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Diverging Diamond Interchange Performance Measures Using Connected Vehicle Data

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Abstract

Since the first Diverging Diamond Interchange (DDI) implementation in 2009, most of the performance studies developed for this type of interchange have been based on simulations and historical crash data, with a small number of studies using Automated Traffic Signal Performance Measures (ATSPM). Simulation models require considerable effort to collect volumes and to model actual controller operations. Safety studies based on historical crashes usually require from 3 to 5 years of data collection. ATSPMs rely on sensing equipment. This study describes the use of connected vehicle trajectory data to analyze the performance of a DDI located in the metropolitan area of Fort Wayne, IN. An extension of the Purdue Probe Diagram (PPD) is proposed to assess the levels of delay, progression, and saturation. Further, an additional PPD variation is presented that provides a convenient visualization to qualitatively understand progression patterns and to evaluate queue length for spillback in the critical interior crossover. Over 7000 trajectories and 130,000 GPS points were analyzed between the 7th and the 11th of June 2021 from 5:00 AM to 10:00 PM to estimate the DDI's arrivals on green, level of service, split failures, and downstream blockage. Although this technique was demonstrated for weekdays, the ubiquity of connected vehicle data makes it very easy to adapt these techniques to analysis during special events, winter storms, and weekends. Furthermore, the methodologies presented in this paper can be applied by any agency wanting to assess the performance of any DDI in their jurisdiction.

Keywords

Diverging Diamond Interchange, Performance Measures, Connected Vehicle, Big Data

1. Introduction

Over the past decade, several Diverging Diamond Interchanges (DDI) have been built in the United States with the objective of reducing construction costs, improving safety, and enhancing traffic operations. A DDI differs from a Conventional Diamond Interchange (CDI) [1] in that it implements directional crossovers on each end of the crossing street. By switching through movements to the left side of the road within the interchange, conflicts between left-turning vehicles and opposing through traffic from the crossing street are eliminated [2] [3].

Although many DDIs have been built around the country, most of the performance analyses have been conducted with simulation models. The objective of this paper is to present analytical techniques for processing commercial probe data to compute quantitative performance measures characterizing the performance of a DDI.

1.1. Literature Review

Currently, most performance analyses of DDIs have been done by means of simulation to provide information on travel times, *v/c* ratios, throughputs, queue lengths, delays, level of service, and number of stops [2] [4]-[12]. Safety performance has been evaluated from historical crash data to assess improvements compared to other types of interchange and to calibrate crash modification factors [13] [14] [15]. Hainen *et al.* made use of high-resolution event data to assess the internal queuing dynamics and the inflow/outflow demand balance within a DDI [16]. An Automated Traffic Signal Performance Measure (ATSPM) [17] was developed by using traffic signal phase data and point sensors to estimate travel time and arrivals on green (AOG) of vehicle trajectories through the intersection. The results of the analysis recommended a change from a two-phase to a three-phase configuration that led to an AOG increase of 39% for the heaviest internal movement.

With the emergence and improvement of commercially available connected vehicle (CV) data, new techniques have been developed to assess operational and safety performance at intersections without the need for costly infrastructure investments. CV hard-braking events have been proven to be a surrogate of crashes [18]. Vehicle trajectories have been used to estimate queue lengths [19] [20]. Traditional travel times [21] [22], Highway Capacity Manual (HCM) Level of Service (LOS) [23] [24] [25] [26] [27], and arrivals on green [24] [25] [26] [28] have also been calculated. In addition, critical analysis on the percentage of vehicles experiencing split failures and downstream blockage can also be derived from CV trajectory data [24] [25]. However, there are no studies that have used this recently available dataset to generate performance measures for DDIs. The advantage of using CV trajectory data to assess DDIs' is stated in the following sub-section.

1.2. Motivation to Use CV Trajectory Data for Characterizing the Performance of DDIs

Estimating performance measures from simulation requires traffic signal timing plans, peak factors, volumes, and model configuration. Usually, this information is not easily accessible and time-consuming data collection is required. Further, the analyst needs to calibrate and validate each simulation based on the personal understanding of the DDI, which can potentially yield different results between different analysts [2]. With regards to data from point sensors to derive ATSPMs, capital and maintenance costs remain a barrier for widespread implementation. Depending on the sensors deployed, some types also cannot distinguish the presence of individual vehicles, queue length, and inflow origins, especially during near- or over-capacity periods.

This study uses commercially available CV trajectory data to generate DDI performance measures. This is particularly important for two reasons:

1) Even with investment in significant traffic sensing infrastructure, there is no robust way for evaluating progression through the two adjacent signals.

2) DDIs are relatively new. The scalability of CV data allows evaluation of a broad cross section of DDIs scattered across the United States to identify best practices for operating these new intersections as well as uniform performance measures.

1.3. Trajectory-Based Performance Measures

An extension of the Purdue Probe Diagram (PPD) is proposed that provides insights on the DDI's levels of delay, progression, and saturation. Further, an additional PPD variation to evaluate critical queue dynamics within the crossover (*i.e.*, internal) storage is presented. Finally, traditional AOG and level of service (LOS), as well as the percentage of vehicles experiencing split failures and downstream blockage, are calculated for different segments of the DDI. By utilizing the presented techniques, agencies can evaluate the performance of any DDI in their jurisdiction to identify movements and time-of-day (TOD) periods that require field adjustments.

1.4. Study Contribution

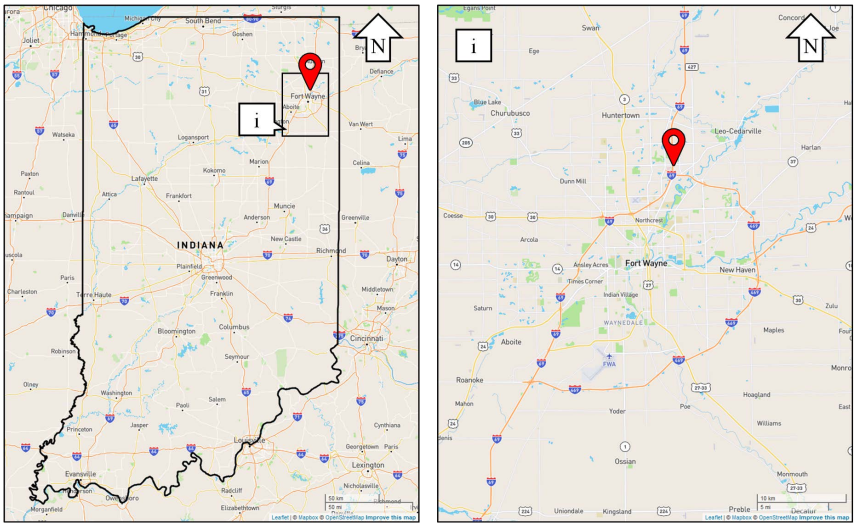
The main contribution of the study is the development of DDI-specific CV trajectory-based performance measures that can provide near-real-time assessments without the need for investing in new traffic signal infrastructure.

2. Study Location and Time Period

To demonstrate the trajectory-based performance measures techniques presented in this study, I-69 at E Dupont Rd, a DDI located in Fort Wayne IN, was analyzed from the 7th to the 11th of June, 2021 (**Figure 1**). This DDI was opened to traffic in 2014 and it has an Annual Average Daily Traffic (AADT) of 56,000 vpd on the interstate and 21,000 vpd on the crossing road.

3. Data Description

Private sector CV trajectory data for the second week of June 2021, with an



(a) (b)

Figure 1. Study location: I-69 at E Dupont Rd. (a) Indiana; (b) Fort Wayne.

estimated penetration rate of 4.6% on IN interstates from the methods presented in [29], was used in this study. The CV trajectory data consists of individual vehicle waypoints with a reporting interval of 3 seconds and a positional accuracy of a 1.5-meter radius. Every waypoint has the following attributes: Speed, heading, GPS location, timestamp, and an anonymous unique trajectory identifier. For this study, over 7 thousand trajectories and 130 thousand GPS points were analyzed.

4. DDI Performance Measures

In this section, DDI terminology, performance measures results naming format, and the proposed graphics to evaluate DDIs are introduced.

Figure 2 shows the analyzed DDI. There are crossover areas at each end of the interchange. The most critical segment of a DDI is crossover storage. If vehicles in this area fail to be discharged efficiently, delays and saturation at the approaches of the entry crossover could be significantly increased [16]. The crossover storage can receive vehicles from the external street and from the interstate exiting ramps. Therefore, the performance of both approaches and the crossover storage needs to be monitored.

When presenting DDIs' performance results, it is important to differentiate two attributes: The source of vehicles, and which crossover signal is being evaluated. To accomplish an effective differentiation of these attributes throughout the paper, the following naming format will be employed: Source Of Traffic_direction Of Travel_movement Type_intersections Crossed. Usage is as follows:

- Source Of Traffic: The source of traffic before entering the DDI. If coming from the external crossing street, represented with an *E*; if coming from the interstate's ramp, represented with an *R*.

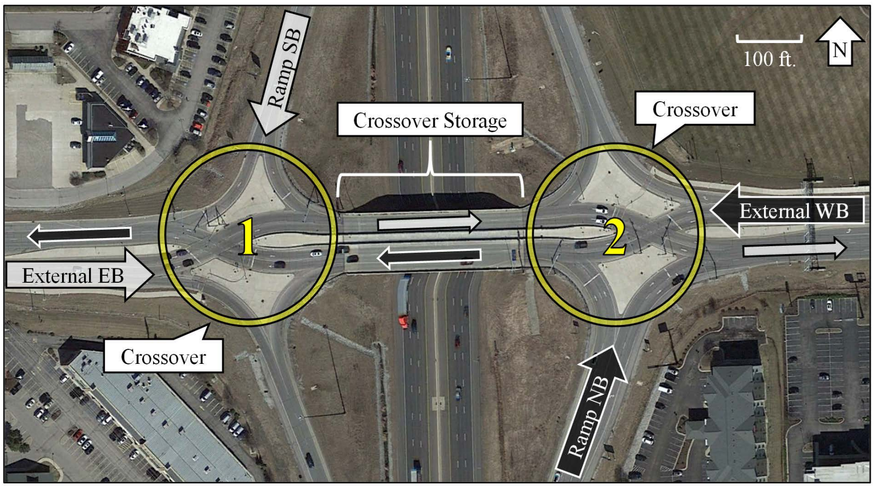


Figure 2. DDI terminology and traffic sources (Map data: Google, IndianaMap Framework Data, Maxar Technologies, U.S. Geological Survey, USDA Farm Service Agency).

- Direction Of Travel: Direction of travel before entering the DDI. Southbound (*SB*), westbound (*WB*), northbound (*NB*), and eastbound (*EB*).
- Movement Type: If through, represented with a *T*; if left, represented with an *L*.
- Intersections Crossed: Which crossover area signals were crossed for the presented results. If only signals in area 1 were crossed, then *1*; if signals in area 1 and then 2 were crossed, then *12*; if only signals in area 2 were crossed, then *2*; if signals in area 2 then 1 were crossed, then *21*.

For example, results for traffic from the NB exit ramp turning left into the DDI, to then cross traffic signals on crossover areas 2 and 1 will be labeled *R_NB_L_21*. The following sub-sections introduce the proposed graphics.

4.1. Diverging Diamond Interchange Purdue Probe Diagram

Since the signals' dynamics between crossover areas 1 and 2 (**Figure 2**) are crucial for the correct operation of DDIs, it is important to provide analytical performance measures (and graphics) that provide insight on the operation status at both locations simultaneously. To accomplish this goal, a variation of the PPD [24] [25] is proposed. A PPD shows the linear-referenced progression of vehicles relative to the far side of an intersection, color-coded by the number of stops. Usually, a PPD provides quantitative information on the experienced delay, progression, split failure, and downstream obstruction of trajectories crossing through a singular traffic signal.

By linear-referencing trajectories of vehicles traveling through both crossover areas in a DDI relative to the far side of the downstream intersection, and by color-coding each upstream trajectory segment based on the number of stops by traffic signal, the Diverging Diamond Interchange Purdue Probe Diagram (DDI PPD) can be plotted. DDI PPDs provide valuable information on the performance state at both crossover signal systems simultaneously.

Figure 3 shows DDI PPDs for the four different traffic sources shown on Figure 2. The location of the traffic signals' is shown with blue lines and labelled 1 and 2. For example, for Figure 3(a), callout i shows the far side of signal 1 and callout ii the far side of signal 2. Each trajectory's upstream segments are connected by a black line, which corresponds to vehicles moving through the traffic signal in the DDI. Additionally, a free-flow trajectory (FFT), which is the theoretical trajectory of a vehicle traveling at the speed limit, is shown with a thick black line for comparison.

In a DDI PPD, as with a traditional PPD, vehicle delay can be assessed by analyzing how far away from the FFT a trajectory approaches the first signalized intersection. The farther away from the FFT a trajectory starts, the longer the experienced delay at the DDI. AOG, a measurement of progression, can be evaluated by comparing the amount of green-colored (no-stops, arrived on green) and non-green-colored (one or more stops) trajectories. The larger the proportion of green trajectories is, the better the progression. Saturation can be assessed

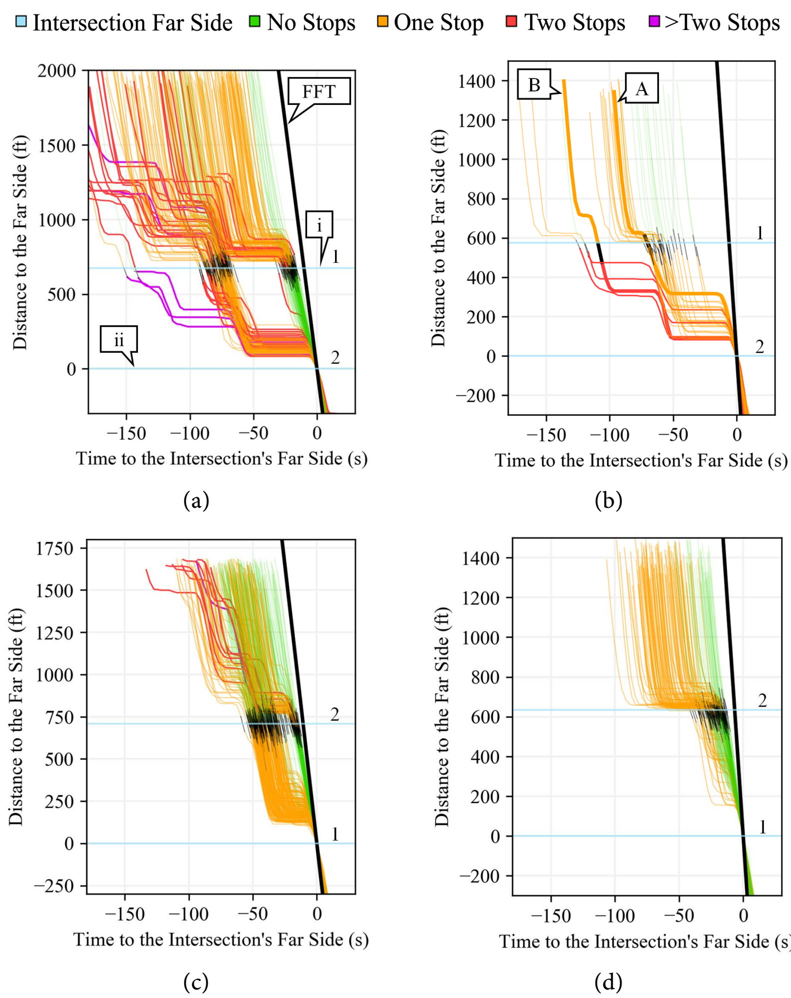


Figure 3. Diverging diamond interchange purdue probe diagrams from 16:00 to 18:00 hrs. between the 7th and 11th of June, 2021. (a) E_EB_T_12 (n = 270); (b) R_SB_L_12 (n = 35); (c) E_WB_T_21 (n = 321); (d) R_NB_L_21 (n = 162).

by identifying the number of trajectories with two or more stops at a traffic signal since those events are indicative of split failures. Finally, downstream blockage, as defined in [24], can be identified by looking at the vehicle's progression immediately after crossing the far side of an intersection (blue lines). The more delay after a vehicle crosses the far side of an intersection, the more likely downstream blockage is experienced. The following qualitative statements can be said from **Figure 3**:

- Trajectories going EB from the external street (**Figure 3(a)**) and NB from the ramp (**Figure 3(b)**) experience the most delay since they approach the intersections the farthest away from the FFT;
- **Figure 3(c)** and **Figure 3(d)** have the highest AOG, and therefore, the best progression;
- Vehicles traveling EB from the external street (**Figure 3(a)**) are experiencing split failures when approaching both, intersections 1 and 2;
- Trajectories traveling NB from the ramp (**Figure 3(b)**) are experiencing split failures when approaching intersection 2;
- Trajectories traveling WB from the external street (**Figure 3(c)**) experience split failures when approaching intersection 2.

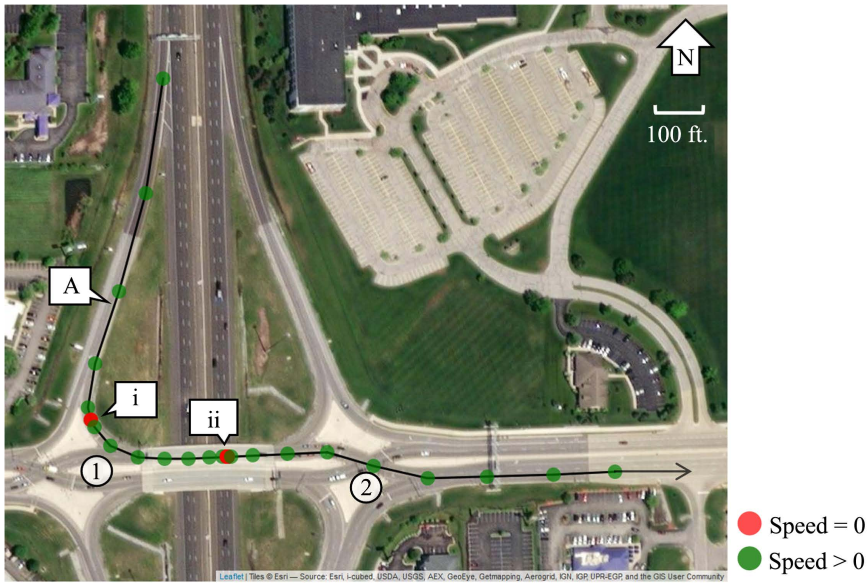
Trajectory Visualization

Figure 4 shows on the studied DDI two of the trajectories exiting from the interstate's ramp, traveling SB, and turning left, that were plotted on the DDI PPD in **Figure 3(b)**.

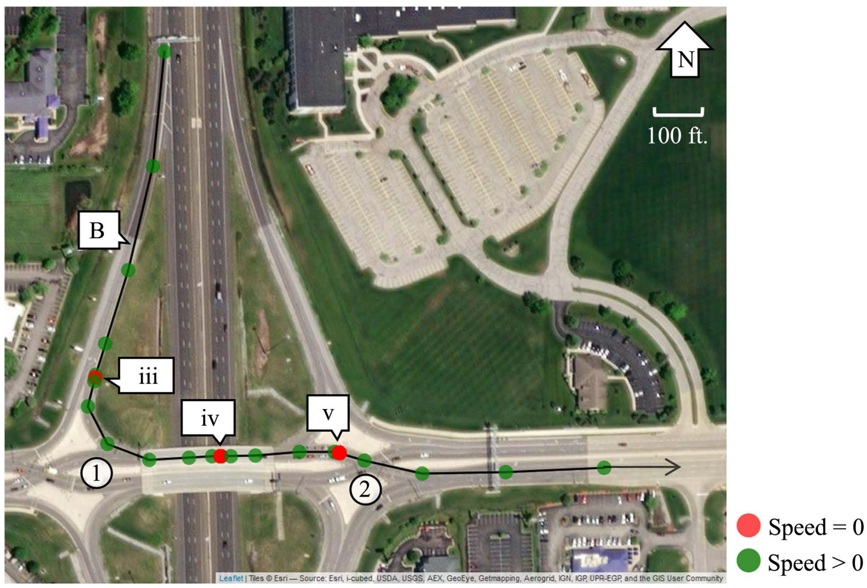
- For trajectory A, it can be seen how the vehicle approaches the traffic signal at the crossover area 1, but before it can make it through the intersection, it has to stop (**Figure 4(a)**, callout i). Then it moves to stop one more time in the middle of the crossover storage (**Figure 4(a)**, callout ii). Finally, it advances again to clear the interchange.
- Similarly, trajectory B stops once before clearing the signal on area 1 (**Figure 4(b)**, callout iii). However, once in the crossover storage, the vehicle had to stop on two different occasions (**Figure 4(b)**, callout iv and v). This is a clear case of a vehicle experiencing a split failure, which is a sign of an oversaturated approach since one cycle length of the traffic signal on area 2 did not provide enough green time to clear the queue. As previously discussed, saturation in the crossover storage needs to be avoided, because if there is queue spillback, both the external and ramp approaches would be affected (of which long queues on the ramp would be of major concern due to the possibility of rear-end crashes on the interstate).

4.2. Crossover Storage Load and Discharge

The most critical segment of a DDI is crossover storage. To facilitate the qualitative assessment of progression patterns, and to evaluate queue length for spillback in the critical interior crossover storage, a DDI PPD variation that provides information on progression by traffic source is presented.



(a)



(b)

Figure 4. R_SB_L_12 trajectories. (a) Trajectory of A (2 stops); (b) Trajectory of B (3 stops).

In this variation of the DDI PPD, trajectories coming from the external street and the interstate ramp, that share lanes on the crossover storage, are superimposed. When doing this, the progression dynamics between signals at the crossover areas 1 and 2 become apparent. Figure 5 shows a progression DDI PPD for the different movements at the study location.

For the EB through and SB left movements (Figure 5(a)), it can be seen that there is a significant number of vehicles coming from both sources stopping when approaching signal 1, as well as stopping when approaching signal 2 (callout i). Most of the traffic in this figure is from the EB through approach, and

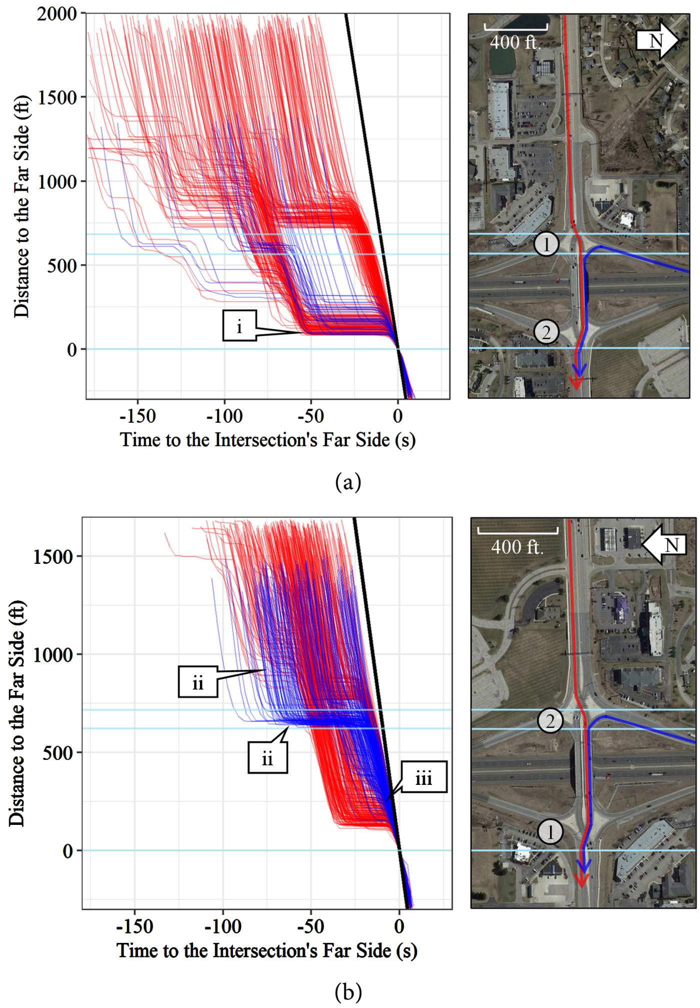


Figure 5. Progression DDI PPD from 16:00 to 18:00 hrs. between the 7th and 11th of June, 2021 (Map data: Google, Indiana Map Framework Data, Maxar Technologies, U.S. Geological Survey, USDA Farm Service Agency). (a) EB through (E_EB_T_12) and SB left trajectories (R_SB_L_12); (b) WB through (E_WB_T_21) and NB left trajectories (R_NB_L_21).

approximately 50% must stop at signal 2. In this case, the EB through and SB ramp have unbalanced. In addition, for the analyzed period, 89% of the trajectories traveled EB through, and only 11% traveled SB left.

For the WB through and NB left movements (Figure 5(b)), it can be observed that there are vehicles from both sources stopping when approaching area 1 (callout ii). However, it is shown how most vehicles coming NB from the ramp can progress without stopping through the signal at 2 (callout iii). This is an indication that the NB left movement has an effective clearance when entering the crossover storage area. Further, for the analyzed period, 66% of the trajectories traveled WB through, and 34% traveled NB left.

5. Summary Performance Measures by Time-of-Day

Apart from the performance graphics presented previously, it is useful for agencies to have graphics that can be used to rapidly understand temporal variations in the performance of all movements at a DDI. To address this need, graphics that provide a summary of performance measures, based in [24] [25], by TOD, in 15-minute periods, are provided. In these graphics, the trajectories' source is specified; further, if individual (1 or 2) or a combination (1 and 2) of traffic signals are analyzed is also indicated. Additional details on how to interpret these graphics are provided below:

- Figure 6: Percentage of sampled vehicles arriving on green. This graphic is useful when assessing the level of progression. From this figure, it is shown how some vehicles traveling SB from the ramp arrive on green at the signal at 1 (callout i), but virtually none do so at 2 (callout ii). On the other hand, some vehicles traveling NB from the ramp have to stop when approaching 2 (callout iii), but most of them progress without stopping at 1 (callout iv).

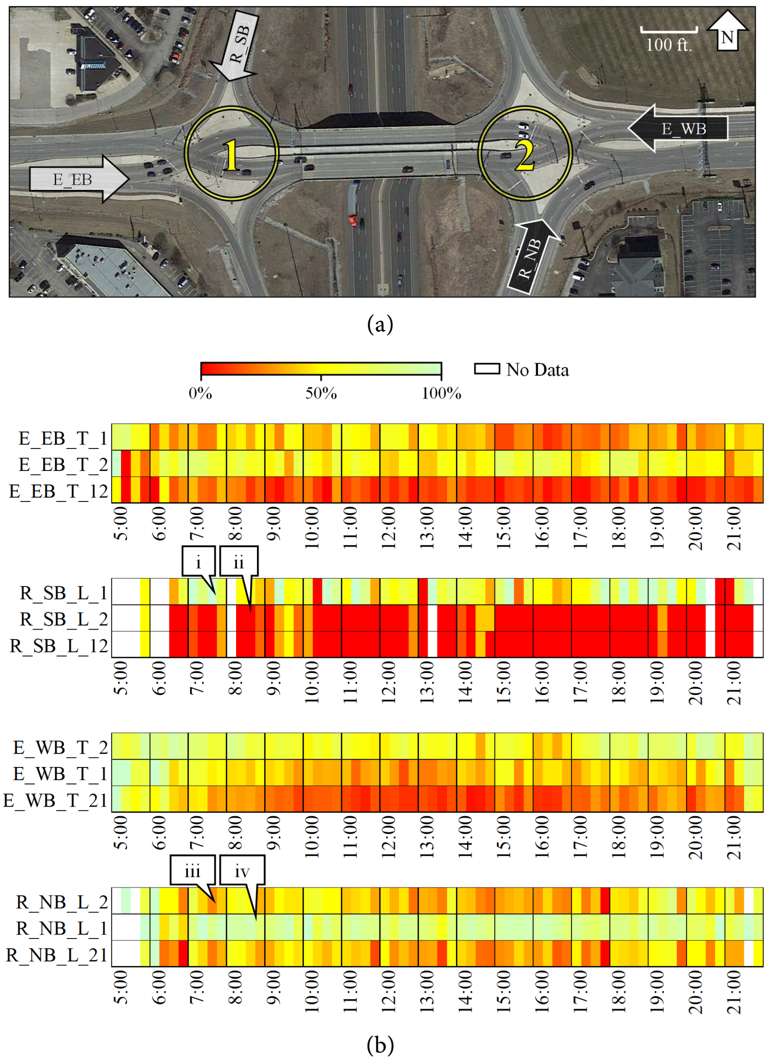
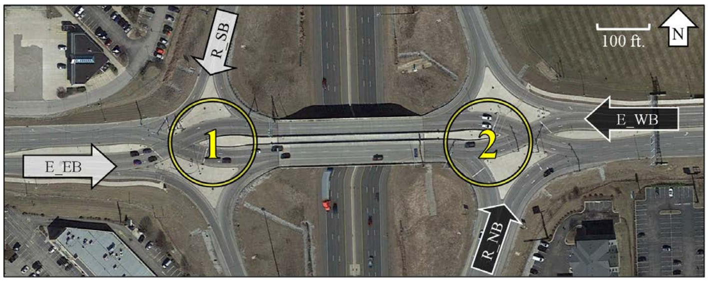
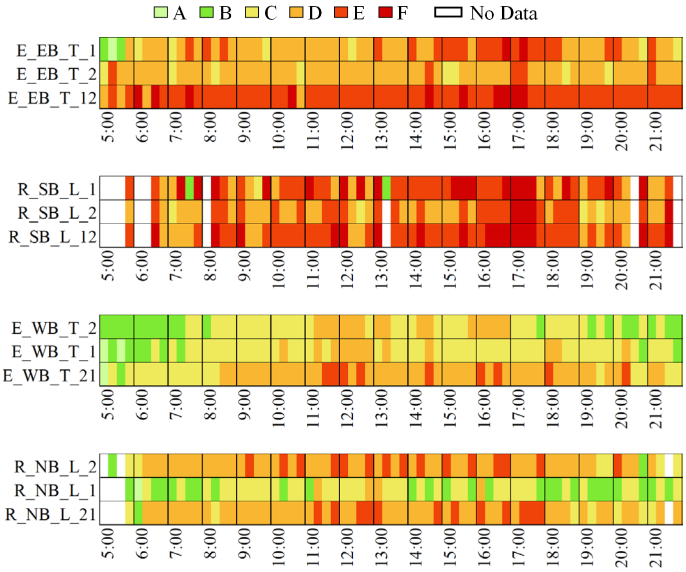


Figure 6. Arrivals on green summary results by movement from 5:00 to 22:00 hrs. between the 7th and 11th of June 2021. (a) Traffic sources (Map data: Google, IndianaMap Framework Data, Maxar Technologies, U.S. Geological Survey, USDA Farm Service Agency); (b) Arrivals on green summary results.

- Figure 7: Weighted average level of service [23]. Even if this graphic is not specifically useful for operational decisions, it provides practitioners with a standard measurement of delay by approach. The color codes used for the LOS in this graphic are based on the Highway Capacity Manual (HCM) [23]. The control delay LOS ranges are shown in Table 1. This graphic can also be adapted to provide alternative numerical scales for delay.



(a)



(b)

Figure 7. Level of service summary results by movement from 5:00 to 22:00 hrs. between the 7th and 11th of June 2021. (a) Traffic sources (Map data: Google, IndianaMap Framework Data, Maxar Technologies, U.S. Geological Survey, USDA Farm Service Agency); (b) Level of service summary results.

Table 1. HCM level of service criteria for signalized intersections [23].

Level of Service	Average Control Delay (sec/vehicle)	Description
A	≤10	Free Flow
B	>10 - 20	Stable Flow (slight delay)
C	>20 - 35	Stable Flow (acceptable delays)
D	>35 - 55	Approaching Unstable Flow (tolerable delay)
E	>55 - 80	Unstable Flow (intolerable delay)
F	>80	Forced Flow (congested and queues fail to clear)

- **Figure 8**: Percentage of sampled vehicles experiencing split failures. This graphic provides an indication of when and where are approaches operating at overcapacity. Those cases are opportunities to rebalance split time. For this performance measure, traffic signals need to be analyzed individually. For the studied location, of special concern are the TOD where vehicles traveling EB from the external street and SB from the ramp experience split failures within the crossover storage (callout i).
- **Figure 9**: Percentage of sampled vehicles experiencing downstream blockage. This graphic is useful to identify a location that is being affected by a downstream queue. For this performance measure, traffic signals need to be analyzed individually. For the studied location, it is shown how the downstream traffic signals are affecting the progression of vehicles entering the DDI traveling SB (callout i) and NB (callout ii).

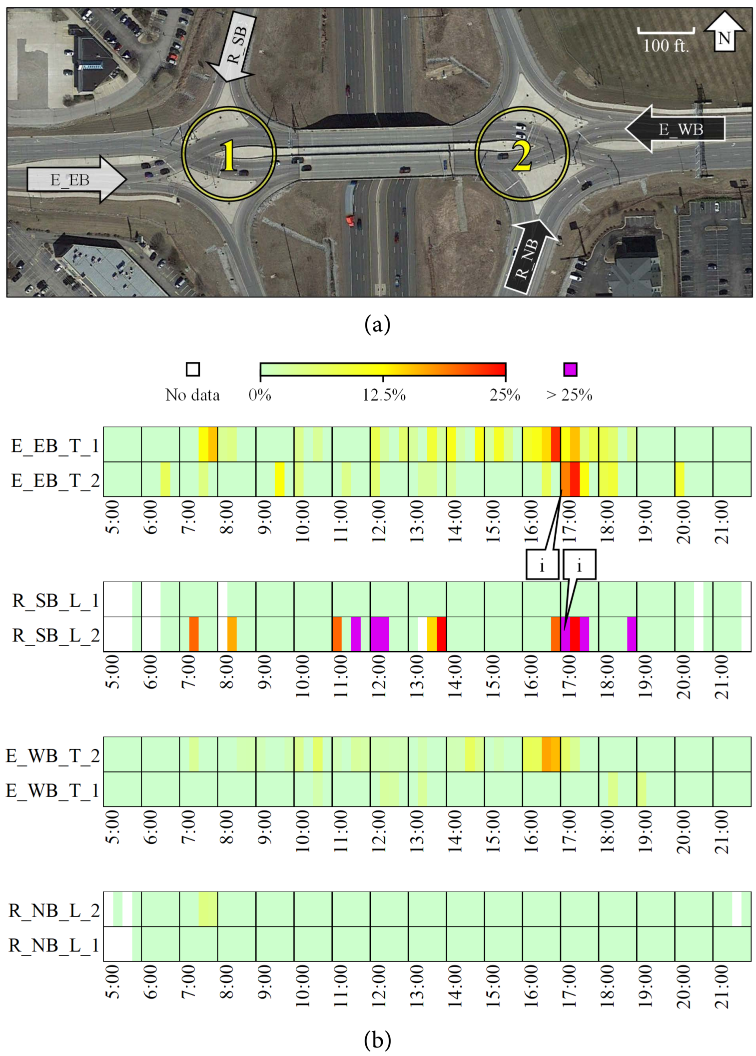


Figure 8. Split failures summary results by movement from 5:00 to 22:00 hrs. between the 7th and 11th of June 2021. (a) Traffic sources (Map data: Google, IndianaMap Framework Data, Maxar Technologies, U.S. Geological Survey, USDA Farm Service Agency); (b) Split failure summary results.

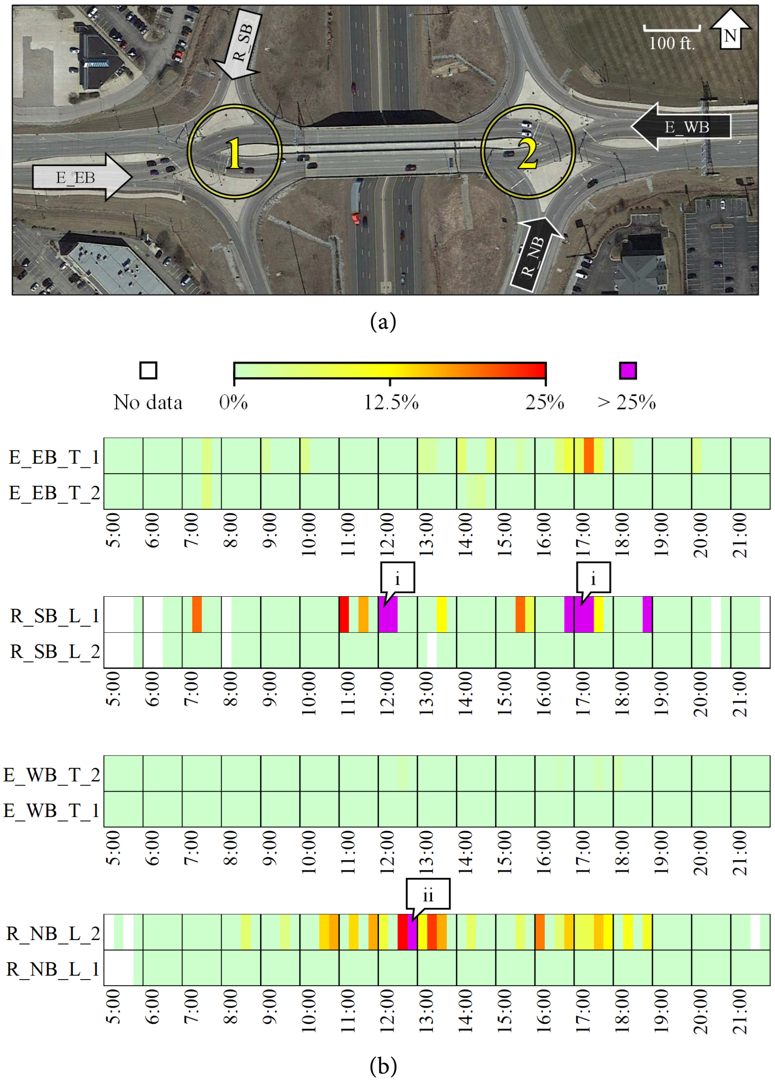


Figure 9. Downstream blockage summary results by movement from 5:00 to 22:00 hrs. between the 7th and 11th of June 2021. (a) Traffic sources (Map data: Google, IndianaMap Framework Data, Maxar Technologies, U.S. Geological Survey, USDA Farm Service Agency); (b) Downstream blockage summary results.

6. Conclusions

This study presented new techniques to assess the performance of Diverging Diamond Interchanges based on CV trajectory data with a 3-second reporting interval. To demonstrate the new methodologies, performance measures of a DDI located in Fort Wayne, IN were calculated. Over 7,000 trajectories and 130,000 GPS points were processed between the 7th and the 11th of June 2021 to generate the following:

- DDI PPD (Figure 3): A new graphic that shows the progression of vehicles coming from a particular approach throughout the entire DDI. Each segment of every crossing trajectory is color-coded based on the number of stops at every traffic signal. This visualization is useful when trying to evaluate delays, progression, and saturation.

- Progression DDI PPD (**Figure 5**): A variation of the DDI PPD that integrates trajectories coming from different approaches that share the same crossover storage. This graphic is useful when evaluating the critical queue dynamics within the crossover storage to ensure the interior crossover remains uncongested and there is no spillback.
- Traditional traffic signal performances such as arrivals on green (**Figure 6**) and level of service (**Figure 7**).
- Convenient graphics summarizing where and when critical split failure (**Figure 8**) and downstream blockage (**Figure 9**) occur.

The methodology presented in this study can be used to assess the performance at any DDI in the world where connected vehicle trajectory data is available. As the construction of DDIs increases, efficiency evaluations are needed to warrant their use and to make adjustments if necessary.

Future research will focus on proposing specialized performance measures for other alternative interchanges, such as single point urban interchanges (SPUIs), closely spaced diamond interchanges, and unsignalized J-Turns.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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