Journal of Sensor Technology, 2023, 13, 1-11
https://www.scirp.org/journal/jst
ISSN Online: 2161-1238
ISSN Print: 2161-122X

Modeling, Simulation and Experimental Studies of Refractometric Fiber Optic Sensor

Supriya S. Patil[1*], Arvind D. Shaligram[2]

[1]Department of Electronic Science, Abasaheb Garware College, Pune, India
[2]Department of Electronic Science, Savitribai Phule Pune University, Pune, India
Email: *patilss121@gmail.com

How to cite this paper: Patil, S.S. and Shaligram, A.D. (2023) Modeling, Simulation and Experimental Studies of Refractometric Fiber Optic Sensor. *Journal of Sensor Technology*, **13**, 1-11.
https://doi.org/10.4236/jst.2023.131001

Received: January 4, 2023
Accepted: March 28, 2023
Published: March 31, 2023

Abstract

Refractometric fiber optic sensors have a number of applications in industry due to advantages like remote sensing ability, compact size, easy to fit, etc. A refractometric sensor contains a pair of parallel fibers and a gap between the sensor probe and reflector, wherein the liquid whose refractive index is to be measured is filled. This paper describes the importance of mathematical modeling of this sensor. Ray tracing approach is used to model the sensor mathematically. This mathematical model is generalized for any scenario which is useful to avoid tedious trial and error techniques to design the sensor prototype. Mathematical modelling is a useful tool to optimize the gap distance for a detection of refractive index of liquid. The model is developed and analyzed rigorously considering adulteration of diesel by kerosene where refractive index varies from 1.44 to 1.46. Simulation experiments are carried out to optimize the gap distance which is found to be 6.8 mm using both models. Experiments are carried out where sensor probe is fabricated and results are analyzed. It is observed that for suggested gap distance sensor output varies almost linear over the entire range.

Keywords

Refractometric Fiber Optic Sensor, Mathematical Modeling, Ray Tracing Technique

1. Introduction

Fiber-based refractive index (RI) sensors have been widely used. They are very compact in size, need only a small sample volume and are used in remote sensing. Refractometric fiber optic sensor consists of sensor probe with a pair of parallel fibers and a reflector is placed at a distance. A liquid whose refractive index is to be measured is filled in the gap between the sensor probe and reflector. Light is launched onto the mirror through the incoming fiber and part of the reflected light is detected by the outgoing fiber and the photodiode [1]. Refractometric extrinsic FOS was developed by Choudhari *et al.* [2] for the measurement of refractive index of liquids based on the reflective intensity modulation.

This paper gives detailed theoretical analysis of refractometric extrinsic type fiber optic sensors. A mathematical model was developed based on the reported analytical method and a novel ray tracing method [3]. It consists of two parallel fibers, one transmitting and other receiving fiber, thus forming a sensor probe with a reflector at a distance. The gap between the sensor probe and reflector is filled with liquid whose refractive index is to be measured. The gap distance is the key parameter which decides the detection ability of the sensor *i.e.* sensitivity.

Experimental trials are found to be useful in determining gap distance. But it is necessary to get an idea of actual value of this distance before fabrication of sensor. This is possible by defining the transfer function or mathematical model of the sensor. Geometrical consideration of model using analytical technique helps in determination of the distance. The ray tracing approach is also used for modeling and optimizing the gap distance value. The results are verified experimentally for sensor probe detecting adulteration of diesel by kerosene.

2. Mathematical Modeling and Simulation

Geometry and operating principal of refractometric fiber optic sensor are as shown in **Figure 1**.

2.1. Operating Principle

Figure 1 shows two parallel fibers having identical dimensions placed at a distance from the reflector *i.e.* mirror. A light is launched through transmitting fibers which expand in a conical shape and enters into the liquid. An incident ray of light entering into the liquid gets reflected back from the reflector and bends more or less depending upon the refractive index of the liquid. Reflected light is then collected by the receiving fiber for further processing.

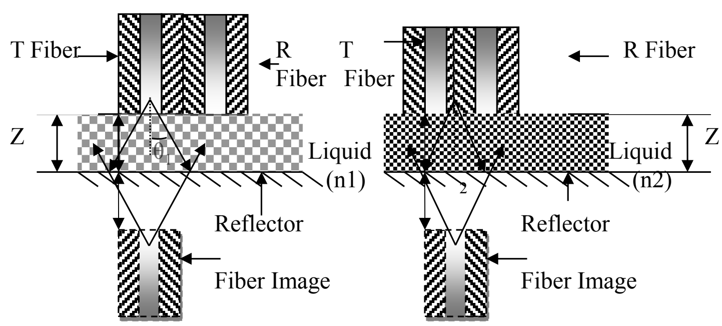

Figure 1. Basic principle of refractometric fiber optic sensor.

The cone of emission from this fiber depends on numerical aperture (NA) of the fiber given by Equation (1)

$$\theta_{NA} = \sin^{-1}\frac{NA}{n1} \tag{1}$$

where θ_{NA} is a cone of emission, NA is the numerical aperture, $n1$ is the refractive index of the liquid medium. For a fixed distance "z" between the fiber end face and the reflector, amount of light reflected from the reflector depends on the refractive index of the medium between the fiber sensor probe and the reflector. The reflected light is collected by the receiving fiber. The output of the receiving fiber varies in accordance with the refractive index $n1$ of the medium.

The region between the fiber end faces and the reflector is filled with liquid having refractive indices $n1$ and $n2$ where $n1 > n2$. Using Equation (1), $\theta_{NA(n1)} < \theta_{NA(n2)}$. With medium having refractive index $n1$, emitted light energy density is more than that for the medium having refractive index $n2$. Thus received light intensity after reflection is more with medium having refractive index $n1$ than for the medium having refractive index $n2$. Hence received light intensity is directly proportional to the change in refractive index of medium.

As explained earlier, gap distance is the most important parameter in refractometric type of fiber optic sensors. A mathematical model is to be developed considering two methods or approaches viz geometrical approach and ray tracing approach.

2.2. Mathematical Model Using Analytical Approach

The intensity of light emitted by the transmitting fiber is modeled as [2] [4] [5]

$$I(r,z) = \frac{2P_E}{\pi\omega^2(z)}\exp\left(-\frac{2r^2}{\omega^2(z)}\right) \tag{2}$$

where $I(r, z)$ is transmitted light intensity, P_E total power irradiated by the transmitting fiber, w_a radius of core of transmitting and receiving fibers, $w(z)$ is beam width and r is radius of cone of emission.

Put $r = 2w_a$ then

$$I(r,z) = \frac{2P_E}{\pi\omega^2(z)}\exp\left(-\frac{8\omega_a^2}{\omega^2(z)}\right) \tag{3}$$

Power collected by receiving fiber is given as

$$P(r,z) = \frac{2P_E\omega_a^2}{\omega^2(z)}\exp\left(-\frac{8\omega_a^2}{\omega^2(z)}\right) \tag{4}$$

From the geometry of the sensor, cone of emission θ_a is given by

$$\theta_a = \sin^{-1}\left(\frac{NA}{n1}\right) \tag{5}$$

where NA is numerical aperture of fiber and $n1$ is the refractive index of liquid filled in the gap.

$$\omega(z) = z\tan\theta_a = z\sin^{-1}\left(\frac{NA}{n1}\right) \tag{6}$$

Substituting this in Equation (4) we get,

$$P(z) = \frac{2P_E w_a^2}{z^2\left(\sin^{-1}\left(\frac{NA}{n1}\right)\right)^2}\exp\left(\frac{-8w_a^2}{z^2\left(\sin^{-1}\left(\frac{NA}{n1}\right)\right)^2}\right) \tag{7}$$

$P(z)$ is function of the refractive index $n1$ of the medium between the fiber end faces and reflector. The sensitivity of sensor is evaluated by taking the differential of $P(z)$ w.r.t. $n1$ at constant z.

Sensitivity Analysis

Sensitivity of sensor is calculated by taking derivative of the $P(z)$ w.r.t. $n1$ keeping Z constant.

$$S = \frac{\mathrm{d}P(n1)}{\mathrm{d}n1} = P(n1)\left(\frac{NA}{n1^2}\right)\left[\frac{-16w_a^2 + 2z^2\left(\sin^{-1}\left(\frac{NA}{n1}\right)\right)^2}{z^2\left(\sin^{-1}\left(\frac{NA}{n1}\right)\right)^3\sqrt{1-\left(\frac{NA}{n1}\right)^2}}\right] \tag{8}$$

For $S = 0$, the minimum value of Z can be calculated. This is the value of Z from which we get the change in the received light intensity as a function of refractive index $n1$.

$$Z_{limit} = \frac{\sqrt{8}w_a^2}{\sin^{-1}\left(\frac{NA}{n1}\right)} \tag{9}$$

2.3. Mathematical Modeling Using Ray Tracing Approach

The beam of light emitted by the source is constituted by number of rays which are incident at different angles on the reflector. One can say the light reflected from the reflector is coming from the image of the source and fiber. Rays emitted by this image are traced and its point of intersection with the receiving fiber plane computed. This is possible for any orientation of source and receiving fiber. This is called ray tracing technique. The technique is useful as mathematical expression modulated as per the sensor geometry. The expression specifically includes the co-ordinates of the transmitting and receiving fibers along with the point of intersections of individual rays on the reflector.

Steps for developing mathematical model for calculating intensity of received light using ray tracing method:

1) The co-ordinates of apparent point source of transmitting fiber $S'(X_s, Z_s)$ is given by equations:

$$X_S = \left(-\left(\frac{f_d}{2}\right)\right)\cos(\alpha 1) + \left(\frac{a}{\tan\left(\frac{\theta_{NA}}{2}\right)}\right)\sin(\alpha 1) - s \tag{10}$$

$$Z_S = \left(-\left(\frac{f_d}{2}\right)\right)\sin(\alpha 1) - \left(\frac{a}{\tan\left(\frac{\theta_{NA}}{2}\right)}\right)\cos(\alpha 1) - 2\left(Z + f_d \sin(\alpha 1)\right) \quad (11)$$

where "f_d" diameter of transmitting fiber as well as receiving fiber;

"s", lateral distance between transmitting and receiving fiber;

"NA", cone of emission of transmitting fiber;

"$\alpha 1$", angle of inclination of transmitting fiber;

"a", radius of core of transmitting fiber;

"Z", distance of mirror from end faces of transmitting and receiving fiber plane.

2) The co-ordinates of point of intersection of line drawn apparent source of image of transmitting fiber to the plane of receiving fiber A (X_i, Z_i) are given by the equations:

$$X_i = \left(-\frac{f_d}{2}\right)\frac{\cos(2\alpha 1)}{\cos\alpha 1 + \tan\alpha 2 \sin\alpha 1} + \frac{a}{\tan\left(\frac{\theta_{NA}}{2}\right)}\frac{\sin(2\alpha 1)}{\cos\alpha 1 + \tan\alpha 2 \sin\alpha 1} + \frac{2\left(Z + f_d \sin\alpha 1\right)\tan\alpha 1}{1 + \tan\alpha 1 \tan\alpha 2} - \frac{s\left(1 - \tan\alpha 1\right)}{1 + \tan\alpha 1 \tan\alpha 2} \quad (12)$$

$$Z_i = s\tan\alpha 2 - X_i \tan\alpha 2 \quad (13)$$

where "$\alpha 2$" angle of inclination of receiving fiber and "Z" is distance between reflector and sensor probe.

Measure of asymmetry β is given by the equation:

$$\beta = |\alpha 1| - |\alpha 2| \quad (14)$$

If $\alpha 1 = \alpha 2$ then $\beta = 0$ and $\alpha 1 \neq \alpha 2$ then β is having some positive/negative value.

3) Effective distance Z_{eff} between the image of transmitting fiber and receiving plane is given by

$$Z_{eff} = \sqrt{\left(X_S - X_i\right)^2 + \left(Z_S - Z_i\right)^2}\cos\beta \quad (15)$$

4) Let X_p and Y_p be the X and Y co-ordinates of rays reflected rays from point apparent source of the image of transmitting fiber. The co-ordinates of these rays are given by the expressions as

$$r_p = \frac{Z_{eff}}{-\sin\theta\cos\varphi\sin\beta + \cos\theta\cos\beta} \quad (16)$$

$$X_p = r_p\left(\sin\theta\cos\phi\cos\beta + \cos\theta\sin\beta\right) \quad (17)$$

$$Y_p = r_p \sin\theta\sin\phi \quad (18)$$

5) Finally received intensity is calculated by considering only those rays which enters the receiving fiber after reflection. The amount of light reflected from the reflector depends on the properties of reflector viz. reflectivity (R) and surface roughness (σ). The reflector with 100% reflectivity and negligible surface roughness are said to have specular reflection of incident light from the surface

and the reflector is called specular reflector. There are other reflectors such as metals or non metals having different reflectivities and surface surface roughness. The light reflected from such surface has multiple reflections due to roughness and hence known to have diffused reflection. The light intensity received by receiving fibers with diffused reflector is a function of reflectivity and surface roughness of reflector.

Sensitivity Analysis

For the refractometric fiber optic sensor, the received intensity is the function of the refractive index, if it is operated on the non linear region of retro-reflective fiber optic sensor as shown in the Figure 2. A remarkable change in the received intensity is observed if this sensor is operated in non linear region of the sensor curve for detection of refractive index of liquid. As the refractive index of the liquid increases the light energy density is concentrated within the small cone of light which increases the received light intensity. The rate of overlap of the reflected light cone with the receiving fiber depends on emitted cone of transmitting fiber which in turn depends on the refractive index of the medium. Larger is cone of emittance, light is spread over the larger area and hence rate of change in the received light intensity is more for smaller values of Z assuming that Z is greater than the peak position value on the sensor curve.

One can also observe that as the refractive index increases peak is shifted towards higher values of Z. This is obvious as effective NA is reduced which in turn makes the total overlap at higher values of Z. In refractometric fiber optic sensors, gap distance is fixed so in order to get better sensitivity, gap distance has to be optimized. The developed model is thus analyzed for sensitivity by taking differential of the Equation (8) and optimized distance Z_{opt} given by Equation (9).

Figure 3 shows the graphical representation of variation in sensitivity for different values of Z for n varying from 1.0 to 1.5. A non linear exponentially decreasing curve depicts that the exponential term in the Equation (8) is dominating for sensitivity.

Generally, in case of detection of refractive index for a particular application viz. measuring purity or concentration etc., the refractive index variation is very small. Considering the small span of variation, the variation in the sensitivity is very much linear. Figure 4 shows sensor response for $n = 1.44$ to 1.46 (pure diesel to pure kerosene) and shows linear increase in the sensor output for the range of refractive index.

From the values in the graph shown in Figure 4, one can consider this to be almost constant over the entire range.

In order to optimize the distance for particular set of values one has to calculate the sensitivity values at each distance and then maximum value of sensitivity is to be found. The value of Z at which maximum sensitivity is obtained should be considered as optimized distance Z_{opt}.

Figure 5 shows the sensitivity values for different Z (4 to 10 mm) values for $n = 1.44$ to $n = 1.46$. From the graph it is observed that $Z = 6.8$ mm is the gap distance at which maximum sensitivity is obtained. Thus for linear detection of variation in refractive index from 1.44 to 1.46 a gap distance of 6.8 mm is optimized.

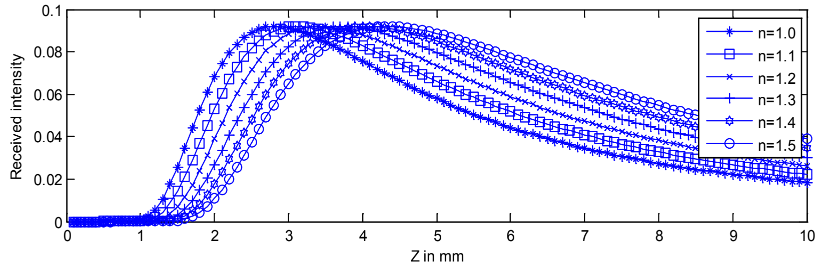

Figure 2. Sensor response.

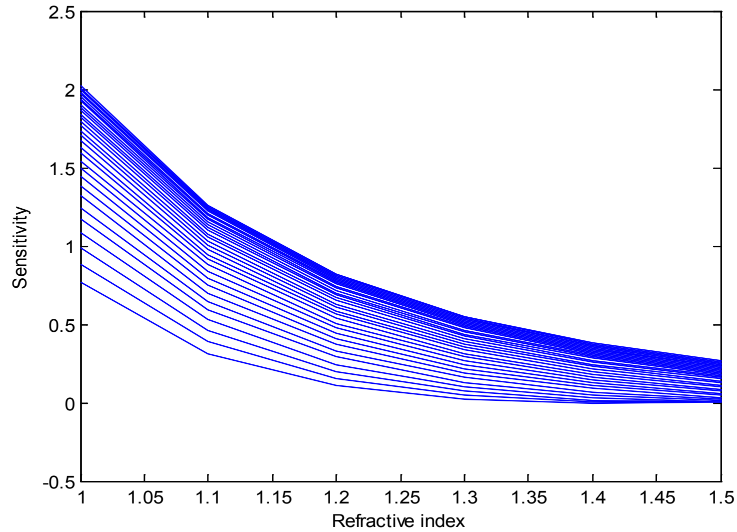

Figure 3. Sensitivity curves for Z = 4.0 mm to 10.0 mm.

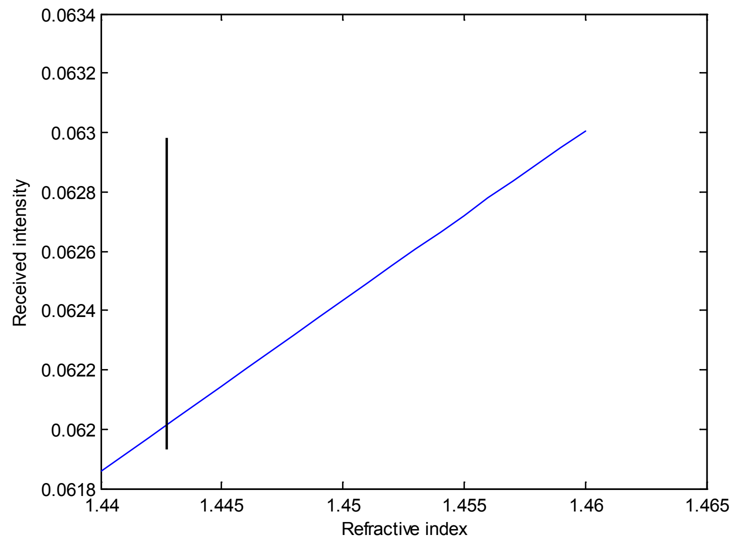

Figure 4. Sensor response for different refractive index.

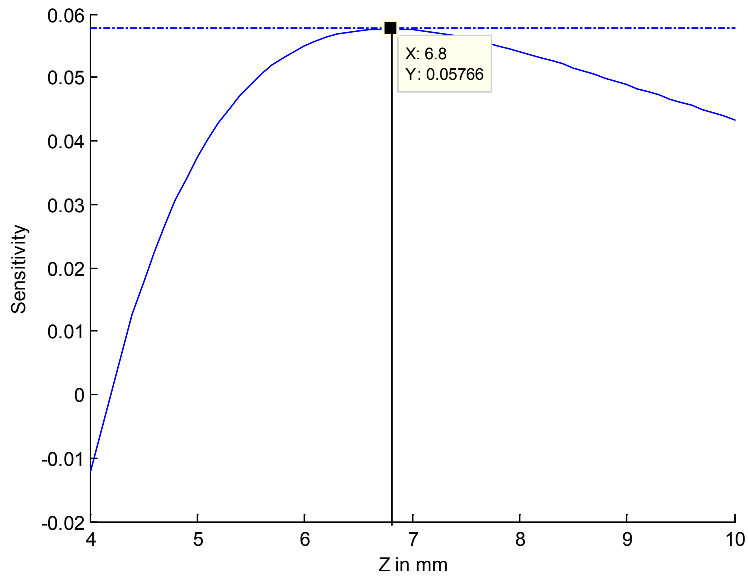

Figure 5. Getting optimized Z value for maximum sensitivity.

3. Simulation

Developed mathematical models are simulated for parallel fiber geometry having following parameters:

Type of fiber: Multimode and plastic

Core radii in mm: 0.488

Cladding thickness in mm: 0.612

Numerical Aperture: 0.47

For analysis, refractive index variation for pure diesel to pure kerosene is considered from 1.44 to 1.46.

Simulation experiments are repeated for mathematical model using geometrical approach and ray tracing approach and results are analyzed for value of optimized distance.

Optimization of gap distance using ray tracing method is as shown in **Figure 6**.

As observed from **Figure 5** and **Figure 6**, the value of optimized distance obtained using geometrical approach and ray tracing approach is found to be same *i.e.* exactly 6.8 mm.

4. Experimental Setup

Experiments are set to verify the simulated models for measuring the adulteration of diesel by kerosene. **Figure 7** shows block diagram of instrument along with sensor probe [6]-[12]. It consists of light source and its driving circuit, photo-detector and its driving circuit, sensor probe and chemical cell. The experiment was carried out for fixed optimized distance between probe and reflector as 6.8 mm. The sensor probe consists of two multimode plastic fibers each of diameter 488 micrometer. Photo detector is a phototransistor and its driving circuit which consists of buffer. Differential amplifier is used to amplify difference between detector output and the reference voltage. This reference voltage is meant for zero-adjust of instrument for pure water. Non-inverting amplifier is used to further amplify the difference with adjustable gain (span adjustment).

The experiment was carried out for 0% adulteration (pure diesel) up to 100% adulteration in pure diesel in interval 10% adulteration level. Sensor probe is dipped into sample of fuel under test. The amount of reflected light received by receiving fiber depends on the refractive index of the fuel and distance between sensor probe and reflector. Keeping the distance constant, we get output proportional to refractive index of fuel depending upon its adulteration level by kerosene. The experimental measurements were carried out with variation in adulteration level of diesel by kerosene. Different quantities of kerosene such as 0% (10 ml pure diesel), 10% (9 ml diesel + 1 ml kerosene), 20% (8 ml diesel + 2 ml kerosene) up to 100% (pure kerosene) are added in diesel to create different adulteration levels. The ZERO adjust potentiometer is adjusted for pure diesel thus making the output voltage zero, indicating 0% adulteration. The span adjustment is done for 100% adulteration *i.e.* only kerosene. Purpose of using micro-controller is to collect and store the data from different fuel pumping stations and compare them with the standard values. Analytical methods are very tedious and may take hours to conduct different tests in the laboratories for detecting adulteration levels and also these methods are not in-situ. Hence such adulteration detector is not only useful for keeping health of the society but also useful for technical persons which are interested in data analysis.

Repeatability of the sensor is the ability of the sensor to maintain the output for repetitive situations. Fiber optic sensor for adulteration level detection is subjected to repeatability test by recording the 60 readings in the experiments for a particular adulteration level. Figure 8 shows mean output voltage variation

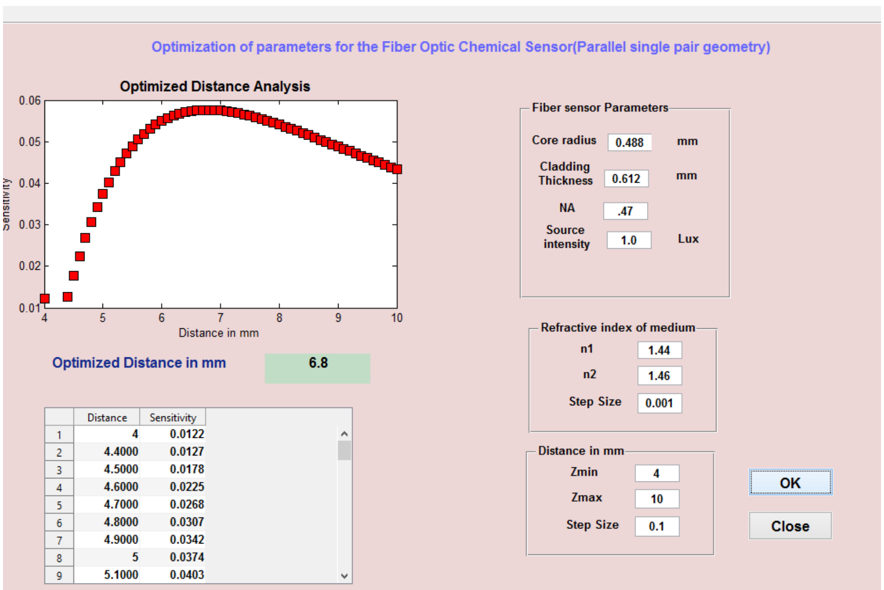

Figure 6. Optimization of gap distance using ray tracing model.

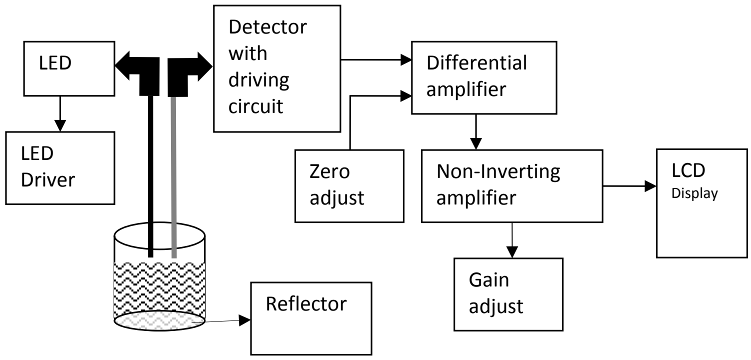

Figure 7. Block diagram for experimental setup.

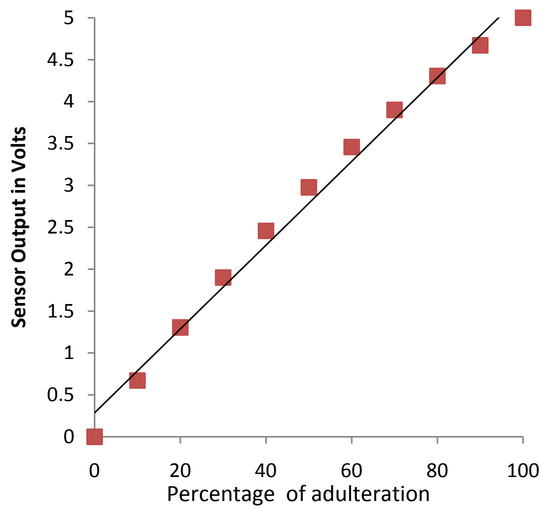

Figure 8. Experimental results.

with increasing adulteration of diesel by kerosene. Though the observations are performed for 0% - 100% range of kerosene the adulteration of diesel will have significance on lower (approximately upto 30%) kerosene concentration side. It is observed that the output voltage of the probe almost shows linear variation in this range.

5. Conclusion

Extrinsic refractometric fiber optic sensor is modeled mathematically using geometrical Gaussian approach and ray tracing approach. This model can be used to design sensor prototype for detection of change in refractive index. There are many applications where such changes in refractive index are observed. Developed mathematical model provides a tool to design the sensor prototype without any tedious experimental trials. From the mathematical and experimental analysis, it is observed that gap distance is a key parameter to design the sensor. Developed mathematical model is providing optimized value of gap distance between the sensor probe and reflector for detection of change in refractive index of liquid.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Laguesse, M. (1988) An Optical Fibre Refractometer for Liquids Using Two Measurement Channels to Reject Optical Attenuation. *Journal of Physics E*: *Scientific Instruments*, **21**, 64-67. https://doi.org/10.1088/0022-3735/21/1/010

[2] Chaudhari, A.L. and Shaligram, A.D. (2002) Multi-Wavelength Optical Fiber Liquid Refractometry Based on Intensity Modulation. *Sensors and Actuators A*: *Physical*, **100**, 160-164. https://doi.org/10.1016/S0924-4247(02)00040-7

[3] Patil, S.S. and Shaligram, A.D. (2011) Modeling and Experimental Studies on Retro-Reflective Fiber Optic Micro-Displacement with Variable Geometrical Properties. *Sensors and Actuators A*: *Physical*, **172**, 428-433. https://doi.org/10.1016/j.sna.2011.10.006

[4] Chaudhari, A.L. and Shaligram, A.D. (2013) Simulation and Experimental Studies of Extrinsic Fiber Optic Sensor for Liquid Refractometry. *Optik—International Journal for Light and Electron Optics*, **124**, 3134-3137. https://doi.org/10.1016/j.ijleo.2012.09.041

[5] Chaudhari, A.L. and Shaligram, A.D. (2012) Fiber Optic Sensor for the Measurement of Concentration and Refractive Index of Liquids Based on Intensity Modulation. *International Journal of Modern Physics*, **6**, 589-593. https://doi.org/10.1142/S2010194512003820

[6] Patil, S.S. and Shaligram, A.D. (2011) Refractometric Fiber Optic Adulteration Level Detector for Diesel. *International Journal of Advances in Engineering & Technology*, **195**, 195-203.

[7] Sengupta, B. (2005) Transport Fuel Quality. Delhi 32 PROBES/78/2000-2001. Central Pollution Control Board Publication.

[8] Yadav, Sh.R., Murthy, K.V., Mishra, D. and Baral, B. (2005) Estimation of Petrol and Diesel Adulteration with Kerosene and Assessment of Usefulness of Selected Automobile Fuel Quality Test Parameters. *International Journal of Environmental Science & Technology*, **1**, 253-255. https://doi.org/10.1007/BF03325839

[9] Wiedemann, L.S.M., d'Avila, L.A. and Azevedo, D.A. (March 2005) Adulteration Detection of Brazilian Gasoline Samples by Statistical Analysis. *Fuel*, **84**, 467-473. https://doi.org/10.1016/j.fuel.2004.09.013

[10] Roy, S. (May 1999) Fiber Optic Sensor for Determining Adulteration of Petrol and Diesel by Kerosene. *Sensors and Actuators B*: *Chemical*, **55**, 212-216. https://doi.org/10.1016/S0925-4005(99)00171-9

[11] Bali, L.M., Srivastava, A.K., Shukla, R.K. and Srivastava, A. (1999) Optical Sensor for Determining Adulteration in a Liquid Sample. *Optical Engineering*, **38**, 1715-1717. https://doi.org/10.1117/1.602223

[12] Govindan, G., Raj, S.G. AND Sastikumar, D. (2009) Measurement of Refractive Index of Liquids Using Fiber Optic Displacement Sensors. *Journal of American Science*, **5**, 13-17.