wool (W/m-K)

\(V\): Volume of the liquid in the pot (m\(^3\))

\(V\): Wind speed (m/s)

\(W\): Pressure cooker width (m)