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Contract Mechanism of Water Environment Regulation for Small and Medium Sized Enterprises Based on Optimal Control Theory

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Abstract

The small and scattered enterprise pattern in the county economy has formed numerous sporadic pollution sources, hindering the centralized treatment of the water environment, increasing the cost and difficulty of treatment. How enterprises can make reasonable decisions on their water environment behavior based on the external environment and their own factors is of great significance for scientifically and effectively designing water environment regulation mechanisms. Based on optimal control theory, this study investigates the design of contractual mechanisms for water environmental regulation for small and medium-sized enterprises. The enterprise is regarded as an independent economic entity that can adopt optimal control strategies to maximize its own interests. Based on the participation of multiple subjects including the government, enterprises, and the public, an optimal control strategy model for enterprises under contractual water environmental regulation is constructed using optimal control theory, and a method for calculating the amount of unit pollutant penalties is derived. The water pollutant treatment cost data of a paper company is selected to conduct empirical numerical analysis on the model. The results show that the increase in the probability of government regulation and public participation, as well as the decrease in local government protection for enterprises, can achieve the same regulatory effect while reducing the number of administrative penalties per unit. Finally, the implementation process of contractual water environmental regulation for small and medium-sized enterprises is designed.

Keywords

Optimal Control Theory, Small and Medium-Sized Enterprises, Water

Environment Regulation, Contract Mechanism

1. Introduction

The economy of Zhejiang is a microcosm of China's economy. Transforming the development model of sacrificing resources and the environment to promote economic growth and improving the ecological resources and environment is an important task under the new normal of Zhejiang and even the national economy. Since the proposal of the "Ecological Province" construction concept in 2003, Zhejiang has completed the transformation from a single ecological environment construction to a comprehensive green Zhejiang construction. Zhejiang Province is forcing enterprises to transform and upgrade due to the construction of ecological civilization. Ecological compensation mechanisms such as environmental taxes, ecological compensation, and emissions trading have been gradually implemented in various parts of Zhejiang [1]. At the beginning of 2013, in the face of the dead pig incident in the Huangpu River caused by livestock and poultry farming in Jiaxing, and the flooding incident caused by the strong typhoon "Fei" in October, the Zhejiang Provincial Government proposed to carry out the "Five Water Co governance" project throughout the province, including sewage treatment, flood prevention, drainage, water supply guarantee, and water-saving [2] [3]. In February 2017, Zhejiang held a comprehensive conference on the eradication of substandard Class V water and issued a military order to completely eradicate substandard Class V water within the year [4]. In 2019, Lishui was designated as the first pilot city in China to implement a mechanism for realizing the value of ecological products [5]. In March 2020, Xi Jinping visited Yucun Village in Zhejiang Province for inspection and pointed out the need to practice the development concept of "green mountains and clear waters are as valuable as gold and silver" [6]. In 2021, the General Office of the Communist Party of China Central Committee and the General Office of the State Council issued the "Opinions on Establishing and Improving the Mechanism for Realizing the Value of Ecological Products" [7], promoting the construction of ecological civilization in Zhejiang to a new level. Based on this background, this article practices the concept of ecological civilization and takes small and medium-sized enterprises in Zhejiang Province as the research object. Based on their pollutant emissions and environmental governance situation, it points out the problems in their ecological environment regulation. Problem-oriented, providing a decision-making basis for solving ecological and environmental governance problems in Zhejiang county economy.

The Zhejiang Provincial Government has continuously explored ways and methods of ecological environment governance in industrial transformation and upgrading and has achieved fruitful results. However, currently, for the pollution control and industry transformation and upgrading of small and medium-sized enterprises, many measures are being taken, such as mandatory shutdown and relocation. Lack of sufficient consideration for the economic characteristics of Zhejiang counties and the characteristics of small and medium-sized enterprises themselves. It cannot provide a good reference for water pollution prevention and control in regions with a large number of small and medium-sized enterprises, such as Zhejiang, Jiangsu, and Fujian. Zhejiang Province is a county-level economy characterized by a private economy as the mainstay and an outward-oriented economy. This county-level economy mainly relies on dispersed processing by small and medium-sized enterprises The "small and scattered" enterprise pattern has formed numerous sporadic pollution sources, hindering the centralized treatment of the water environment, and increasing the cost and difficulty of treatment. As the most important papermaking base in Zhejiang Province, Fuyang Paper Park has not fundamentally transformed its extensive production mode. How enterprises make reasonable decisions on their water environment behavior based on the external environment and their own factors is of great importance for scientifically and effectively designing water environment regulation mechanisms The significance of.

The research in this article has three objectives: firstly, to study a scientific industrial water environment supervision mechanism, so that polluting enterprises can voluntarily achieve the standard discharge of water pollutants, actively carry out clean production and emission reduction technology innovation, and reduce the impact of information asymmetry. Quantitatively determine from a theoretical perspective how local governments can design industrial water environment supervision mechanisms, encourage polluting enterprises to actively disclose real private information, avoid "moral hazard" and "adverse selection" issues, and achieve the ultimate goal of water environment protection. Secondly, through the design of incentive mechanisms in industrial water environment supervision, the production and operation of enterprises, pollutant reduction, and illegal discharge behavior are closely linked, stimulating the initiative and social responsibility of enterprises, effectively connecting traditional command and control methods with advanced environmental and economic management policies, and promoting the sound and unified environmental management methods. Thirdly, focusing on the diversified interest pattern in the industrial water environment supervision system, we need to transform the current situation of local governments as the sole regulatory body, so that China's industrial water environment supervision system includes judicial and administrative supervision by the central government, as well as social supervision by the public, environmental NGOs, experts and scholars, and news media. This weakens the motivation of local governments to protect enterprises from illegal pollution discharge and establishes a system consisting of the central government, local governments, and others.

2. Literature Review

The theoretical research on the regulatory mechanism of industrial water environment mostly focuses on the impact of different environmental and economic policies on the reduction of water pollutants by enterprises, treating them as passive economic entities and considering their environmental behavior solely to meet environmental regulatory requirements. Viaggi *et al.* (2009) [8] constructed a principal-agent model for the main participants in environmental regulation, simulating agricultural environmental regulation problems. Ren *et al.* (2010) [9] designed a quantitative model incorporating regulatory preference factors to study the coordinated regulatory system of the environment. Li Guoping *et al.* (2014) [10] analyzed the optimal environmental regulations and their fluctuations in local governments from the perspective of optimal contract design. These studies overlook the strategic behavior that businesses, as independent economic entities, can adopt to seek maximum benefits, namely their environmental behavior. It is the response of enterprises to government environmental policies and public environmental preferences in order to achieve their own development goals (including positive and negative reactions, as well as proactive environmental behavior of enterprises), and is an important component of environmental management behavior in the enterprise strategic management system [11]. How enterprises can make reasonable decisions about their water environment behavior based on the external environment and their own factors, and what kind of industrial water environment regulation system the government can formulate to guide enterprises to achieve conscious pollution control, all of which require research on the basis of clarifying the mechanism of enterprise water environment behavior. Liu Guangzhong *et al.* (2001) [12] established a dynamic control model and optimal control model for the total amount of pollutants based on the goal of controlling the total amount of pollutants and total output. They provided the optimal trajectory for the total amount of pollutants and total output, as well as the corresponding optimal pollution discharge fee standards. Liu Ke *et al.* (2005) [13] proposed the concept of operational regulation from the perspective of the interests of enterprises themselves, which means that individuals of enterprises adopt the optimal way of operating to obtain profits in the face of external regulations, and external regulations achieve the purpose of regulation by adjusting relevant parameters. Moon (2008) [14] regarded the environmental behavior of enterprises as a discrete process and used diffusion theory to analyze the voluntary degree of enterprise participation in green lighting plans from early to late stages. Li Shoude *et al.* (2009) [15] used optimal control theory to establish a dynamic control model for firm pollution control investment based on intertemporal emissions trading and studied firm pollution control investment control strategies under emissions trading conditions. Lin *et al.* (2010) [16] considered the gap in financial and human resources between small and medium-sized enterprises and large enterprises and studied the environmental behavior decision-making of small and medium-sized enterprises under environmental uncertainty through a questionnaire survey method. This article draws on the concept of operational regulation proposed by Liu Ke, and overcomes the concept of treating enterprises as passive entities based on the participation of multiple stakeholders such as the government, enterprises, and the public. It introduces the strategic behavior of enterprises and uses optimal control theory to analyze the optimal water pollutant emission control strategies that enterprises will adopt under different policy conditions. It studies the design of industrial water environment regulation mechanisms.

Enterprise water environment behavior refers to the measures and means taken by enterprises in the face of environmental pressure from the government, the public, and the market, based on achieving their own development goals, in response to government water environment policies and public environmental preferences, such as macro strategic and institutional changes, and internal specific production adjustments [17] [18]. In the past, the design and implementation of industrial water environment supervision mechanisms were carried out from top to bottom, treating enterprises as passive economic entities, which resulted in policies not being effectively implemented, increasing the cost of environmental supervision and reducing implementation effectiveness [19]. This policy-making model overlooks two issues: on the one hand, as an independent brokerage individual, enterprises can take corresponding strategic actions to seek maximum benefits, that is, to regulate the water environment operation of enterprises. Water environment operation supervision refers to the optimal operation of individual enterprises in the face of external supervision to obtain profits, and external supervision achieves the purpose of supervision by adjusting relevant parameters [20] [21]. On the other hand, different types of enterprises have different reactions to environmental policies: some enterprises mainly aim to meet the basic requirements of environmental laws and regulations, and have a relatively negative attitude towards environmental policies, viewing them as non-important factors affecting corporate decision-making; Some companies carry out production wastewater treatment to avoid environmental law enforcement penalties that may affect their corporate image and prevent pollution accidents from posing a threat to their business performance; Some companies regard energy conservation and emission reduction as opportunities for innovation, seeking opportunities to improve social reputation and market position; Some enterprises integrate the environment with their business objectives appropriately, and environmental protection has become an organic component of business management. From management philosophy to decision-making, enterprises have regarded environmental protection as an important target factor.

The strategic behavior that enterprises will adopt under the constraints of industrial water environment supervision mechanism depends not only on internal factors such as their own water pollutant treatment cost, strategic positioning, and scale strength, but also on external factors such as the national environmental protection system environment, local government environmental supervision mechanism, and social supervision, as shown in **Table 1**.

Table 1. Analysis of factors influencing enterprise water environment behavior.

	Specific influencing factors	factor analysis
External factors	National Environmental Protection Policy	Command and control environmental policy or market incentive environmental policy
	Local government environmental supervision	The environmental monitoring capacity and enforcement effectiveness of local governments
	Social supervision	Environmental pressure from upstream and downstream enterprises, customers, and surrounding residents
Internal factors	The cost of water pollutant treatment for enterprises	Production function, pollution control technology, equipment and personnel, etc
	Enterprise strategic positioning	Positioning as a long-term strategy or a short-term strategy
	Enterprise scale and strength	Large, medium, and small enterprises in terms of scale and strength

The choice of corporate water environment behavior is not only a manifestation of corporate environmental strategy, but also an important component of overall strategic management of the enterprise. At the same time, research on corporate water environment behavior has always been closely related to water environment supervision mechanisms. Many scholars have incorporated environmental strategy into the scope of corporate political strategy and proposed many classification methods for the attitude of enterprises toward dealing with environmental issues. There are mainly two types of classification methods [22]. The first classification method is based on the interval of water environment behavior. This method starts from the perspective of time cutoff, studying the positioning of enterprises in a series of selection sets, with the aim of analyzing the differences in water environment behavior choices of different enterprises at the same time node. The main basis for dividing the interval of water environment behavior is what attitude enterprises adopt to respond to water environment regulation. For example, Roome (1992) [23] divides the water environment behavior of enterprises into five levels in order based on their level of initiative in energy conservation and emission reduction: Non-obedience, obedience, obedience to increase, optimal business and environment, and leadership advantage. Sharma (2000) [24] defined a company's water environment behavior as a continuous range of changes from obedience to conscious compliance, based on the degree of compliance with water environment laws and regulations. The second classification method is based on the process of enterprise water environment behavior. This method starts from the time dimension and studies the progress of corporate water environment behavior, with the aim of analyzing the process of selecting the same corporate water environment behavior at different time nodes. The main basis for dividing the process of water environmental behavior is the water treatment capacity and technological innovation capacity of enterprises. Hart (1995) [25] classified four strategic choices based on the gradual improvement of enterprise environmental technology: end of pipe treatment, pollution prevention, link monitoring, and sustainable development. Aragon Correa (1998) [26] divided the water environmental behavior of enterprises into corrective and preventive measures, namely traditional end-of-pipe treatment methods and modern clean production technologies.

Based on the characteristics of the two classification methods, the selection space for water environment behavior is determined from the perspectives of water environment supervision and enterprise environmental technology. Each type of environmental strategy reflects different behavioral characteristics. When facing different pollutant discharge standards, such as the first and second level standards in the comprehensive sewage discharge standard GB8978-1996, enterprises will make different water environment behavior decisions. This article divides the selection of corporate water environment behavior into four types: cooperative, opportunity, risk, and adaptive, as shown in **Figure 1**.

		Standards for the discharge of water pollutants	
		Standard high	Low standard
Enterprise water treatment costs	High cost	Risk oriented	Adaptive type
	Low cost	Collaborative type	Opportunity oriented

Figure 1. Collection of enterprise water environment behavior choices.

The "risk-based" water environment behavior choice is a difficult and risky response adopted by enterprises with high water pollutant treatment costs to cope with higher sewage discharge standards. The cost of treating water pollutants in enterprises is high, and if they still choose to meet their own unbearable sewage discharge standards, they will incur higher water treatment costs, which may even affect the company's financial chain and ultimately affect its production and operation. Therefore, enterprises with higher treatment costs for such water pollutants are more willing to choose lower water pollutant discharge standards, adopt "adaptive" water environment behaviors, and obtain emission rights that meet their own needs by entering emission trading markets and other economic means, reducing the operating costs of enterprises.

The opportunistic choice of water environment behavior is the opportunistic response of enterprises with low water pollutant treatment costs to lower sewage discharge standards. Enterprises have low costs for treating water pollutants and have an instinctive motivation to choose when facing lower sewage discharge standards. However, enterprises that maximize their own interests, from a rational perspective, will comprehensively consider their own water treatment cost types and the additional benefits that selling pollution discharge rights will bring. Therefore, companies with lower treatment costs for water pollutants will ultimately present a corporate image of fulfilling corporate social responsibility, and choose "cooperative" water environment behavior, thereby obtaining additional benefits from selling pollution discharge rights.

3. A Model of Optimal Control Strategies for Enterprises under Contractual Water Environment Regulation

The regulatory mechanism for the water environment actually belongs to a contractual arrangement between the government and enterprises. In the early 1980s, China began to implement an environmental protection responsibility system in environmental management. By formulating environmental administrative contracts, the responsibilities, rights, and interests of environmental managers and polluters in environmental protection were confirmed. However, due to the fact that the environmental administrative contract system in China has not yet been formed, there are a large number of contractual relationships in the field of environmental management, and the theoretical research on environmental management contracts is very weak. Therefore, the following text continues the management concept of environmental administrative contracts and proposes a contractual water environment regulation, which is an environmental regulation method that the government and enterprises reach consensus through negotiation. It is beneficial for enterprises to meet pollution standards, reduce regulatory execution costs, and provide decision-making support for scientifically and efficiently solving water environment management problems. This model is designed for environmental regulation issues in small and medium-sized enterprises, and can also be used as a reference for large enterprises.

3.1. Model Assumptions and Parameter Settings

When a company has a motive for illegal discharge, it will discharge a portion of its production wastewater before entering the water treatment facility, in order to reduce treatment costs.

To study how enterprises can achieve optimal operation under local government water environment supervision, a model of optimal control strategy for enterprises is constructed, assuming the following:

Assumption 1: Assuming that the industrial water environment regulation model of the local government is that the enterprise signs a regulation contract with the local government, which is formulated by the local government, including environmental administrative penalties, environmental supervision and law enforcement, and public participation, $\left[F_t, P, \lambda\right]$. F_t is the amount of punishment that enterprises will face after illegally discharging sewage at t time, which is a measure of environmental administrative penalties. $P\left(0 \le P \le 1\right)$ is the probability of local government water administrative departments supervising enterprises, which is a measurement standard for environmental supervision and law enforcement. $\lambda(0 \le \lambda \le 1)$ is the probability that a company's illegal discharge of sewage or local government shelter behavior will be discovered and reported by the public through public participation

Assumption 2: Enterprises have their own water pollutant treatment facilities. Under the industrial water environment regulation system, enterprises have two behavioral choices. When enterprises comply with the law and discharge pollutants, they will inject all production wastewater into water treatment facilities for pollutant reduction, in order to achieve standard discharge. Enterprises have a motive for illegal discharge, and in such cases, they may discharge a portion of their production wastewater before entering water treatment facilities to reduce treatment costs.

Assumption 3: The goal of enterprises is to maximize their own interests under the regulation of industrial water environment by local governments. Local governments have a willingness to protect companies from polluting due to economic development considerations. $\eta(0 \le \eta \le 1)$ is the probability that local governments will provide shelter to polluting enterprises (monitoring them for illegally discharging sewage without punishment), and is a measure of dereliction of duty supervision. When regulatory agencies fail to conduct spot checks, there is a possibility that the illegal discharge behavior of enterprises may be discovered and reported by the public.

In order to describe the optimal control strategy model of enterprises under industrial water environment regulation, the following parameters are specifically defined:

The state variable is $x(t)$, which represents the amount of wastewater produced by the enterprise at time *t*.

The control variable is $u(t)$, which is the rate at which the production wastewater of the enterprise does not enter the water treatment facility and is discharged illegally at time *t*.

The external function is $Q(t)$, which is the rate of production of wastewater generated by the enterprise at time *t* (set to be constant).

Parameter: ρ is a non-negative constant discount rate; ω is an accompanying variable; δ is the loss coefficient of water leakage during the water treatment process of the enterprise; φ_1 and φ_2 is the Lagrange multiplier; P is the probability of supervision by the local government's water administration department on enterprises ($0 \le P \le 1$); F_t is the penalty amount that the enterprise will face after illegally discharging sewage at time *t*, and f is the unit water pollutant penalty amount; C_t is the water pollutant treatment cost of the enterprise at time *t*, and c is the cost coefficient; $\left[Q(t)-u(t)-\delta \cdot x(t)\right]$ is the water treatment rate at time *t*.

The water pollutant treatment cost of the enterprise is C_t at time *t*, which is the product of the cost coefficient and the amount of wastewater treated per unit time, *i.e.* $C_t = c \cdot \left[Q(t)-u(t)-\delta \cdot x(t)\right]$.

The penalty amount F_t is a company will face after illegally discharging sewage at time *t*, which is the product of the penalty amount and probability of being punished for the company's illegal discharge of water pollutants per unit, and the company's illegal discharge rate $u(t)$ at time *t*. that is:

$$F_t = f \cdot \left\{ P \cdot \left[\eta \cdot \lambda + (1-\eta) \right] + (1-P) \cdot \lambda \right\} \cdot u(t) = f \cdot M \cdot u(t) \tag{1}$$

In the formula, the probability of being punished is composed of two parts $P \cdot \left[\eta \cdot \lambda + (1-\eta) \right]$ and $(1-P) \cdot \lambda$. The first part is the probability P of being punished for sheltering enterprises that are discovered and exposed by public participation $\eta \cdot \lambda$ in the implementation of regulations by local governments, as well as the probability $(1-\eta)$ of being fined for strict implementation of regulations by the government. The second part is the probability $(1-P)$ that during the period when local governments do not regulate, the illegal emission behavior of enterprises will be discovered and exposed by public participation, resulting in fines.

3.2. Construction of Optimal Control Strategy Model for Enterprises

We assume that enterprises make behavioral decisions based on the parameter design of industrial water environment regulation mechanisms. Within the time interval $[0, T]$, the objective function of the enterprise is:

$$\max_{u(t) \in \Omega(t)} \left\{ \begin{aligned} J &= \int_0^T -\mathrm{e}^{-\rho t} \cdot \left\{ F_t + C_t \right\} \mathrm{d}t \\ &= \int_0^T -\mathrm{e}^{-\rho t} \cdot \left\{ f \cdot M \cdot u(t) + c \cdot \left[Q(t) - u(t) - \delta \cdot x(t) \right] \right\} \mathrm{d}t \end{aligned} \right\}, \quad i = 1, 2 \tag{2}$$

The change rate of water pollutant reduction in enterprises can be represented by the following model:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = g\left(t, x(t), u(t)\right) = Q(t) - u(t) - \delta \cdot x(t),\ x(0) = x_0 = 0 \tag{3}$$

Satisfying the constraint is $0 \le u(t) \le Q(t)$, that is $u(t) \ge 0$, $Q(t) - u(t) \ge 0$. Where $x(0) = x_0 = 0$ is given, and $x(t) \ge 0$. ρ is the discount factor.

Therefore, the optimal control strategy model for enterprises under government industrial water environment regulation is:

$$\begin{aligned} &\max_{0 \le u(t) \le Q(t)} \left\{ J = \int_0^T -\mathrm{e}^{-\rho t} \cdot \left\{ f \cdot M \cdot u(t) + c \cdot \left[Q(t) - u(t) - \delta \cdot x(t) \right] \right\} \mathrm{d}t \right\} \\ &\text{s.t. } \frac{\mathrm{d}x(t)}{\mathrm{d}t} = Q(t) - u(t) - \delta \cdot x(t),\ x(0) = x_0 = 0 \\ &\quad u(t) \ge 0,\ Q(t) - u(t) \ge 0,\ x(t) \ge 0 \end{aligned} \tag{4}$$

3.3. Solving the Optimal Control Strategy Model for Enterprises

Using the maximum principle in optimal control to solve the optimal decision-making problem of water pollutant treatment capacity in enterprises. Firstly, construct the immediate value Hamiltonian function:

$$H = e^{-\rho t}\left\{-f\cdot[M]\cdot u(t) - c\cdot\left[Q(t) - u(t) - \delta\cdot x(t)\right]\right.$$
$$\left. + \omega(t)\cdot\left[Q(t) - u(t) - \delta\cdot x(t)\right] + \varphi_1\cdot u(t) + \varphi_2\cdot\left[Q(t) - u(t)\right]\right\} \quad (5)$$
$$= e^{-\rho t}\tilde{H}$$

We call $\tilde{H}$ the present value the Hamiltonian function. $\omega(t)$ is the Hamiltonian multiplier of the present value. $\omega(t)$ represents the marginal value of the cumulative treatment capacity $x(t)$ of the enterprise's production wastewater by the enterprise's water treatment facilities at time *t*. It represents the optimal value change caused by adding one unit to the state variable at time *t*. Also known as the shadow price of state variables.

The optimal trajectory of this model must satisfy the following equation

$$\frac{\partial\tilde{H}}{\partial u} = c - f\cdot M - \omega(t) + \varphi_1 - \varphi_2 = 0 \quad (6)$$

In the formula, φ_1 and φ_2 satisfies the complementary relaxation condition:

$$\varphi_1 \geq 0,\ u(t) \geq 0,\ \varphi_1\cdot u(t) = 0 \quad (7)$$

$$\varphi_1 = 0,\ Q(t) - u(t) \geq 0,\ \varphi_2\cdot\left[Q(t) - u(t)\right] = 0 \quad (8)$$

If $\varphi_2 > 0$, then $Q(t) - u(t) = 0$, thus $\varphi_1 = 0$. At this point, the optimal condition can be written as: $\varphi_2 = c - f\cdot M - \omega(t)$. That is, when $c - f\cdot M - \omega(t) > 0$, the optimal control variable is $u(t) = Q(t)$.

If $\varphi_1 > 0$, then $u(t) = 0$, thus $\varphi_2 = 0$. At this point, the optimal condition can be written as: $\varphi_1 = \omega(t) - c + f\cdot M$. That is, when $c - f\cdot M - \omega(t) < 0$, the optimal control variable is $u(t) = 0$.

If $\varphi_1 = 0$ and $\varphi_2 = 0$, the value of $u^*(t)$ is uncertain. At this point, the optimal condition can be written as $c - f\cdot M - \omega(t) = 0$. That is, when $c - f\cdot M - \omega(t) = 0$, the value of the optimal control variable $u^*(t)$ is uncertain. This is the moment of singular control.

According to the Hamiltonian function, the adjoint vector satisfies the following differential equation.

$$\frac{d\omega}{dt} = \rho\cdot\omega(t) - \frac{\partial\tilde{H}}{\partial x} = (\rho + \delta)\cdot\omega(t) - c\cdot\delta \quad (9)$$

Based on the cross-sectional conditions $\omega(T) = 0$, it can be concluded that

$$\omega(t) = \frac{c\cdot\delta}{\rho + \delta}\left(1 - e^{(\rho+\delta)(t-T)}\right) \quad (10)$$

From $0 \leq t \leq T$, we can obtain $0 \leq \omega(t) \leq \frac{c\cdot\delta}{\rho + \delta}\left(1 - e^{-(\rho+\delta)T}\right)$, then:

$$\left\{c - f\cdot M - \omega(t)\right\} \in \left[c\left(\frac{\rho}{\rho + \delta} + \frac{\delta}{\rho + \delta}e^{-(\rho+\delta)T}\right) - f\cdot[M], c - f\cdot[M]\right] \quad (11)$$

Therefore, the following conclusion can be drawn:

1) When $c - f\cdot M - \omega(t) > 0$, $u^*(t) = Q(t)$, the following conditions must

be met:

$$c\left(\frac{\rho}{\rho+\delta}+\frac{\delta}{\rho+\delta}e^{-(\rho+\delta)T}\right)-f\cdot M>0 \quad (12)$$

However, it is obvious that the optimal control rate of illegal emissions by enterprises at this time is not the expected behavior of environmental protection.

2) When $c-f\cdot M-\omega(t)=0$, $u^*(t)$ is uncertain, which is the moment of singular control. This situation is also not the expected corporate behavior for environmental protection.

3) When $c-f\cdot M-\omega(t)<0$, $u^*(t)=0$, $c-f\cdot M<0$ is required. At this point, the optimal control of the illegal emission rate $u^*(t)=0$ for the enterprise is also the expected behavior of environmental protection and the ultimate goal of designing incentive mechanisms. Therefore, finding the f at this point can force the enterprise to choose zero illegal discharge of pollutants. Based on the conditions for the establishment of the optimal pollution control strategy, the quantity method of f is theoretically derived. Take the critical value and solve it to obtain the government's penalty amount for enterprises illegally discharging unit water pollutants, as follows:

$$f=\frac{c}{P\cdot\left[\eta\cdot\lambda+(1-\eta)\right]+(1-P)\cdot\lambda} \quad (13)$$

From the perspective of corporate behavior, a reasonable design of the corresponding penalty amount f for the illegal discharge of water pollutants by enterprises can constrain them to stop their illegal discharge behavior. And f is a variable that can be controlled by the government's industrial water environment regulation mechanism. Therefore, when the probability of water environment regulation P, the probability of shelter η, and the degree of public participation λ are constant, and the conditions shown in Equation (13) are met, the administrative penalty amount f for unit pollutant discharge in the industrial water environment regulation mechanism can be calculated. At this point, the enterprise has a unique optimal dynamic emission control strategy for water pollutants, $u^*(t)=0$, to maximize the expected benefits.

4. Empirical Study on a Paper-Making Enterprise in Zhejiang Province and Regulation Implementation Process Design

4.1. Empirical Study on a Paper-Making Enterprise in Zhejiang Province

The cost data of water pollutant treatment for a paper manufacturing enterprise in Zhejiang Province were selected to conduct empirical numerical analysis on the above model.

Firstly, assuming that the probability of asylum is constant, simulate the impact of government regulation probability P and public participation level λ on the penalty amount f of unit water pollutants. The basic data values are shown in **Table 2**.

First, assuming that the probability η of asylum is constant, simulate the impact of government regulation probability P and public participation level λ on the penalty amount f per unit of water pollutant.

Table 2. Basic data value table.

Basic data value	Cost of water pollutant treatment per enterprise unit	Public participation level λ	Regulatory probability P	Shelter probability η
Working condition 1	1.28 yuan/m^3	0.1	0.1	0.1
Working condition 2	1.28 yuan/m^3	0.3	0.3	0.1
Condition 3	1.28 yuan/m^3	0.5	0.5	0.1
Working condition 4	1.28 yuan/m^3	0.7	0.7	0.1

Substituting the data from **Table 2** into Equation (15) yields, the values of penalty amount f for unit water pollutants under different operating conditions are obtained, as shown in **Figure 2**.

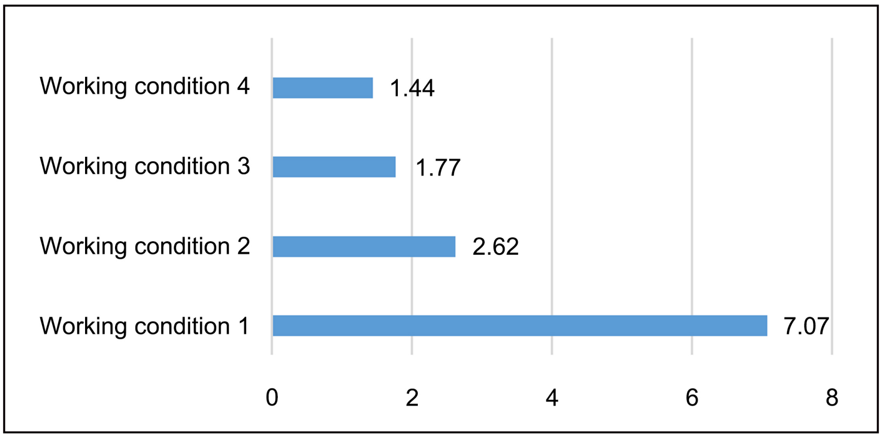

Figure 2. Value of penalty amount f for unit water pollutants under different operating conditions.

In addition, using Matlab 7.0 for data simulation, the following figure is obtained:

From the results of **Figure 2** and **Figure 3**, it can be concluded that there is an inverse correlation between the degree of public participation and the probability of regulation and the amount of punishment required. That is, the smaller the degree of public participation and the frequency of regulation, the greater the amount of punishment required for enterprises to meet pollution standards. Conversely, the larger the degree of public participation and the frequency of regulation, the smaller the amount of punishment required for enterprises to meet pollution standards. When the frequency of regulation and the level of public participation are both 0, the penalty amount reaches its maximum value.

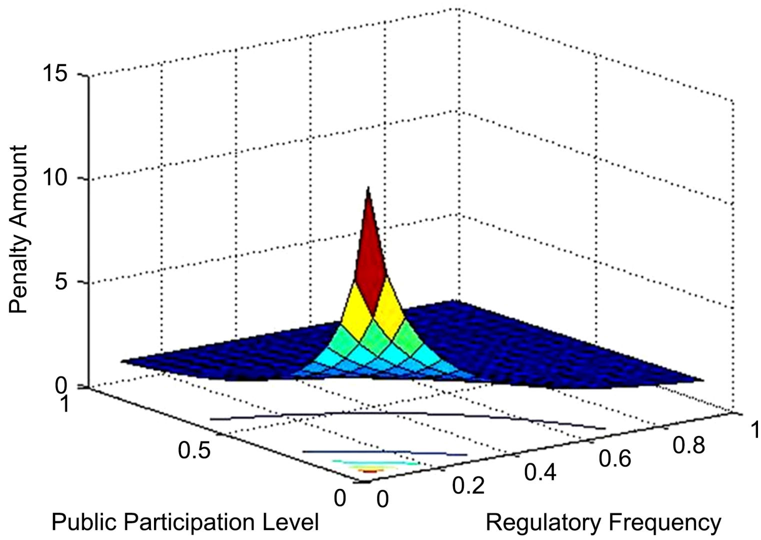

Figure 3. Relationship between public participation level, regulatory frequency, and penalty amount.

Secondly, assuming the probability of regulation $P = 0.1$ and the degree of public participation $\lambda = 0.1$. Simulate the impact of asylum probability η on the penalty amount *f*.

The simulation results in **Figure 4** show that under the same regulatory effect, a decrease in the probability of asylum can lead to a decrease in the amount of punishment.

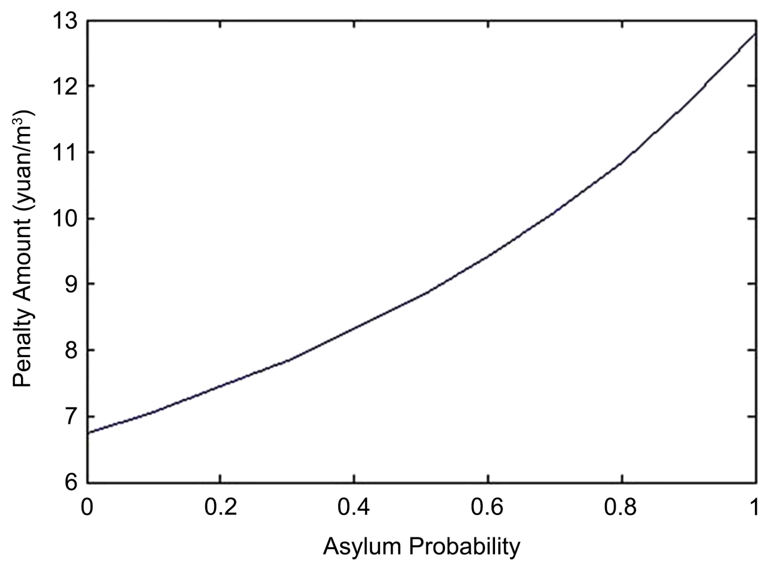

Figure 4. Correlation between the probability of asylum and the amount of punishment.

Based on the below, **Table 3** shows the results.

The above simulation results show that an increase in the probability of government regulation *P* and the degree of public participation λ, as well as a decrease in the probability of local governments sheltering enterprises η, can achieve the same regulatory effect by lowering the administrative penalty amount f for illegal discharge of water pollutants per unit. A lower f is beneficial for reducing conflicts of interest between enterprises and local governments. Therefore, to reduce illegal pollution, the traditional approach of "increasing punishment" is not the only option, as this approach can easily generate resistance. The effective implementation of the regulatory mechanism for industrial water environment can be achieved through increasing the frequency of regulation and policy support, improving public participation, strengthening the supervision mechanism for regulatory dereliction of duty, and other auxiliary measures.

Table 3. Comparative static analysis of optimal control strategy model parameters for enterprises

External environmental parameters of the enterprise	Parameter changes	The penalty amount for enterprises illegally discharging water pollutants from units
Regulatory probability P	↑	↓
	↓	↑
The probability of local governments sheltering enterprises η	↑	↑
	↓	↓
Public participation level λ	↑	↓
	↓	↑

4.2. Implementation Process of Water Environment Regulation Contract

To fully demonstrate and leverage the effectiveness of the water environment regulation contract mechanism, it is necessary to base it on a detailed design of the implementation process of the regulation contract. The implementation of a contractual regulatory mechanism refers to the signing of regulatory contract texts $[F_t, P, \lambda]$ between the environmental administrative department and paper enterprises. This text clearly states: at time t, the punishment amount F_t that the enterprise will face after illegally discharging sewage; The probability of local government water administrative departments supervising enterprises is P $(0 \le P \le 1)$; The probability of enterprises illegally discharging sewage or local government sheltering behavior being reported by the public is λ $(0 \le \lambda \le 1)$. The implementation process of the water environment regulation contract mechanism is shown in **Figure 5**.

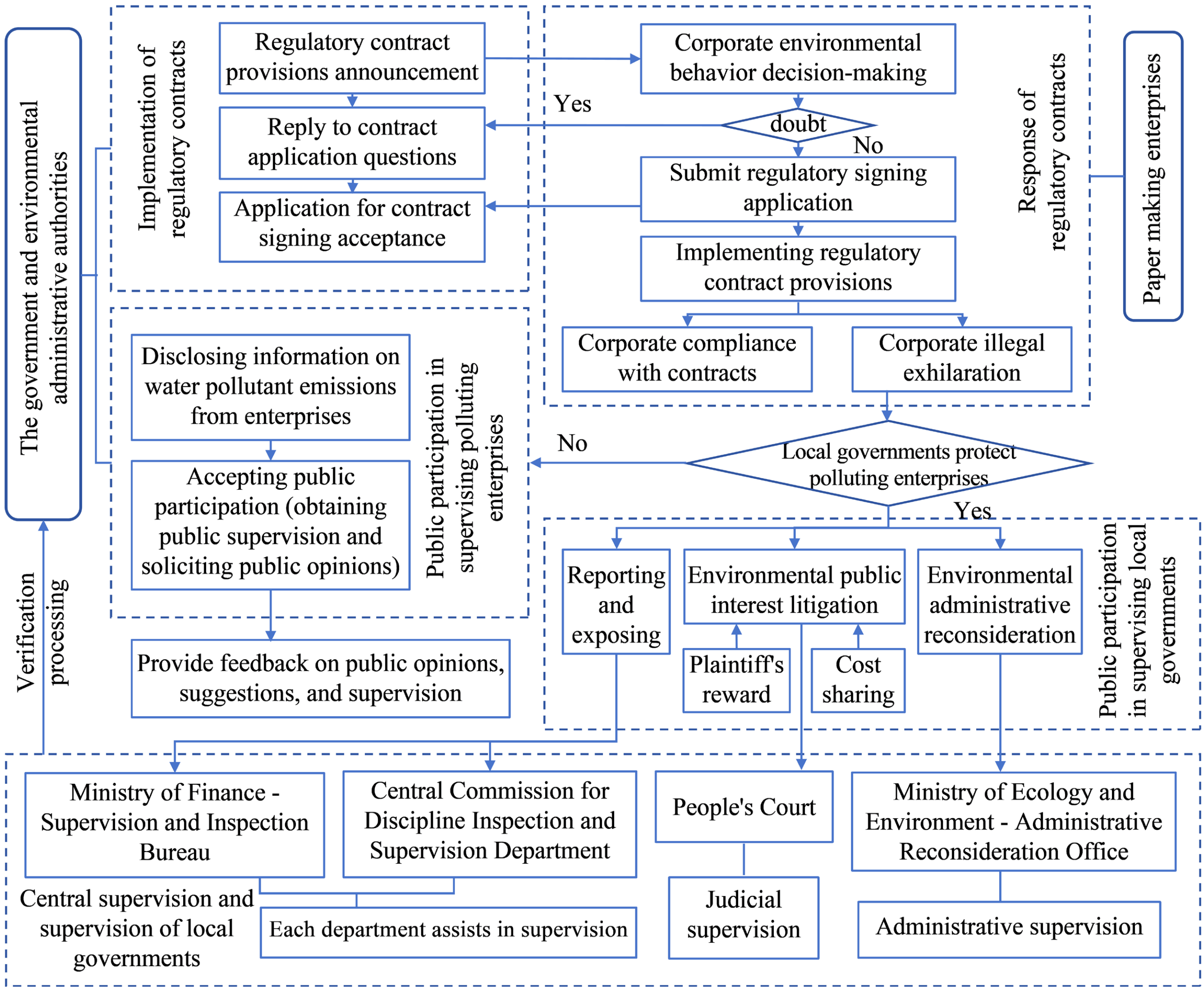

Figure 5. Implementation process of water environment regulation contract.

As shown in **Figure 5**, the implementation process of the water environment regulation contract is as follows:

Firstly, the government implements contractual water environment regulations. The local government and water environment administrative department shall publish detailed regulations on contractual water environment regulation through government bulletins and government websites, including relevant laws and regulations, application and acceptance procedures for contract signing, and other comprehensive information related to the implementation of contractual regulations; Applicants of enterprises participating in contractual regulations who have doubts about the matters, basis, conditions, procedures, deadlines, and other contents related to contractual regulations announced by the water environment administrative department may request explanations from the water environment administrative department. At this time, the competent department should provide scientifically accurate information;

Secondly, when enterprises engage in illegal pollution discharge, the environmental administrative department of the local government conducts routine supervision on the enterprises using Introduction P. When the government strictly enforced the law on enterprises, they were subject to environmental administrative penalties. But when enterprises engage in illegal discharge during non-supervisory periods, social supervision groups such as the public, in conjunction with local governments, supervise and manage the illegal discharge of water pollutants by enterprises; When the government sheltered the illegal discharge behavior of enterprises, social supervision groups such as the public discovered this sheltering behavior and will cooperate with the central judicial administrative supervision to supervise and manage the environmental fraud behavior of local governments.

In summary, under the contract-based water environment regulation mechanism, a core regulatory level composed of government and enterprises has been formed, as well as a peripheral regulatory level composed of social regulatory groups such as the central government and the public. Furthermore, a multi-subject joint prevention and control network for water environment regulation has been established, where local governments and the public jointly regulate enterprises, the public and the central government jointly regulate local governments, and enterprises supervise each other.

5. Conclusions

The research object of this paper is the design of contractual mechanisms for water environmental regulation of small and medium-sized enterprises in Zhejiang Province. Based on the paper industry and wastewater discharge situation, as well as the overview of water environmental governance, the problems existing in water environmental regulation are proposed. The enterprise is regarded as an independent economic entity that can adopt optimal control strategies to maximize its own interests. Drawing on the concept of operational regulation proposed by Liu Ke, based on the multi-agent participation of government, enterprise and public, using optimal control theory, a model of optimal control strategy for enterprises under contractual water environmental regulation is constructed. By solving the model, the optimal emission rate of enterprises under different conditions is obtained, and the unit pollutant penalty amount that should be set by the government subject when the emission rate is zero is found. The quantitative method of penalty amount is given in theory. And select the cost data of water pollutant treatment of a paper-making enterprise in Zhejiang Province to conduct empirical numerical analysis on the model. Therefore, in the implementation process design of contractual water environment regulations, the role of government regulation probability, public participation level, and local government protection level for enterprises should be highlighted. Form a core regulation level consisting of local governments and polluting enterprises, as well as a peripheral supervision level consisting of central government and public social regulation groups. Realize a multi-agent joint prevention and control water environment regulation system in which local governments and the public jointly supervise enterprises, the public and the central government jointly supervise local governments, and enterprises supervise each other.

The design of industrial water environment regulation mechanisms and the selection of water environment behaviors by enterprises are influenced by various factors such as society, economy, and daily life. This article is only a preliminary attempt in this field, attempting to construct a more realistic optimal control strategy model for enterprises based on mathematical analysis. In future research, factors such as environmental subsidies should be considered to make the problem research more realistic.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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