

# Millimeter-Scale Liquid Droplet Migration on Solid Surface with Temperature Gradient: A Simulation Investigation

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**How to cite this paper:** Zheng, J.Y. (2020) Millimeter-Scale Liquid Droplet Migration on Solid Surface with Temperature Gradient: A Simulation Investigation. *Modern Mechanical Engineering*, 10, 34-38.  
<https://doi.org/10.4236/mme.2020.103004>

**Received:** December 12, 2019

**Accepted:** August 7, 2020

**Published:** August 10, 2020

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## Abstract

In this paper, we established a time-dependent model that investigates the migration behavior of a millimeter-scale liquid droplet on a solid surface with temperature gradient. Both fluid mechanics and heat transfer are incorporated in the model. The Navier-Stokes equation is employed both inside and outside the droplet. Size variation is observed in the transient simulation. Results show that the velocity of the migration is about 1.7 mm/s under a temperature gradient of 30 K/mm. The model is consistent with results with previous literatures.

## Keywords

Liquid Droplet, Fluid Mechanics, Navier-Stokes Equation

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## 1. Introduction

Millimeter-scale liquid droplet has profound applications in various fields, such as evaporating or electrowetting behavior in bio-medical industry [1] [2], dielectrophoresis or electrochemical reduction in chemistry and chemical engineering [3] [4], combustion characteristics or thermal conductivity in heat transfer [5] [6], scattering or absorptive properties in electromagnetics [7] [8], etc. Specifically, the migration behavior which requires the joint investigation in both heat transfer and fluid flow region is particularly interesting. However, a simplified model that simulates this phenomenon is not present to the knowledge of the authors.

Tseng has done a fundamental study on the movement of various sized micro-liter droplets on a surface subjected to temperature gradients. The histories

of droplet movement are recorded by high-speed CCD camera and are simulated by numerical methods based on first principle equations. His study indicates that temperature gradients, the change of dynamic receding/advancing contact angles across the droplets, and the flow fields inside the droplet are the key parameters determining the moving behavior of the micro-droplet driven by Marangoni and capillary effects.

Based on Tseng's work, we established a simplified model to investigate the migration behavior of a millimeter-scale liquid droplet on a solid surface, where the underlying surface is assumed to have a temperature gradient along which the droplet migrates. Finite element simulation is applied to our work.

## 2. Theoretical Background

The description of the fluid flow is based on the Navier-Stokes equations, which in their most general form read as the following [9] [10].

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (1)$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = \nabla \cdot (-p \mathbf{I} + \boldsymbol{\tau}) + \mathbf{F} \quad (2)$$

$$\rho C_p \left[ \frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T \right] = -(\nabla \cdot \mathbf{q}) + \boldsymbol{\tau} : \mathbf{S} - \frac{T}{\rho} \frac{\partial \rho}{\partial T} \left[ \frac{\partial p}{\partial t} + (\mathbf{u} \cdot \nabla) p \right] + Q \quad (3)$$

The first is the continuity equation and represents conservation of mass. The second is the vector equation which represents conservation of momentum. The third describes the conservation of energy, formulated in terms of temperature. This is an intuitive formulation that facilitates boundary condition specifications.  $\rho$  is the density (SI unit: kg/m<sup>3</sup>),  $p$  is pressure (SI unit: Pa),  $\mathbf{u}$  is the velocity vector (SI unit: m/s),  $\boldsymbol{\tau}$  is the viscous stress tensor (SI unit: Pa),  $\mathbf{F}$  is the volume force vector (SI unit: N/m<sup>3</sup>),  $C_p$  is the specific heat capacity at constant pressure (SI unit: J/(kg·K)),  $T$  is the absolute temperature (SI unit: K),  $\mathbf{q}$  is the heat flux vector (SI unit: W/m<sup>2</sup>),  $Q$  contains the heat sources (SI unit: W/m<sup>3</sup>).

$$\boldsymbol{\tau} = 2\mu \mathbf{S} - \frac{2}{3}\mu(\nabla \cdot \mathbf{u}) \mathbf{I} \quad (4)$$

$$\mathbf{S} = \frac{1}{2}[\nabla \mathbf{u} + (\nabla \mathbf{u})^T] \quad (5)$$

$\boldsymbol{\tau}$  is the viscous stress tensor (SI unit: Pa).  $\mathbf{S}$  is the strain-rate tensor. The dynamic viscosity,  $\mu$  (SI unit: Pa·s), for a Newtonian fluid is allowed to depend on the thermodynamic state but not on the velocity field. All gases and many liquids can be considered Newtonian.

## 3. Model Definition

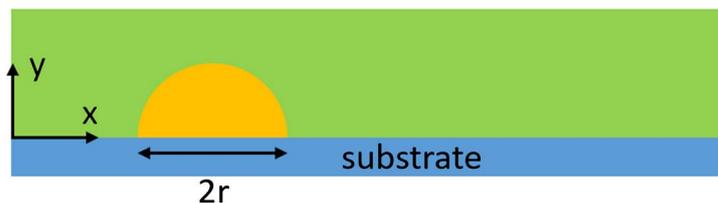
A schematic illustration of the millimeter-scale liquid droplet migration on solid surface problem under study is shown in **Figure 1**. The initial radius of the hemisphere is 0.5 mm. The temperature of the upper boundary of the simulation re-

gion is set to be the ambient temperature. The temperature of the lower boundary of the simulation region is set to change in a linear way with expression  $(493-30 * x[1/\text{mm}])$  [K].

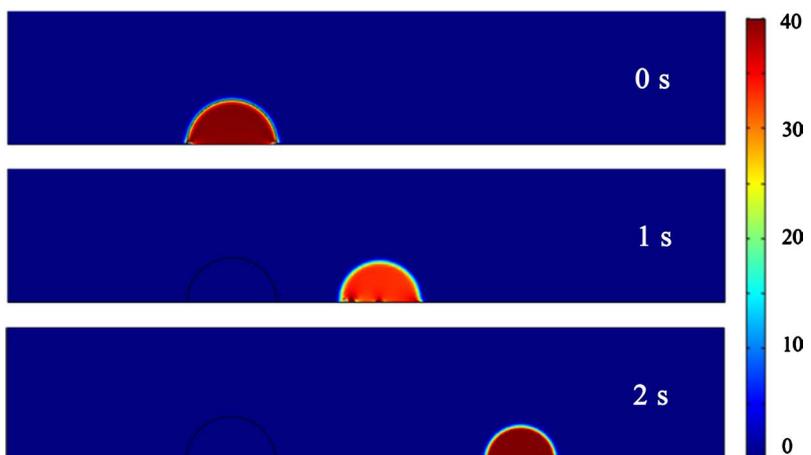
#### 4. Results and Discussions

**Figure 2** shows the pressure distribution of the entire simulation domain. The region of the liquid remains at 32 ~ 38 Pa during the whole simulation time. To confirm the moving of the droplet, we also plot the volume ration of liquid in **Figure 3** and it shows that there is no volume change. Since there is no absolute boundary between liquid and air, we define the region where the volume ration is greater than 0.3 to be liquid. The results in **Figure 2** clearly show that the millimeter-scale liquid sphere is migrating with a speed of about 1.7 mm per second. To ensure the accuracy of the finite element simulation, we set the maximum element size to be 0.04 mm during mesh process.

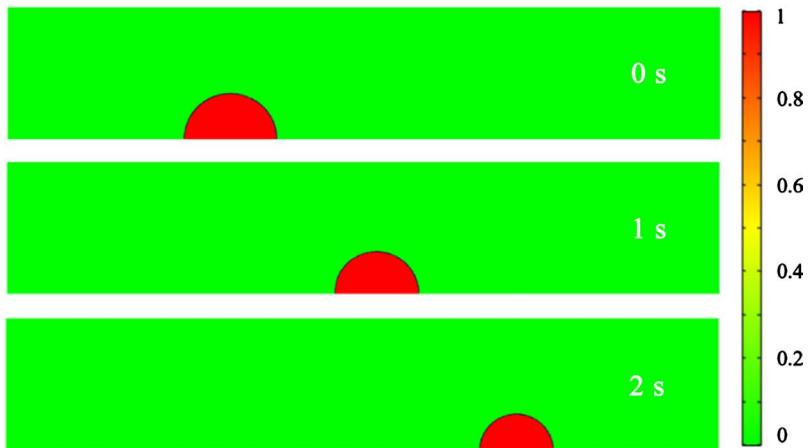
In order to give more details of the velocity field, the streamline distribution indicating instantaneously tangent to the velocity vector of the flow is presented in **Figure 4**. The largest curvature occurs at the edge of the liquid. The temperature field also changes with the influence of the liquid migration, with contribution both from the heat transfer from the liquid itself and the ambient air flow (also shown in **Figure 4**). The simulation results are consistent with previous experimental results in **Figure 5** [11].



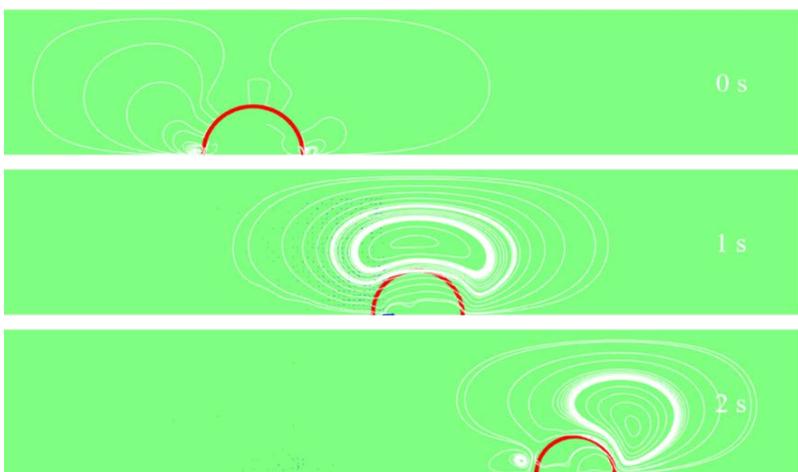
**Figure 1.** Schematic of the millimeter-scale liquid droplet migration on solid surface problem under study.



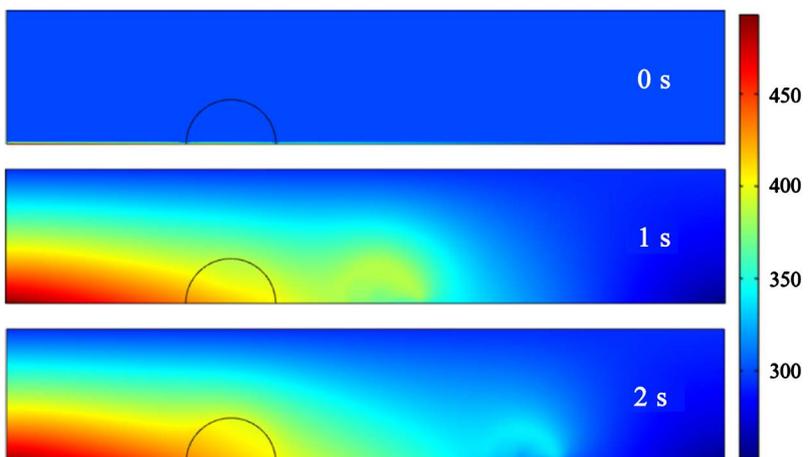
**Figure 2.** Pressure distribution of the entire simulation region.



**Figure 3.** Volume of liquid (larger than 0.3) at different times.



**Figure 4.** Streamline distribution plot.



**Figure 5.** Temperature field distribution of the entire simulation region.

## 5. Conclusion

In conclusion, we firstly build a model that simulates the millimeter-scale droplet migration both in heat transfer and fluid mechanics domain with finite ele-

ment simulation. Results show that the velocity of the migration is about 1.7 mm/s under a temperature gradient of 30 K/mm and the largest curvature occurs at the edge of the liquid. The principle can be referred to Tseng's work. We hope that our model finds a way for studying millimeter-scale droplet migration phenomenon under thermal gradient activation. Future work can be done to implement the model in three dimensions that require much more computing power and memory.

### Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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