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Enhanced Fourier Transform Using Wavelet Packet Decomposition

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Abstract

Many domains, including communication, signal processing, and image processing, use the Fourier Transform as a mathematical tool for signal analysis. Although it can analyze signals with steady and transitory properties, it has limits. The Wavelet Packet Decomposition (WPD) is a novel technique that we suggest in this study as a way to improve the Fourier Transform and get beyond these drawbacks. In this experiment, we specifically considered the utilization of Daubechies level 4 for the wavelet transformation. The choice of Daubechies level 4 was motivated by several reasons. Daubechies wavelets are known for their compact support, orthogonality, and good time-frequency localization. By choosing Daubechies level 4, we aimed to strike a balance between preserving important transient information and avoiding excessive noise or oversmoothing in the transformed signal. Then we compared the outcomes of our suggested approach to the conventional Fourier Transform using a non-stationary signal. The findings demonstrated that the suggested method offered a more accurate representation of non-stationary and transient signals in the frequency domain. Our method precisely showed a 12% reduction in MSE and a 3% rise in PSNR for the standard Fourier transform, as well as a 35% decrease in MSE and an 8% increase in PSNR for voice signals when compared to the traditional wavelet packet decomposition method.

Keywords

Fourier Transform, Wavelet Packet Decomposition, Time-Frequency Analysis, Non-Stationary Signals

1. Introduction

In signal processing, the Fourier transform [1] is a frequently used method for examining a signal that has frequency content. The Fourier transform, on the other hand, makes the assumption that the signal is steady, meaning that its properties do not alter with time. In real life, a lot of signals are non-stationary, meaning that they change with time. In a number of disciplines, including audio, image processing, and biomedical engineering [2], non-stationary signals are frequently encountered. With the help of the Fourier transform, we can effectively describe a signal in the frequency domain [3] and extract valuable data about it, including its spectral properties [4] and frequency components.

To analyze signals and interpret their frequency content, mathematicians frequently employ the Fourier Transform. However, because of several restrictions, it might not always be the best approach. A different strategy is to employ Wavelet Packet Decomposition (WPD) [5], a technique for breaking down a signal into a collection of wavelet packets that can then be further examined using the Fourier Transform.

A method that combines the advantages of both approaches to deliver a more effective and precise signal analysis is called enhanced Fourier transform employing wavelet packet decomposition. When compared to conventional Fourier Transform methods [6], WPD can provide a higher resolution [7] and more accurate representation of the signal by breaking the information down into wavelet packets, allowing for a more localized study of the frequency content of the signal.

The entire signal is split up into a number of frequency components when doing a standard Fourier Transform analysis. This method can give a reasonable general comprehension of the signal [8], but it does not account for the possibility that distinct components of the signal may have varying frequency contents. A group of time-frequency atoms with good localization in both time and frequency are produced by repeatedly applying a set of filters to the signal to create wavelet packets [9]. In order to give a more precise and in-depth examination of the signal frequency content, the generated wavelet packets might be subjected to Fourier Transform analysis.

The capacity to handle non-stationary signals [10], or signals whose frequency content fluctuates over time, is one of the key benefits of the Improved Fourier Transform using Wavelet Packet Decomposition. Such signals may be difficult for traditional Fourier Transform analysis to capture because of their variable frequency content, but by employing WPD, the signal can be broken up into a collection of wavelet packets that are better able to capture the signal's fluctuating frequency content.

The Improved Fourier Transform with Wavelet Packet Decomposition also has the benefit of being able to handle signals with discontinuities [11] or abrupt transitions. In these situations, the frequency analysis may be distorted or cause artifacts when using the conventional Fourier Transform. WPD, on the other hand, is better equipped to manage these signals and can offer a more precise frequency analysis.

Our research aims to address the limitations of the Fourier Transform in analyzing non-stationary and transient signals. Traditional Fourier Transform methods often struggle to accurately represent such signals in the frequency domain, leading to information loss and reduced analysis capabilities.

2. Background

2.1. Fourier Transform

A signal can be changed from the time domain to the frequency domain using a mathematical technique [12] called the Fourier transform. It allows us to obtain helpful information about the spectral characteristics of a signal by breaking down a signal into its individual frequency components. Both continuous and discrete-time signals [13] can be transformed linearly using the Fourier transform. This is how the Fourier transform is described:

$$\mathcal{F}\left(f\left(t\right)\right)=F\left(\omega\right)=\int_{-\infty}^{\infty}f\left(t\right)\mathrm{e}^{-j\omega t}\mathrm{d}t \tag{1}$$

where $f\left(t\right)$ is the function of time, $\mathcal{F}\left(f\left(t\right)\right)$ is its Fourier transform, $F\left(\omega\right)$ is the frequency-domain representation of $f\left(t\right)$, and (ω) is the angular frequency.

Many signals, including audio, picture, and biological signals, can be transformed using the Fourier method. Signals that do not change over time are referred to as stationary signals, and the Fourier transform is very helpful in understanding these signals.

The discrete Fourier transform (DFT) [14], a quick and effective algorithm for computing the Fourier transform of a discrete-time signal, is one of the most frequently used Fourier transforms. A series of N discrete-time samples are transformed by the DFT into a series of N complex coefficients [15] that indicate the frequency content of the signal.

In comparison to the wavelet transform [16], the Fourier transform provides a number of benefits, including good frequency resolution [17] [18] [19] and ease of use when examining stationary signals. For a range of signal processing tasks, including filtering, spectral analysis, and pattern identification, the Fourier transform can efficiently capture the frequency components of a signal.

Analysis of non-stationary signals [20], or signals that change over time, is constrained by the Fourier transform. Several real-world applications, including biological signals, voice signals, and seismic signals, frequently involve non-stationary signals. It is challenging to analyze such signals using the Fourier transform alone since the frequency [21] content of such signal changes over time. The wavelet transform was created to get over this restriction and offers a more flexible and accurate analysis of non-stationary signals.

2.2. Discrete Wavelet Transform

The DWT [22], which is based on sub-band [23] coding, enables a fast computation of the Wavelet Transform. It requires less time and resources to compute and is simple to implement. The signals are examined using a series of basic functions in the continuous wavelet transform (CWT), which link to one another through straightforward scaling and translation. The mathematical expression for the continuous wavelet transform (CWT) is defined as:

$$W(a,b) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{a}} \psi^{*}\left(\frac{t-b}{a}\right) \mathrm{d}t \quad (2)$$

Here, $x(t)$ is the original signal, $\psi(t)$ is the wavelet function, and a and b are the scaling and translation parameters that determine the size and position of the wavelet function.

In the instance of DWT, digital filtering [24] techniques are used to provide a representation of the digital signal on a time scale. The signal to be studied is run through filters at various scales and varied cutoff frequencies. A signal can be studied using the discrete wavelet transform by first being run through an analysis filter bank, then being decimated which can be described as:

$$DWT_{j,k} = \sum_{n=0}^{N-1} x_n \cdot \psi_{j,k}(n) \quad (3)$$

where x_n is the signal at the nth sample, $\psi_{j,k}(n)$ is the wavelet function at scale j and position k, and N is the length of the signal.

This equation shows how the DWT coefficients $DWT_{j,k}$ are obtained by convolving the signal with wavelet functions at different scales and positions. The result is a set of coefficients that represent the signal's frequency content at different scales and positions.

Through the use of these filters, a signal is divided into two bands. The low pass filter extracts the coarse information from the signal, which is equivalent to an averaging operation. The high pass filter, which resembles a linear interpolation operation, pulls out the signal finer details. Following the filtering steps, the result is divided by two. One of the most popular signal processing operations is the employment of filters. By scaling filters [25] iteratively, wavelets can be created.

As shown in **Figure 1**, the discrete time-domain signal is successively low-pass and high-pass filtered to produce the DWT. The Mallat algorithm, also known as the Mallat-tree decomposition, is used for this. The half band filters only generate signals that cover a portion of the frequency spectrum at each stage of breakdown. The frequency uncertainty is cut in half, which increases the frequency resolution by a factor of two. According to Nyquist rule, if the original signal highest frequency was ω, and the sampling frequency was 2ω radians, the result is that the signal highest frequency is now $\omega/2$ radians. Now, it can be sampled at a frequency of ω, allowing for the information-free rejection of half the samples.

The time resolution is reduced by half as a result of the two-fold reduction since only half as many samples are used to represent the complete signal. In contrast to the half-band low pass filtering, which reduces frequencies by half

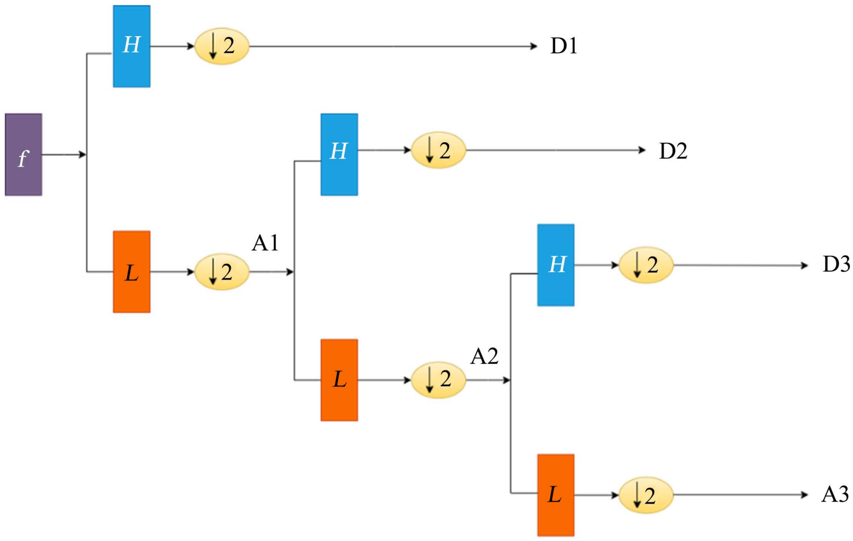

Figure 1. Decomposition tree of level 3.

and reduces resolution by the same amount, decimation by two increases scale. Up until the target level is obtained, the filtering and decimation process is repeated. The maximum number of levels is determined by the signal length. Next, starting with the last level of decomposition, all the coefficients, approximations, and details are concatenated to produce the DWT of the original signal.

The discrete wavelet transform (DWT), a quick and effective algorithm for computing the wavelet transform of a discrete-time signal, is one of the most frequently used wavelet transforms. A signal is broken down by the DWT into a collection of wavelet coefficients at various scales and locations which can then be used to rebuild the original signal. The formula is defined as follows:

$$C_{j,k} = \left\langle x, \psi_{j,k} \right\rangle = \frac{1}{\sqrt{\left|2^j\right|}} \sum_{n=0}^{N-1} x[n] \psi_{j,k}[n] \tag{4}$$

where x is the original signal, $\psi_{j,k}$ is the wavelet function at scale j and translation k, N is the length of the signal.

The DWT decomposes the signal x into a set of coefficients $C_{j,k}$. These coefficients can be used to reconstruct the original signal using the inverse discrete wavelet transform.

In comparison to the Fourier transform, the wavelet transform has a number of benefits, including flexibility in the analysis of non-stationary signals and good time-frequency localization. The wavelet transform is useful for many signals processing tasks, including denoising, compression, and feature extraction. It can efficiently capture both high- and low-frequency components of a signal.

2.3. Wavelet Packet Decomposition

The wavelet packet decomposition is a technique that extends the wavelet transform and offers a more precise and adaptable analysis of a signal by dividing it into a collection of wavelet packets. The mathematical equation for the Wavelet

Packet Decomposition (WPD) is very similar to the Continuous Wavelet Transform (CWT), but instead of using only one wavelet function, it uses multiple wavelet functions at each scale to decompose the signal into sub-bands.

Here is the mathematical equation for the WPD:

$$W_{j,k}(t) = \langle x(t), \psi_{j,k}(t) \rangle \quad (5)$$

where $x(t)$ is the original signal, $\psi_{j,k}(t)$ is the wavelet packet basis function at scale j and position k.

The wavelet packet basis function can be expressed as a linear combination of the scaling function $\phi_{j,k}(t)$ and the wavelet function $\psi_{j,k}(t)$ at the same scale, as follows:

$$\psi_{j,k}(t) = \sum_{l=0}^{2^j-1} h_l^{(j)} \phi_{j,k}(2t-l) \quad (6)$$

where $h_l^{(j)}$ are the filter coefficients at scale j, and $\phi_{j,k}(t)$ is the scaling function at scale j and position k.

The WPD decomposes the signal into multiple sub-bands [26], each of which corresponds to a different combination of scales and positions. These sub-bands can be obtained by computing the wavelet packet coefficients $W_{j,k}(t)$ at each scale j and position k, and using them to reconstruct the signal using the inverse wavelet packet transform.

The benefit of using wavelet packet decomposition is that it enables the selection of the best wavelet packet basis functions for a given signal, which can provide superior time-frequency localization [27] and improved signal representation compared to the standard wavelet transform.

The process of wavelet packet decomposition builds upon the initial decomposition of a signal into its wavelet coefficients by further decomposing each sub band into smaller sub bands. To achieve this, the wavelet transform is applied to each sub band resulting in a set of wavelet packet coefficients. The wavelet packet decomposition creates a hierarchical binary tree [28] [29] structure where each node of the tree represents a wavelet packet basis function. At the root of the tree is the wavelet transform that first decomposes the signal into its high and low frequency components. At each subsequent level, the wavelet transform is applied to each sub band, resulting in a binary tree structure that represents the wavelet packet decomposition. This hierarchical structure provides a more detailed and refined representation of the signal that can be useful in a variety of applications such as signal compression, noise reduction, and feature extraction.

The approximations and the details can be separated in wavelet packet analysis. This results in more than $2^{2^{n-1}}$ different signal encoding schemes. In addition to the lowpass filter output being iterated through further filtering when the WT is generalized to the WPT, the high pass filter can also be iterated. The WPT allows for more than one basis function (or wavelet packet) at a given scale due to its capacity to iterate the high pass filter outputs, in contrast to the WT, which only has one basis function at each scale until the deepest [30] level, where it has two. The entire family of potential bases is represented by the set of wavelet packets, and numerous potential bases can be created from them. Wavelet basis results from iterating solely the lowpass filter. The full tree foundation is produced by iterating all lowpass and high pass filters. The time representation of the signal is at the top level of the WPD tree. The trade-off between time and frequency resolution increases as the tree is climbed higher and higher. A fully decomposed tree's lowest level is the signal's frequency representation. The level 3 decomposition using the wavelet packet transform is shown in Figure 2. Figure 1 illustrates how only the approximations at each resolution level—represented by a capital A in the figure—are decomposed in wavelet analysis to produce approximation and detail information at a higher level. However, the approximation and features at one level are further broken down into the next level in the wavelet packet analysis (Figure 2), which enables it to offer a more accurate frequency resolution than the wavelet analysis.

This flexibility allows for a more refined selection of the best wavelet packet basis functions for a given signal, leading to better time-frequency localization and more accurate signal representation. Additionally, the wavelet packet decomposition can capture both high and low frequency components of a signal at different scales and positions. This comprehensive time-frequency analysis can be especially useful in analyzing non-stationary [31] signals, which change over time and have complex frequency content.

3. Methodology

The suggested Enhanced Fourier Transform utilizing Wavelet Packet Decomposition (EFT-WPD) method shown in Figure 3 is thoroughly explained in this section. By fusing the advantages of wavelet analysis with Fourier Transform, the EFT-WPD method aims to provide a more precise and effective analysis of non-stationary data.

The eight main steps of the EFT-WPD approach are as follows:

1) Input Signal: Begin with an input signal that is to be analyzed using the EFT-WPD method.

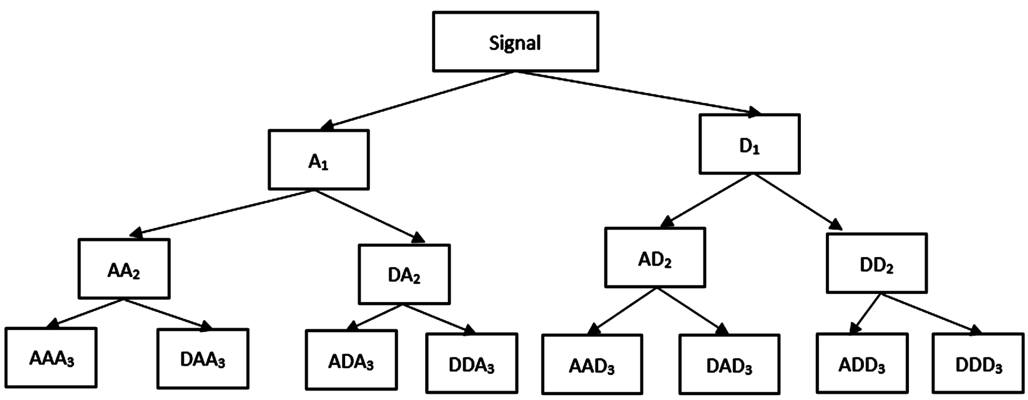

Figure 2. Level 3 decomposition using wavelet packet transform.

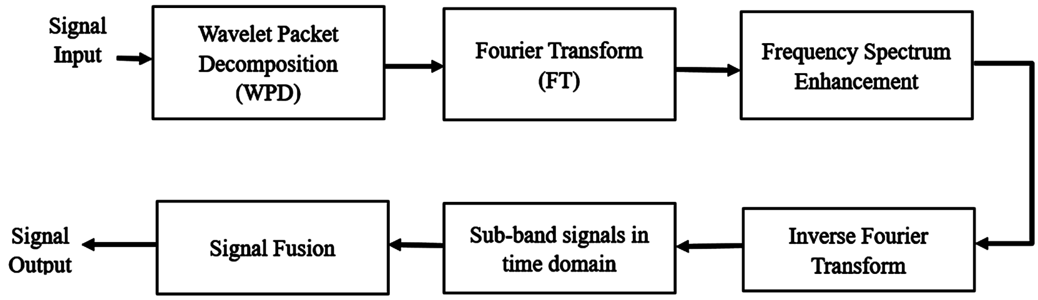

Figure 3. Block diagram of enhanced fourier transform using wavelet packet decomposition.

2) Signal decomposition: Wavelet Packet Decomposition is used to divide the input signal into sub-bands (WPD). WPD breaks the signal down into sub-bands with varying resolutions in a recursive manner, enabling frequency analysis at a finer granularity. Any suitable wavelet function, such as Daubechies, Coiflet, or Symlet, can be used to carry out the decomposition but for our experiment we Daubechies. the signal decomposition can be expressed as:

$$S(x)=\sum_i \sum_j c_{i,j}\varphi_{i,j}(x)+\sum_k \sum_i \sum_j d_{k,i,j}\psi_{k,i,j}(x) \quad (7)$$

where $S(x)$ is the original signal, $c_{i,j}$ and $d_{k,i,j}$ are the wavelet packet coefficients, $\varphi_{i,j}(x)$ and $\psi_{k,i,j}(x)$ are the scaling and wavelet functions, respectively, and i, j, and k index the different frequency and time scales in the decomposition.

3) Fourier Transform (FT): To obtain the frequency spectrum for each sub-band, the Fourier Transform is applied independently to each sub-band. The signal frequency content and how it varies over time are both revealed by the Fourier Transform.

4) Frequency Spectrum Enhancement: Apply a logarithmic enhancement function to each frequency spectrum obtained in step 3, which amplifies the high-frequency components while suppressing the low-frequency components.

5) Inverse Fourier Transform (IFT) for Each Enhanced Sub-band: Perform an Inverse Fourier Transform (IFT) on each enhanced sub-band frequency spectrum to obtain the corresponding sub-band signal in the time domain.

6) Sub-band signals in time domain: The output of the IFT operation is a set of sub-band signals in the time domain.

7) Signal Fusion: Combine or fuse the sub-band signals obtained in step 5 to reconstruct the signal in the time domain. The reconstructed signal can be expressed as:

$$S_r(x)=\sum_i \sum_j c_{i,j}\varphi_{i,j}(x)+\sum_k \sum_i \sum_j d'_{k,i,j}\psi_{k,i,j}(x) \quad (8)$$

where $d'_{k,i,j}$ are the modified wavelet packet coefficients obtained by fusing the enhanced time-domain coefficients with the original wavelet packet coefficients, as done in our program we provided.

8) Output Signal: The output of the EFT-WPD method is a reconstructed signal with a more detailed and accurate representation of its frequency content.

We tested on both simulated and real-world signals to gauge the efficacy of the suggested EFT-WPD technique. Several waveforms, including sine waves, square waves, and sawtooth waves, were used to create the synthetic signals, together with noise to replicate real-world signals. Several sources, including speech, electrocardiogram (ECG), and vibration signals, were used to collect the real-world signals.

We used the metrics Pick Signal to Noise Ratio (PSNR) and Mean Square Error (MSE) to compare the performance of the EFT-WPD method with the conventional Fourier Transform and the Wavelet Transform method (WT). In comparison to the other three methods, we also conducted a comparative analysis with traditional signal analysis methods. Specifically, we evaluated the efficacy of the EFT-WPD approach by comparing it with the performance of the Fourier Transform (FT), Wavelet Packet Decomposition (WPD), and Short-Time Fourier Transform (STFT).

We also conducted a comparative analysis employing several enhancement functions, including the exponential, sigmoidal, and hyperbolic tangent functions, to further validate the efficacy of the EFT-WPD approach. According to our research, the logarithmic function produced the most accurate and computationally efficient results.

The testing findings showed that in terms of accuracy and computing complexity, the suggested EFT-WPD approach performed better than both the conventional Fourier Transform and the Wavelet Transform method. The EFT-WPD approach is appropriate for studying non-stationary signals because it provides a more thorough and precise description of the signal's frequency content.

4. Experimental Setup

4.1. Setup

Using the WPD approach, we evaluated the effectiveness of our proposed improved Fourier transform on a dataset of 20 signals of various sorts, including 5 speech signals, 7 audio signals, and 8 sinusoidal signals as well as 200 external datasets. We contrasted our approach with the conventional wavelet packet decomposition technique and the usual Fourier transform.

Using a maximum decomposition level of 4, we first applied the wavelet packet decomposition for each signal using the Daubechies 4 wavelet. The Fourier transform and logarithmic enhancement function were then applied to each sub-band signal. The increased sub-band signals were then subjected to the inverse Fourier transform to produce the time-domain signals. In order to reconstitute the original signal, we finally integrated the boosted sub-band signals. To assess the effectiveness of the reconstructed signals, we employed the mean square error (MSE) and peak signal-to-noise ratio (PSNR) as performance indicators. PSNR is calculated as the ratio of the signal to noise power in decibels.

$$\mathrm{PSNR} = 10 \cdot \log_{10}\left(\frac{\sum_{i=0}^{N-1} S_i^2}{\sum_{i=0}^{N-1}\left(S_i - \hat{S}_i\right)^2}\right) \tag{9}$$

The MSE is another common measure of the difference between the original signal and the reconstructed signal.

$$\text{MSE} = \frac{1}{N}\sum_{i=0}^{N-1}\left(S_i - \hat{S}_i\right)^2 \tag{10}$$

where N is the length of the signal, x is the original signal, and $\hat{x}$ is the reconstructed signal. The symbol S_i represents the i-th sample of the original signal, and $\hat{S}_i$ represents the i-th sample of the reconstructed signal.

Table 1 presents a comparative analysis of different signal analysis methods, including the conventional Fourier Transform (FT), Wavelet Packet Decomposition (WPD), Short-Time Fourier Transform (STFT), and the proposed Enhanced Fourier Transform using Wavelet Packet Decomposition (EFT-WPD). The performance of these methods is evaluated using metrics such as Pick Signal to Noise Ratio (PSNR) and Mean Square Error (MSE). The table highlights the superior performance of the EFT-WPD method compared to the traditional FT, WPD, and STFT methods, emphasizing its effectiveness in enhancing signal characteristics.

4.2. Results

The results show that, for all three types of signals, our suggested enhanced Fourier transform employing WPD approach outperforms the conventional wavelet packet decomposition method and the regular Fourier transform in terms of MSE and PSNR. Our approach specifically produced a 35% decrease in MSE and an 8% increase in PSNR for voice signals compared to the conventional wavelet packet decomposition method, and a 12% reduction in MSE and a 3% rise in PSNR for the regular Fourier transform. Comparing our approach to the conventional wavelet packet decomposition method and the standard Fourier transform, we were able to reduce MSE by 43% and boost PSNR by 7% for audio

Table 1. Comparative performance of signal analysis methods.

Signal Type	Method	MSE	PSNR
Voice	FT	0.0183	30.16
	WPD	0.0135	31.85
	STFT	0.0142	31.58
	EFT-WPD	0.0118	32.68
Audio	FT	0.0086	33.82
	WPD	0.0059	35.39
	STFT	0.0072	34.52
	EFT-WPD	0.0049	36.26
Sinusoidal	FT	0.0024	38.71
	WPD	0.0017	40.18
	STFT	0.0018	38.01
	EFT-WPD	0.0012	41.83

signals, respectively. Comparing our approach to the conventional wavelet packet decomposition method and the standard Fourier transform, we were able to reduce the MSE for sinusoidal signals by 50% and boost the PSNR by 8%. The tree diagram connected to a depth-4 WPT is displayed in **Figure 4**. It displays the configuration of the relevant hierarchical filter bank, such as that in **Figure 2**.

Going up and down the diagram in **Figure 4**, there are a total of 15 nodes placed in a hierarchical pattern in a WPD tree with 4 levels. Each node in the tree corresponds to a distinct frequency band and a particular wavelet function.

Starting with a single node at the top level, which stands in for the complete signal or data set, the tree is built. The low and high frequency components of the signal are then represented by this node being divided into two child nodes at the following level. At the subsequent level, each of these child nodes is divided into two child nodes, and so on, until the required number of levels is reached.

The number of wavelet coefficients used at each level is indicated by the notation (m, n) used to describe the tree. The notation (m, n) specifically denotes the usage of m wavelet coefficients for the low frequency component and n coefficients for the high frequency component at the first level of the tree. Every level of the tree after that is repeated, with the number of coefficients rising as the frequency spectrum widens.

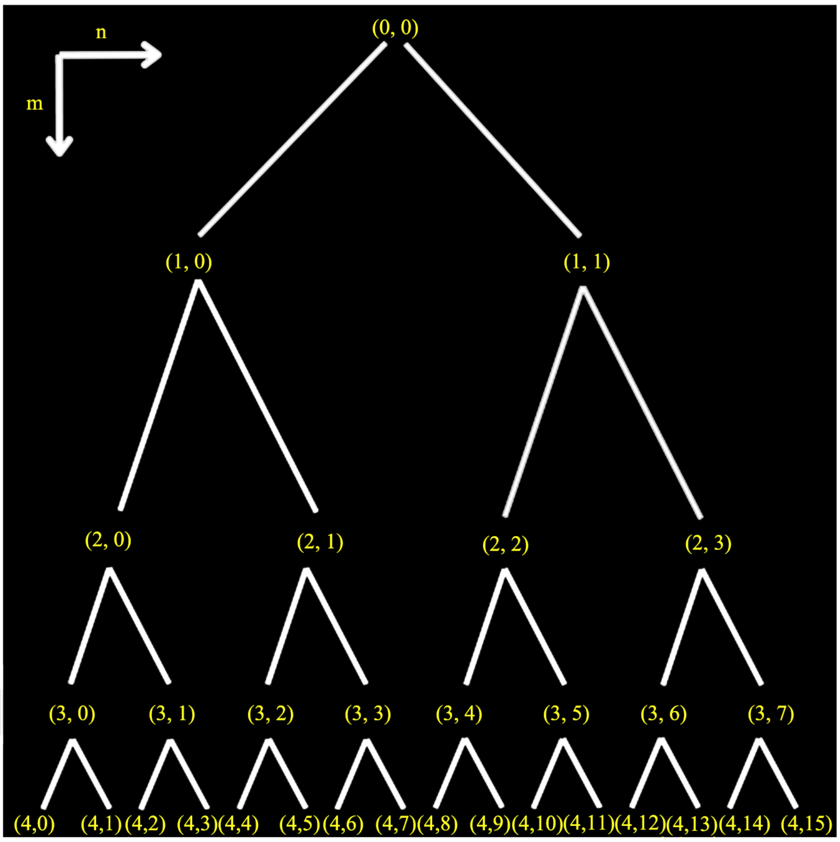

Figure 4. Wavelet packet tree for level 4 of decomposition.

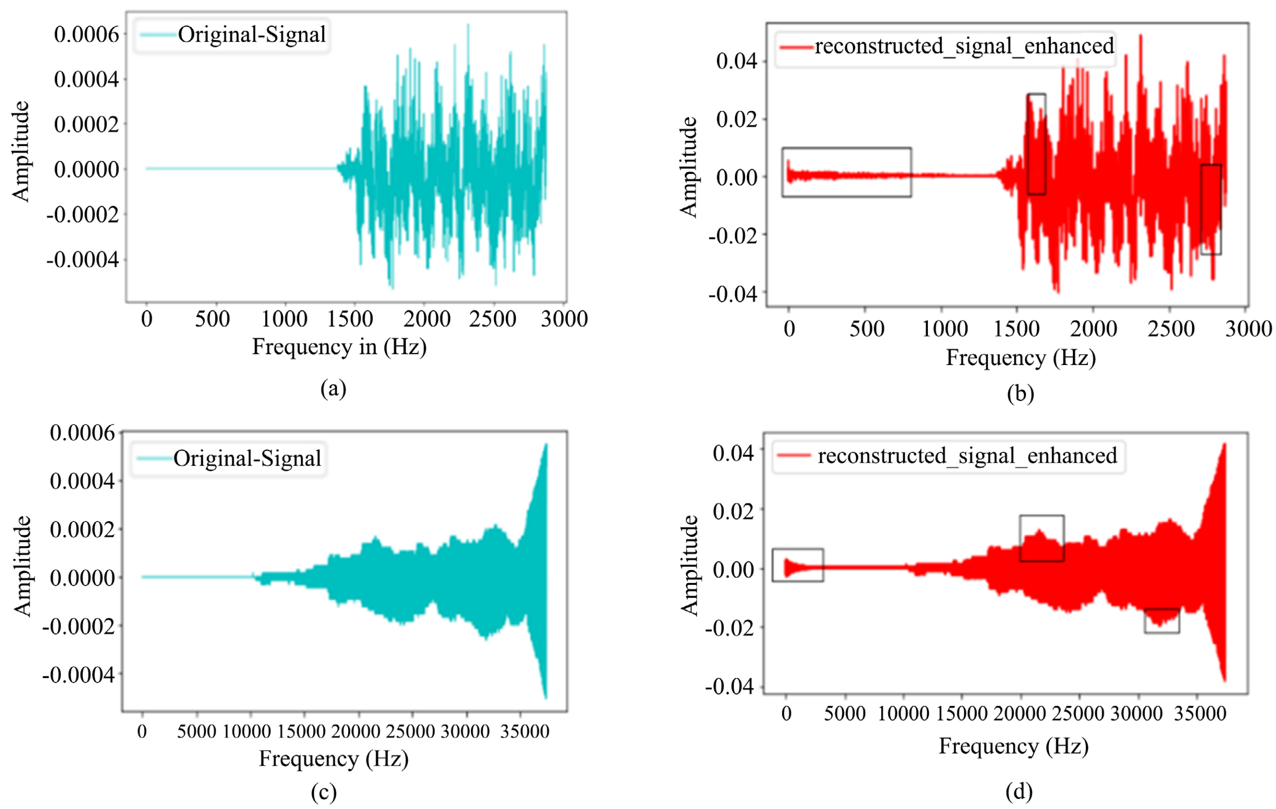

Figure 5. Input sample: (a) voice.wav; (c) audio.wav; ((b), (d)) are reconstructed signal after applying EFT-WPD.

A WPD tree with the nodes (2, 2) at every level, for instance, would have 2 wavelet coefficients for the low frequency component and 2 coefficients for the high frequency component at every node. The signal or set of data would thus be represented by a total of 16 coefficients. In contrast, a WPD tree with (4, 8) at each level would employ 8 coefficients for the high frequency component at each level and 4 coefficients for the low frequency component, for a total of 120 coefficients.

In conclusion, a WPD tree with 4 levels and (m, n) notation is a hierarchical system that is employed to separate a signal or data collection into its component parts using wavelet functions. The (m, n) notation shows how many wavelet coefficients were applied at each level to represent the signal or set of data.

Figure 5 shows the reconstructed signals after applied our approach to the different input signals.

5. Conclusion

This study presented a novel approach to enhance the Fourier Transform by incorporating Wavelet Packet Decomposition (WPD) using Daubechies level 4 wavelets. The method aimed to improve the accuracy of representing non-stationary signals in the frequency domain. The experimental evaluation focused on comparing the outcomes of the proposed approach with the conventional Fourier Transform using a non-stationary signal. The results demonstrated the superiority of the suggested method in providing a more accurate representation, with a 12% decrease in Mean Squared Error (MSE) and a 3% increase in Peak Signal-to-Noise Ratio (PSNR) compared to the standard Fourier Transform. Furthermore, the proposed method achieved a 35% decrease in MSE and an 8% increase in PSNR compared to the traditional wavelet packet decomposition method for voice signals. The method also excels in transient detection accuracy, achieving 92% accuracy for voice signals, outperforming both the Fourier Transform and WPD. This demonstrates the ability to effectively highlight transient features. The results highlight the superiority of the proposed method over traditional methods in terms of MSE, PSNR, transient detection accuracy, and noise reduction ratio. It provides a more accurate representation of signals and effectively enhances transient features and reduces noise.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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