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Errors in Presenting and Writing Mathematics Symbols Trigger Learners' Misconception of Mathematics Concepts

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Abstract

The focus of this study was to understand how errors in presenting and writing mathematics language pave way to learners' misconception of mathematics concepts. This came to our attention during community service. We observed that the way two learners in grade seven copied a mathematics question the teacher presented on the blackboard as an example was different. The copies were not even close to what the teacher presented. This led us to arrange a follow up in the form of a continuous development activity with the teacher. To understand where the teacher stands and alleviate the challenge learners encounter in mathematics language, the teacher was observed teaching and engaged in an interview. To further support our observation in order to understand *how errors in presenting and writing mathematics symbols trigger learners' misconception in mathematics* we did an analysis of some of the learners' exercise books and of the textbook the teacher used in mathematics. Some of the revelation from the data generated from this qualitative study on why learners committed errors in presenting and writing mathematics symbols is as a result of being ill equipped in the mathematics language brought about during presenting and writing mathematics.

Keywords

Mathematics, Errors, Misconception

1. Introduction

To understand how errors in presenting and writing mathematics symbols trigger learners' misconception of mathematics concepts, we present first the problem which was observed and then the context of this study in the section which follows. In the section of the problem encountered, which was observed and decided that this study needed to be carried out we also state the research question. Thereafter to discuss is the context.

In the context, we bring Vygotsky's (1986) sociocultural theory where he points the importance of the two social levels. According to Vygotsky (1978), cited in Vygotsky (1986), the two social levels involved in knowledge construction are interpsychological and intrapsychological. These levels signal how language enables one to be accepted in a mathematics community of practice (COP) which requires one to be fluent in the language. Lave and Wenger (1991) understand a COP as a collection of who can be experts or practitioners who have the same concern for a particular cultural practice they do often and learn to handle it fairly as they get into social intercourse. In so doing, the participants develop professionally and personally. In our case, we focused on how errors in presenting and writing mathematics language constrain fluency in mathematic language acquisition in order to enter into a COP. For this reason, learners are not easily and quickly accepted into the mathematics community of practice. Also, since our view is in line with what other theorists view that mathematics is a language (Bharuthram & McKenna, 2012) and language is a component of culture, we bring the view of Tylor (1871). Tylor (1871) proposes that culture reflects episteme since he defines culture as an intricate whole made up of epistemes, views, craftsmanship, ethics, rules, tradition, and any other competences and practices assimilated by man as an adherent of a community of practice. Thereafter, we present the methodology which we used. Finally, the data is presented, analyzed to answer the research question and recommendations are done.

2. Problem Encountered

While engaging in community service, helping some few learners in our neighbouring community we observed that the learners had written in their mathematics exercise books using different ways to represent decimal numbers. Some had used a comma (,) as a decimal separator referred to as a decimal comma while others used a point as a decimal separator referred to as a decimal point. Also observed in learners' exercise books was the positioning of the decimal comma. It did not meet what is required when writing decimal numbers using a decimal comma. It is recommended that when the decimal comma is used to separate an integer and the decimal part, the fractional part, a space must not be left between the two parts. When a space is left between digits or numbers it signals that the numbers or digits are being listed. Another alternative to list numbers is through the use of a semicolon (;). This makes this study not to be similar to that of Makonye and Fakude (2016) who concentrated on errors in addition and subtraction and their symbols. This study however looked into misconceptions emerging as a result of misunderstanding symbols used in writing decimal numbers and listing numbers.

The reason which could have compounded this is the curriculum the teacher followed; such idea of use of mentioned symbols is absent. Also, in other curricula of institutes responsible for equipping teachers with content knowledge, pedagogical content knowledge and technical content knowledge Shulman (1987) proposes as important, the ideas to use those symbols is absent. Another reason which might have heightened the problem the learners were presenting is a lack of an international agreed standardization/standard of the use of separators. These intricate symbols as tools in the mathematics language, a social structure as Archer (1995) suggests if not properly used on account of not being standardized trigger learners' misconception to understand mathematics concepts. A lack of an internationally agreed standardization of these symbols as tools used in a COP if not mastered fully disable one to be a participant in a given COP, in this case a mathematics community of practice. Even in the antiquate, cultural practice indigenous community used were agreed upon in a given area. For instance, to standardize cultural practices, body parts were used as tools to measure length, cultural artefacts were used to measure time, the sun was used for this. Even before reaching an agreement to use a tool to measure length, there was still an element of standardization. For instance, the chief's arm was taken as the standard measure to measure a one-meter length. If found measuring one meter length while not comparing what you measure with a standard measure taken from the length of the arm of the chief it was taken as disobedience and the consequence was a punishment. As a result, we aim to recommend that there be an agreed standard way of presenting and writing decimal separator symbols as a way to address *errors encountered by learners in presenting and writing symbols to avoid learners' misconception in mathematics.*

3. Context

When writing units for physical quantities, for instance, mass, the kilogram (kg) is the symbol, distance, the meter (m) is the symbol and time, the second (s) is the symbol. These symbols for the units of these three fundamental physical quantities that inform all other units of other quantities were agreed upon by the International System of Units (ISI). This is not the same scenario observed with those symbols used as separators and others used for listing when ordering, writing decimal numbers or listing numbers in a sequence. Literature suggests that particular countries can use either a decimal coma or a decimal point in presenting decimal numbers. For instance, both Namibia uses and the United States of America use a comma (,) while Europeans use a full stop (.) to represent a decimal number. Such an approach has decelerated the movement of learners into those different membership levels of a community of practice and which was evident during our engagement in community service.

The idea to engage in community service, in this case charity work was to uplift learners and ensuring that teaching and learning of mathematics developed learners' skills, thus promoting deep learning (Marton & Säljö, 1976; Biggs & Tang, 2007). Learners' membership into mathematics community of practice (COP) can be as; peripheral, inbound, insider, boundary or outbound (Wenger, 1998). An attainment of peripheral status is for those not fully involved and is novice in the activities of the COP. This is where learners can be placed as they are still grappling to acquire mathematics language. Archer (1995) suggests that being a novice enables an individual to acquaint herself or himself with the intricate symbols and tools in the language, a social structure. When they finally acquaint themselves with the tools and symbols of the COP they can become legitimate members and have the identity of the community of practice. Learners are novices as they grapple to understand cultural practices which are also social structures other community members engage with. Cultural development at this interpsychological level starts with interacting with the teacher and pears as they socialize. However, on account of the mentioned symbols presented in different formats in the recommended textbooks of the curriculum, learners are left grappling to understanding them. As a result these are written inconsistently each time the learners use them. This violates the genetic law of cultural development which Vygotsky (1986) views as vital in the acquisition of a culture, language and in this case the mathematics language. This constricts them (learners) from entering the next membership level, the inbound since intrapsychological activities which still fall under the genetic law of cultural development and require higher mental functions are underdeveloped. In other words, the confusion promoted by different systems in different countries regarding learners' expression of mathematical language prevents learners from developing higher mental functions and/or processes in mathematics.

Inbound in a COP is to be a fully participating member. Inbound membership status signals movement towards the acquisition of symbols, tools and rules in the mathematics COP to attain the required identity and become fully-fledged members. A learner acquiring some skills regarding certain beliefs of a particular community serves as a typical example of one who is an inbound-being inducted into the goings-on of a community of practice. This gives him or her identity of the mathematics community of practice.

An insider member of a COP is an individual who has become a fully participating member, equipped with all tools of the language of mathematics. A typical example of an insider member is a teacher who brings inventive strategies; sees and avoids use of errors in curriculum materials in his teaching philosophy. From the view of Schönwetter, Sokal, Friesen and Taylor (2002), teaching philosophy is "a systematic and critical rationale that focuses on the important components defining effective teaching and learning in a particular discipline and/or institutional context" (p. 84). A teaching philosophy acts as a guide to lead a teacher's desire to touch the life of his/her learners forever (Truter, 2014). A good teaching philosophy allows the teacher not to import curriculum materials as Boughey (2001) and Schweisfurth (2011) suggest, but analyze concepts and situate them to/in the context to bring about language acquisition leading learners to enter the mathematics COP.

Individuals on the boundary are not fully participating members of mathematics community of practice. However, their partial participation brings a different set of skills or services to the COP. An individual doing community service can serve this role, for instance, after seeing poor presentation and errors in learners' exercise books, the individual on community service can give feedback to the teacher responsible for teaching the learners that subject. The aim of the boundary member of a COP is to ensure that cultural practices of these learners resonate with what is practiced in the mathematics COP. In his view, Sillitoe (2000) suggests that cultural practices as social structures are "flexible, adaptable and innovative" (p. 4).

In the analysis, we say an outbound individual in a community of practice is an individual preparing to leave the community. In our case we see that these are the learners who give up the learning of mathematics because they have failed to enter into any of those strands explained above. However, in their case they remain in the teacher's class but they are not participating since they lack the tools. Finally, a member of a COP who desires to see that proper symbols of the language are used is an insider member and in our case, we believe they are few on account of the lack of how decimal separators are used.

4. Methodology

The focus of this qualitative study was to understand how errors in presenting and writing mathematics symbols trigger learners' misconception of mathematics concepts. This interpretive study achieved responding to the research question of "*how errors in presenting and writing symbols trigger learners' misconception in mathematics*" through engaging with learners and a teacher at a school in a given circuit.

Even though there are a number of schools in the circuit and region with learners doing grade seven mathematics, only one school was selected. The reason to do so was that all learners use one mathematics textbook prescribed by the curriculum and presents mathematics in the same way. That is if a teacher does not wear an analytical eye in order to be a reflective practitioner, who uses reflexivity and praxis skills, these textbook errors might be repeated while teaching in all the other schools.

The same reason was used to select participants who were learners and the mathematics teacher from that given population at the school. This also allowed narrowing our terrain to operate. As a result, in sampling these participants we only selected one class where one teacher teaches mathematics to Grade seven learners. Even though the teacher had other Grade seven mathematics classes he teaches, one class was selected since we saw that it allowed using extrapolation to see what also happens in other classes and in other schools. Even though in our community service program, an activity where one does charity work to uplift communities of learners without remuneration, we have more than two learners we assist, In this study only two learners were selected to be part of the sample.

The instruments used to generate data were observation of the teacher and interview. This was followed by analysis of documents. The documents were those of the learners and the prescribed textbook the teacher and learners use during teaching and learning respectively. For the purpose of triangulation, we also interviewed the teacher. The three instruments used allowed us to generate data which when analyzed helped to answer the research question.

During data analysis, data that emerged congruent views were used to come up with themes. The data generated was presented in **Table 1** and **Figures 1-10**. Commenting of the themes also provided answers for the research question of this study.

To ensure that research ethics were adhered to, we sought permission from the participants. Since learners whose exercise books were analyzed are not adults we went to seek permission from their guardians. We also assured these learners and their guardians that the data generated will not reveal their identity. That is, pseudonyms would be used to refer to what they would have contributed. The section which follows present the data generated.

4.1. Data Presentation

In the following section we first present data from the two learners. This is followed by presenting data from the teacher. Finally, data obtained from analysis of textbook used in the curriculum is presented.

4.2. Data from the Two Learners

The diagrams above, **Figure 1** and **Figure 2** show how the learners presented their work. Due to scarcity of Grade seven mathematics textbooks at the school, learners had to copy the task from the chalkboard. Their work is illustrated in **Figure 1** and **Figure 2** above.

The pictures presented show some of the misconception which came out.

Table 1. Data generated from interviewing the teacher.

Question	Response	Theme
Is there a format used to list numbers?	No standard format	Absence of standardization
What should be used when writing an integer and the decimal part to separate them?	It depends on the teacher	Absence of standardization
Is there a standard format to use when writing decimal numbers?	No standard format	Absence of standardization
Is there a standard format to list numbers?	No standard format	Absence of standardization

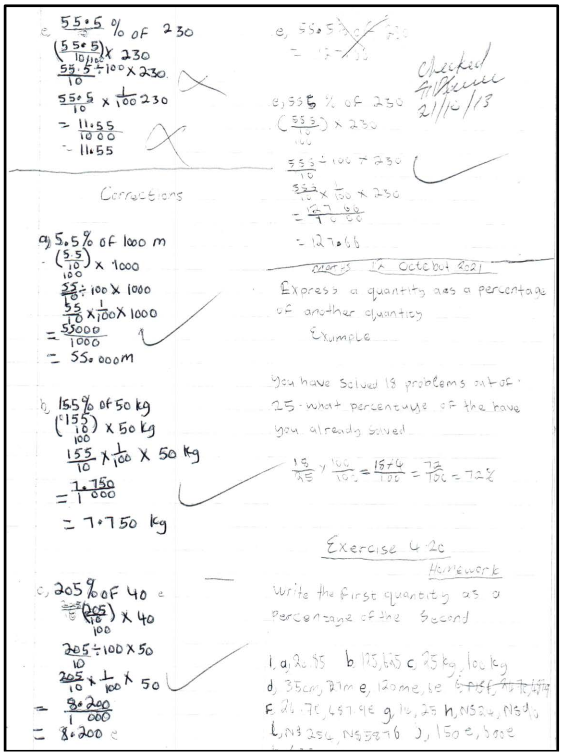


Figure 1. Work of learner one.

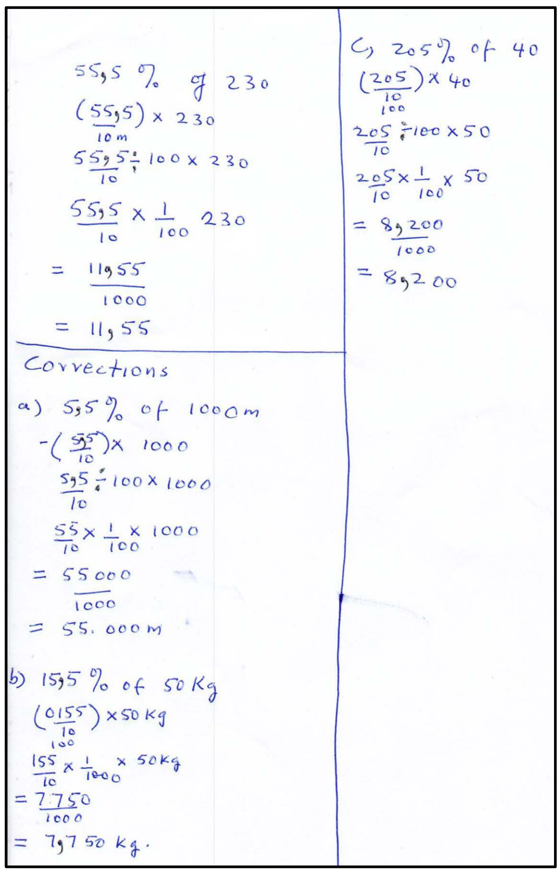


Figure 2. Work of learner two.

These misconceptions brought about by poor knowledge of mathematics knowledge will be discussed in detail under data discussion. In the next presentation is data from the teacher.

4.3. Data from the Teacher

In this section data presented is to do with how the teacher presented work on the chalkboard. Also presented is data which was generated during interviews.

The diagram of the chalkboard shows how the teacher listed the numbers. This generated data acted as baseline data and triggered the need to know how the teacher thinks when listing numbers and led to bring an interview as a tool to probe the teacher further.

The data obtained from the two instruments will be discussed further in the discussion of data when it is being analyzed in the next section. The next and last data to present is that from the analysis of the textbook.

4.4. Data Generated from Analyzing the Textbook

In this section data related to how the author uses commas or semi colons in the presentation of numbers is presented. Also presented are sections showing how numbers are listed. A number of sections were taken from the pages of the book to show that the errors in listing numbers is not a mistake but a case influenced by lack of having a standard way of listing numbers using a comma or a semicolon.

4.5. Data Analysis and Themes Drawn from the Data Generated

Data generated was analyzed as follows. First the data from learners; second, data generated from teacher and finally, the data generated from textbook analysis. In each case themes were drawn from the data and we looked into how generated themes possibly answered the research question of *how errors in presenting and writing symbols trigger learners' misconception in mathematics*.

4.6. Data from the Learners

The work of the two learners, show that they used different ways to write decimal numbers. One learner used a comma (,) as a decimal separator, also known as a decimal comma. The other learner went to use a point (.), also known as a decimal point and as a decimal separator. Failure of these learners to communicate using similar language, the mathematics symbols which are a component of the social structure (Archer, 1995; Wenger, 1998) suggests a lack of identity to fully and quickly enter into a mathematics community of practice. On account of failure to understand how these symbols are used, learners are partially in the inbound membership status since they have not yet acquired the symbols, tools and rules (Lave & Wenger, 1991). We see this as a lack of a standard format which mathematics language comes with and as a result it invokes misconceptions.

Namibia, unlike the United States of America which uses a decimal point as a decimal separator is expected to use a decimal comma as a decimal separator. As we see in **Figure 1** and **Figure 2** where one learner used a comma and the other a decimal point as decimal separators. Such a mismatch is brought by errors in the representation of symbols and this brings misconceptions of mathematics concepts. For instance, in future when a comma is used as a symbol for multiplication, such learners will fail to accommodate this easily thus entering into the mathematics community of practice at a slow pace. Lack of uniform representation suggests that learners lack the culture that reflects mathematics language episteme (Tylor, 1871). From the learners we were unable to see how they would list numbers since the learners had not yet done an activity in this area even though the teacher had taught it in a class we had observed as discussed below.

4.7. Analysis of Data Generated with Teacher

It is in this data generated from the teacher that we see whether or not symbols are used to separate numbers which are listed. From **Figure 3**, it is evident that the teacher does not, possibly because the teacher does not have the pedagogical content knowledge (Shulman, 1987). This is further supported in the data generated from the interview where the theme that emerged is that there is no standard format to list numbers. We see this as an error which will lead learners to have misconceptions in understanding mathematics concepts. As a result of this retards learners' entrance into those stages of mathematics COP namely; peripheral, inbound, insider, boundary or outbound (Wenger, 1998). We see this as attributed to a lack of developing learners' skills to do deep learning (Marton & Säljö, 1976; Biggs & Tang, 2007).

The data generated from interviewing the teacher also support why the learners do not use a uniform format to write numbers which uses decimal separators, see **Table 1**. This is revealed from the theme which emerged when the teacher responded that *that depends on the teacher's choice when responding to the question*, *what should be used when writing an integer and the decimal part to separate them*? In the other question in the interview it is also revealed that there is no standard format to use when writing decimal numbers.

Such an approach constrains learners' mathematics language development as

Figure 3. Data showing how the teacher presented work.

it violates the genetic law of cultural development (Vygotsky, 1986). Failure to be consistent in the use of decimal separators or symbols used to list numbers results in constraining the development of the interpsychological level that starts with interacting with the teacher and peers as they socialize. If the interpsychological level where language is constructed is not developed properly it will imply that the intrapsychological level where a learner engages in cognitive activities to accommodate or assimilate mathematics concepts becomes impaired as this depends on what exists in the schema of the learner (Piaget, 1953).

A study by Makonye and Fakude (2016) suggests that failure to adhere to a consistent format as a result of a lack of a standardized format to list numbers or to write decimal numbers using decimal separators brings about logical, procedural and strategical errors. We see this as hampering "conceptual progression and coherence" character (Kriek & Basson, 2008: p. 63). That is to say, concepts are introduced at a grade level and then are revisited in the next grade. However, failure to achieve this implies that when learners proceed to register in higher grades, they will keep on grappling to understand mathematics concepts. This is also attributed to that these errors hamper deep learning (Marton & Säljö, 1976; Biggs & Tang, 2007) which is pivotal in mathematics concept understanding. As a result, in the next grade level, progress cannot take place successfully. Our understanding is, each time the concepts are revisited they need to serve as prior knowledge for the other concepts to be introduced by the teacher. It is most probably that these challenges are attributed to how the textbook recommended in the Grade seven curriculum presents these ideas. These will be seen as we do an analysis of the textbook below.

4.8. Analysis of Data Generated from the Textbook

The theme extracted from analyzing sections in the pages of the textbook used in grade seven curriculum shows that there is no consistency with the way decimal numbers are presented. This is also the same scenario observed when numbers are being listed for any particular purpose. There is no particular standard which is followed. **Figures 4-10** reveal this. In **Figure 4** the author uses a point also known as a decimal point to present a number. In **Figure 5**, the author uses a decimal comma in one occasion and in another occasion, a semicolon. That is, a mixture of symbols is used on one page when listing numbers. In **Figures 6-10** the author uses a decimal comma to list numbers. However, in **Figures 7-9** the learner uses a semicolon to list numbers. Can learners be expected to come up with a pattern which is a nature of mathematics?

We see this as a practice which should not occur since this has a high chance of bringing errors as learners fail to understand these symbols used in the mathematics language a social structure (Archer, 1995). As a result this constrains the mathematics language acquisition as it retards learners to be enrolled in the membership levels of mathematics COP namely; peripheral, inbound, insider, boundary or outbound (Wenger, 1998).

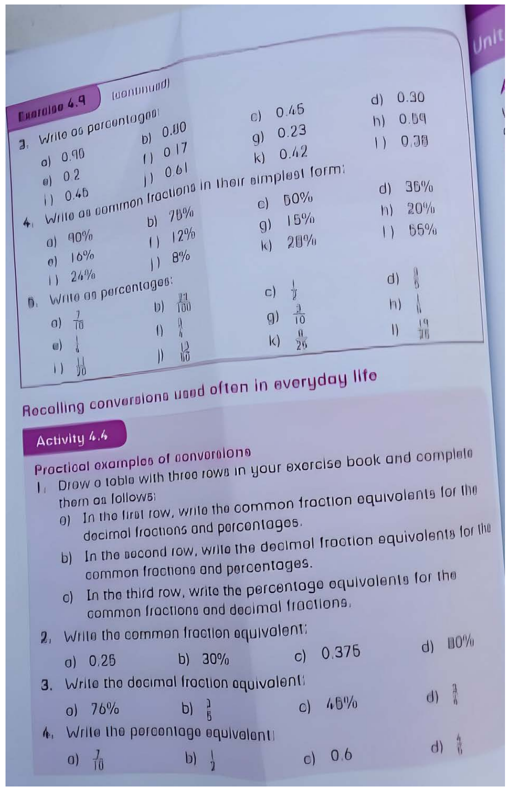


Figure 4. Representation of decimal numbers in the textbook.

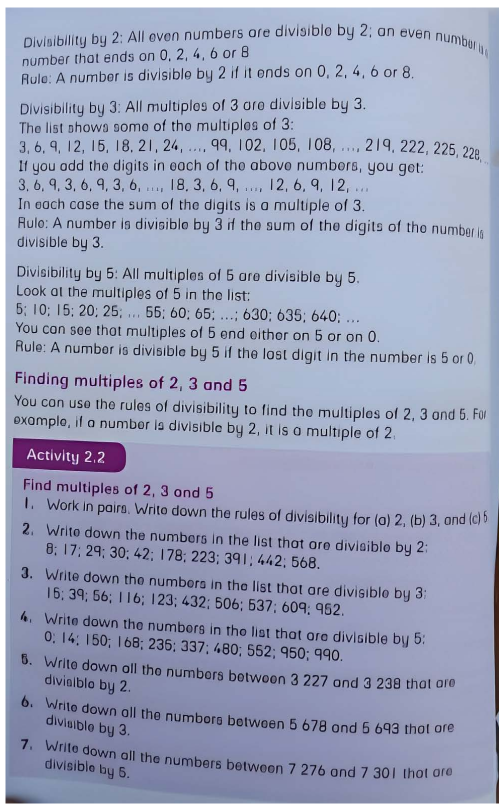
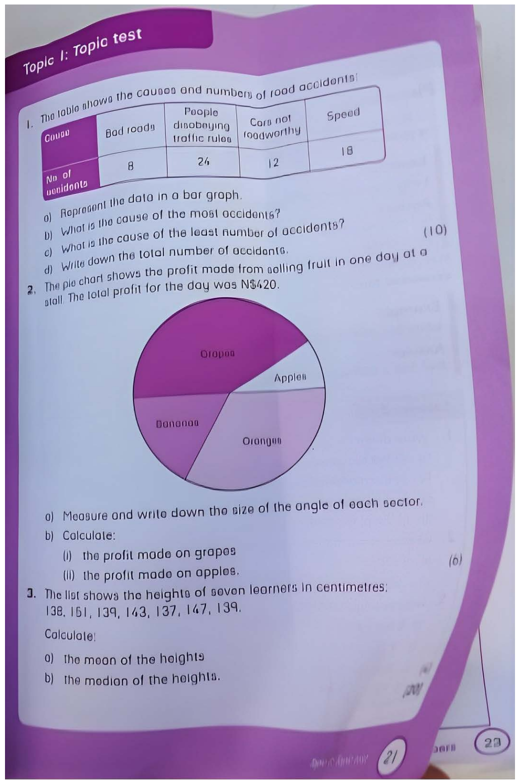


Figure 5. Listing of numbers in a textbook.



Topic 1: Topic test

1. The table shows the causes and numbers of road accidents:

Cause	Bad roads	People disobeying traffic rules	Cars not roadworthy	Speed
No. of accidents	8	24	12	18

a) Represent the data in a bar graph.
b) What is the cause of the most accidents?
c) What is the cause of the least number of accidents?
d) Write down the total number of accidents. (10)

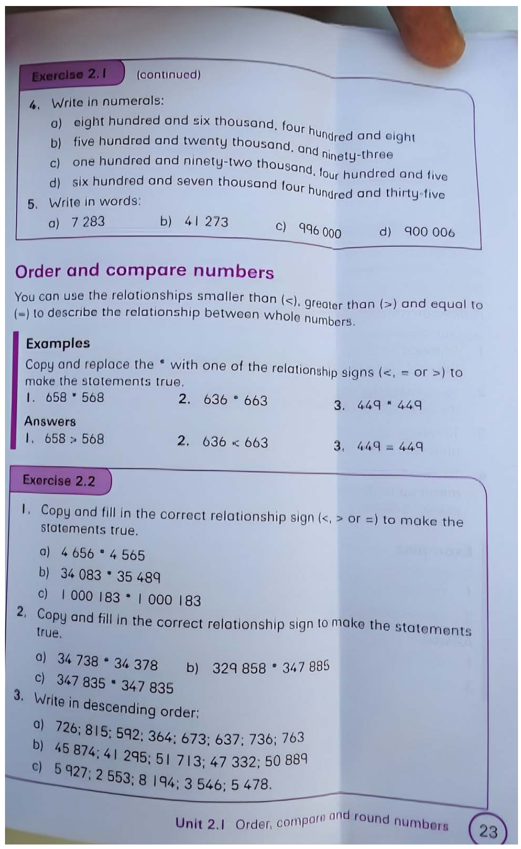
2. The pie chart shows the profit made from selling fruit in one day at a stall. The total profit for the day was N$420.

a) Measure and write down the size of the angle of each sector.
b) Calculate:
(i) the profit made on grapes
(ii) the profit made on apples. (6)

3. The list shows the heights of seven learners in centimetres:
138, 151, 139, 143, 137, 147, 139.
Calculate:
a) the mean of the heights
b) the median of the heights.
(20)

21

Figure 6. Listing of numbers in a textbook.



Exercise 2.1 (continued)

4. Write in numerals:
a) eight hundred and six thousand, four hundred and eight
b) five hundred and twenty thousand, and ninety-three
c) one hundred and ninety-two thousand, four hundred and five
d) six hundred and seven thousand four hundred and thirty-five
5. Write in words:
a) 7 283 b) 41 273 c) 996 000 d) 900 006

Order and compare numbers

You can use the relationships smaller than (<), greater than (>) and equal to (=) to describe the relationship between whole numbers.

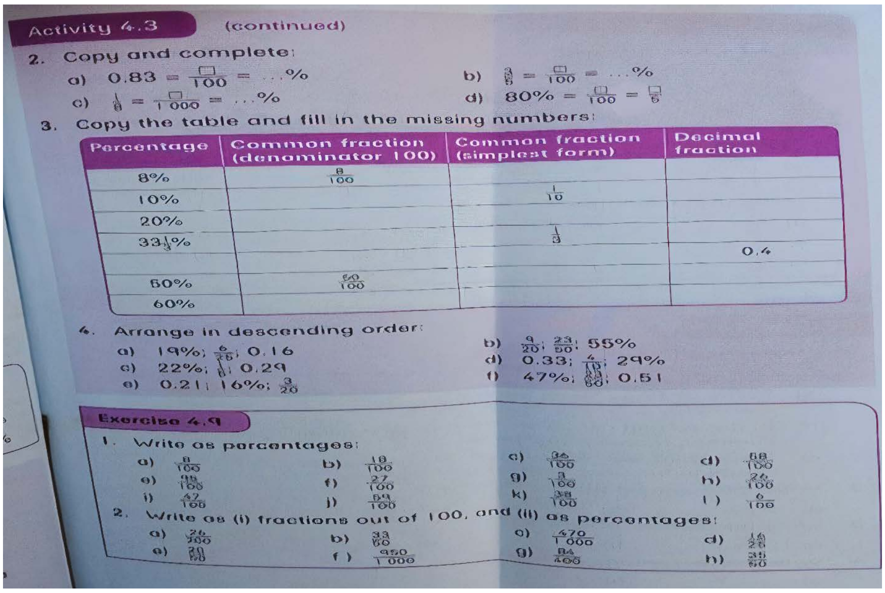
Examples
Copy and replace the • with one of the relationship signs (<, = or >) to make the statements true.
1. 658 • 568 2. 636 • 663 3. 449 • 449
Answers
1. 658 > 568 2. 636 < 663 3. 449 = 449

Exercise 2.2

1. Copy and fill in the correct relationship sign (<, > or =) to make the statements true.
a) 4 656 • 4 565
b) 34 083 • 35 489
c) 1 000 183 • 1 000 183
2. Copy and fill in the correct relationship sign to make the statements true.
a) 34 738 • 34 378 b) 329 858 • 347 885
c) 347 835 • 347 835
3. Write in descending order:
a) 726; 815; 592; 364; 673; 637; 736; 763
b) 45 874; 41 295; 51 713; 47 332; 50 889
c) 5 927; 2 553; 8 194; 3 546; 5 478.

Unit 2.1 Order, compare and round numbers 23

Figure 7. Listing of numbers to be ordered.



Activity 4.3 (continued)

2. Copy and complete:
   a) $0.83 = \frac{\square}{100} = \ldots\%$
   b) $\frac{3}{5} = \frac{\square}{100} = \ldots\%$
   c) $\frac{1}{8} = \frac{\square}{1\,000} = \ldots\%$
   d) $80\% = \frac{\square}{100} = \frac{\square}{5}$
3. Copy the table and fill in the missing numbers:

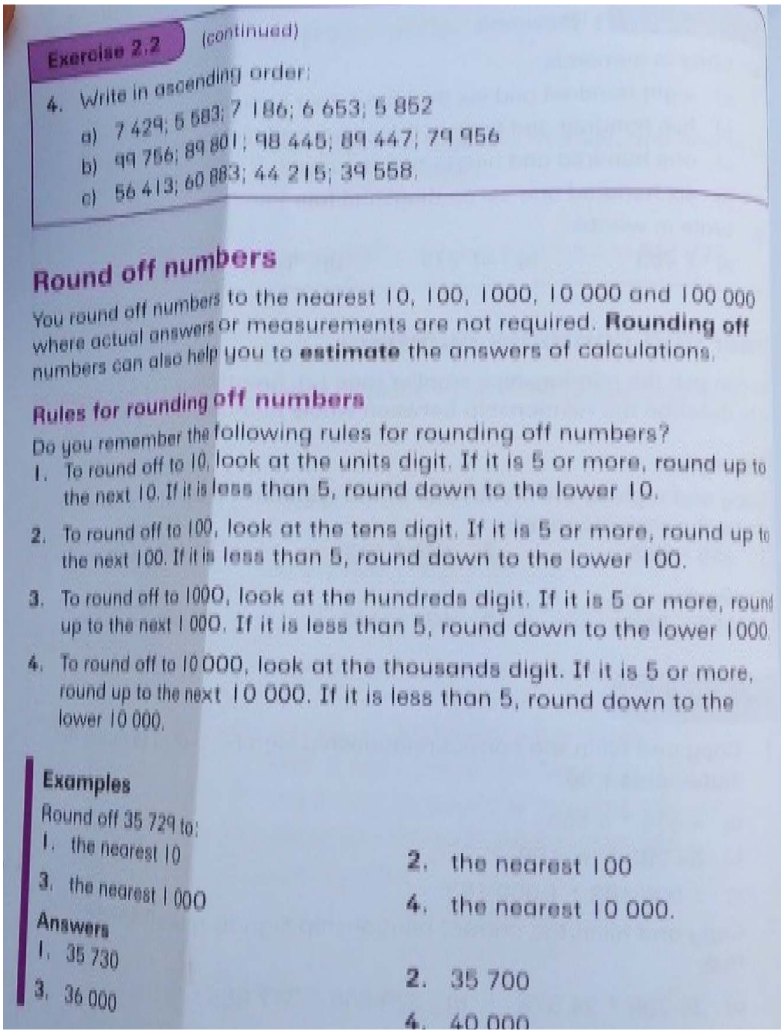
Percentage	Common fraction (denominator 100)	Common fraction (simplest form)	Decimal fraction
8%	$\frac{8}{100}$		
10%		$\frac{1}{10}$	
20%			
$33\frac{1}{3}\%$		$\frac{1}{3}$	
			0.4
50%	$\frac{50}{100}$		
60%			

4. Arrange in descending order:
   a) 19%; $\frac{6}{25}$; 0.16
   b) $\frac{9}{20}$; $\frac{23}{50}$; 55%
   c) 22%; $\frac{1}{4}$; 0.29
   d) 0.33; $\frac{4}{10}$; 29%
   e) 0.21; 16%; $\frac{3}{20}$
   f) 47%; $\frac{33}{60}$; 0.51

Exercise 4.9

1. Write as percentages:
   a) $\frac{8}{100}$ b) $\frac{18}{100}$ c) $\frac{36}{100}$ d) $\frac{58}{100}$
   e) $\frac{99}{100}$ f) $\frac{27}{100}$ g) $\frac{3}{100}$ h) $\frac{26}{100}$
   i) $\frac{47}{100}$ j) $\frac{59}{100}$ k) $\frac{38}{100}$ l) $\frac{6}{100}$
2. Write as (i) fractions out of 100, and (ii) as percentages:
   a) $\frac{26}{200}$ b) $\frac{33}{60}$ c) $\frac{470}{1\,000}$ d) $\frac{16}{25}$
   e) $\frac{20}{50}$ f) $\frac{950}{1\,000}$ g) $\frac{84}{400}$ h) $\frac{35}{50}$

Figure 8. Listing of numbers to be ordered.



Exercise 2.2 (continued)

4. Write in ascending order:
   a) 7 429; 5 583; 7 186; 6 653; 5 852
   b) 99 756; 89 801; 98 445; 89 447; 79 956
   c) 56 413; 60 883; 44 215; 39 558.

Round off numbers

You round off numbers to the nearest 10, 100, 1000, 10 000 and 100 000 where actual answers or measurements are not required. **Rounding off** numbers can also help you to **estimate** the answers of calculations.

Rules for rounding off numbers

Do you remember the following rules for rounding off numbers?

1. To round off to 10, look at the units digit. If it is 5 or more, round up to the next 10. If it is less than 5, round down to the lower 10.
2. To round off to 100, look at the tens digit. If it is 5 or more, round up to the next 100. If it is less than 5, round down to the lower 100.
3. To round off to 1000, look at the hundreds digit. If it is 5 or more, round up to the next 1 000. If it is less than 5, round down to the lower 1000.
4. To round off to 10 000, look at the thousands digit. If it is 5 or more, round up to the next 10 000. If it is less than 5, round down to the lower 10 000.

Examples

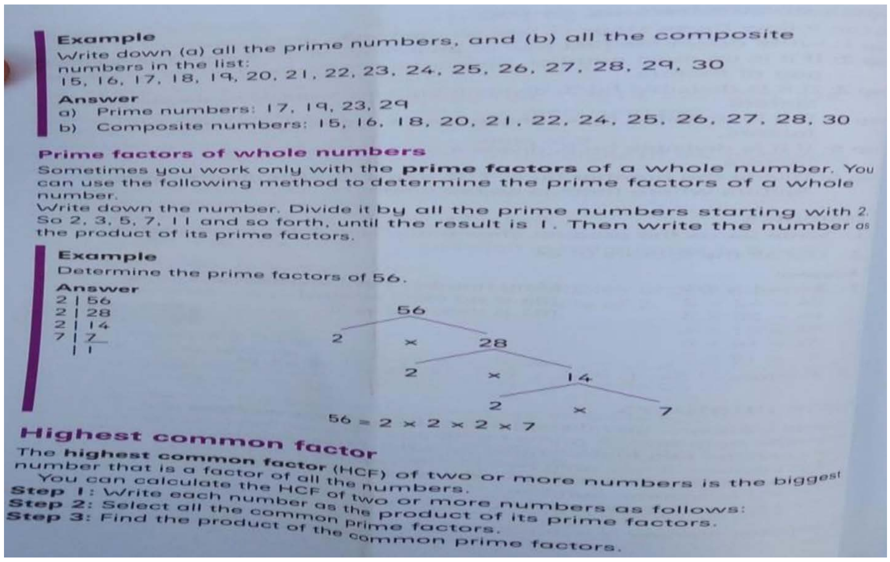
Round off 35 729 to:

1. the nearest 10
2. the nearest 100
3. the nearest 1 000
4. the nearest 10 000.

Answers

1. 35 730
2. 35 700
3. 36 000
4. 40 000

Figure 9. Listing of numbers to be ordered.



Example
Write down (a) all the prime numbers, and (b) all the composite numbers in the list:
15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30

Answer
a) Prime numbers: 17, 19, 23, 29
b) Composite numbers: 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30

Prime factors of whole numbers
Sometimes you work only with the **prime factors** of a whole number. You can use the following method to determine the prime factors of a whole number.
Write down the number. Divide it by all the prime numbers starting with 2. So 2, 3, 5, 7, 11 and so forth, until the result is 1. Then write the number as the product of its prime factors.

Example
Determine the prime factors of 56.

Answer
2 | 56
2 | 28
2 | 14
7 | 7
| 1

56 = 2 × 2 × 2 × 7

Highest common factor
The **highest common factor** (HCF) of two or more numbers is the biggest number that is a factor of all the numbers.
You can calculate the HCF of two or more numbers as follows:
Step 1: Write each number as the product of its prime factors.
Step 2: Select all the common prime factors.
Step 3: Find the product of the common prime factors.

Figure 10. Listing of numbers to be selected.

Mathematics as a language as Bharuthram and McKenna (2012) opine, is like any other language and needs to have consistency in how the symbols are used. In other languages if a symbol as a component of a language has a dual or many uses, the context will be different. For instance, in Physics, "v" can be used as a symbol for representing velocity or speed. These two physical quantities convey different contexts to the language speaker and listener. Velocity has direction and magnitude whereas speed has only magnitude. On the other hand, η a symbol for efficiency of a machine can be in another context be a symbol for viscosity of a liquid.

The teacher is the interpreter of the curriculum material but surprisingly he only imports the concepts which are presented in the textbook (Boughey, 2001; Schweisfurth, 2011). This makes teachers not insider members of a mathematics COP. If skills of innovativeness are in the teacher such misrepresentation can be detected and rectified. However, this can happen with one who engages in reflexive practice in his/her teaching philosophy to make some adjustment on the curriculum and this emerges quality through self-evaluation (Boughey, 2001). Sillitoe (2000) supports the idea of making adjustment in the curriculum as a way of perfecting a teacher's cultural practices. In doing so it alleviates the problem of misrepresentation of the said symbols. The life of learners cannot be changed forever (Truter, 2014) if the teacher does not allow them to acquire the proper mathematics language through presenting the symbols in it properly.

The data generated from different tools have some similarities which contribute to answer the research question *how an error in presenting and writing symbols trigger learners' misconception in mathematics*? A look at data generated from learners' work show that it is the lack of standardization which contributes to learners to present decimal numbers using a decimal point and a decimal comma that is not to consistent. This scenario is also observed in the data generated from the teacher. This data emerges the same theme that lack of an agreed upon format of presenting decimal numbers and listing numbers brings about misconception. Finally, the data generated from the textbook also brings the idea of absence of a standard format to write decimal numbers and to list numbers. This is evident from the fact that the author was not selective in using a decimal comma and a decimal point to write decimal numbers. This also applies to when the author was listing numbers for any particular purpose that could be ordering, selection of prime numbers or any activity to be done to help the learners to develop mathematics language to enter those membership levels of mathematics community of practice. In this regard we found it necessary to recommend the following.

5. Recommendations

The rationale of verbal communication when done verbal (or through use of texts) is to signal one to take action of what the message says. For this communication to be successful the communicator and the receiver needs to be aware of what the verbal or text signal says. However, when they do not knock sense in someone, that is to influence one, misconceptions may occur. This is also what is happening when symbols in mathematics are not used consistently. To avoid this, some form of standardization of these mathematical symbols should be done. Just like those units for particular physical quantities as mentioned before, for instance, mass the kilogram (kg) is the symbol, distance the meter (m) is the symbol and time the second (s) is the symbol, they were agreed upon by the International System of Units (SI). This is a stance which also needs to be taken regarding mathematical symbols. This ensures that misconceptions in the acquisition of mathematics concepts do not occur in learners and teachers.

When such an agreement is reached, then some professional development programs need to be carried out. These will allow teachers to become aware of how decimal numbers can be written and listed. Teacher programs at institutes where they are equipped must have in the curriculum some sections which discuss the concepts of decimal number writing and how numbers can be listed. This recommendation is based on the fact that these ideas are absent in many curricula in institutes where teachers are developed. Currently, the expectation is trainee teachers become aware by observing patterns in the mathematics literature. However, this is not shown in mathematics textbooks, hence the absence of consistency in writing and listing decimal numbers.

6. Conclusion

In conclusion, errors emerging on account of mathematics language not conforming to standardization can range from procedural, strategical and logical. All these types of errors result in learners developing misconceptions thus exacerbating learners' failure to realize deep learning.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

Archer, M. S. (1995). *Realist Social Theory: The Morphogenetic Approach.* Cambridge University Press. https://doi.org/10.1017/CBO9780511557675

Bharuthram, S., & McKenna. S. (2012) Students' Navigation of the Uncharted Territories of Academic Writing. *Africa Education Review, 9,* 581-594. https://doi.org/10.1080/18146627.2012.742651

Biggs, J. B., & Tang, C. (2007). *Teaching for Quality Learning at University* (3rd ed.). Open University Press.

Boughey, C. (2001). Evaluation as a Means of Assuring Quality in Teaching and Learning: Policing or Development? In *Proceedings of a Regional Conference for Staff from Tertiary Institutions from SADC Countries.* Rhodes University.

Kriek, J., & Basson, I. (2008). Implementation of the New FET Physical Sciences Curriculum: Teachers' Perspectives. *African Journal of Research in Mathematics, Science and Technology Education, 12,* 63-76. https://doi.org/10.1080/10288457.2008.10740642

Lave, J., & Wenger, E. (1991). *Legitimate Peripheral Participation.* Cambridge University Press. https://doi.org/10.1017/CBO9780511815355

Makonye, J. P., & Fakude, J. (2016). A Study of Errors and Misconceptions in the Learning of Addition and Subtraction of Directed Numbers in Grade 8. *SAGE Open, 6.* https://doi.org/10.1177/2158244016671375

Marton, F., & Säljö, R. (1976). On Qualitative Differences in Learning: I—Outcome and Process. *British Journal of Educational Psychology, 46,* 4-11. https://doi.org/10.1111/j.2044-8279.1976.tb02980.x

Piaget, J. (1953). *The Origins of Intelligence in Children.* Basic Books.

Schönwetter, D. J., Sokal, L., Friesen, M., & Taylor, K. L. (2002). Teaching Philosophies Reconsidered: A Conceptual Model for the Development and Evaluation of Teaching Philosophy Statements. *The International Journal for Academic Development, 7,* 83-97. https://doi.org/10.1080/13601440210156501

Schweisfurth, M. (2011). Learner-Centred Education in Developing Country Contexts: From Solution to Problem? *International Journal of Educational Development, 31,* 425-432. https://doi.org/10.1016/j.ijedudev.2011.03.005

Shulman, L. S. (1987). Knowledge and Teaching: Foundations of the New Reform. *Harvard Educational Review, 57,* 1-23. https://doi.org/10.17763/haer.57.1.j463w79r56455411

Sillitoe, P. (2000). Let Them Eat Cake: Indigenous Knowledge, Science, and the 'Poorest of the Poor'. *Anthropology Today, 16,* 3-7. https://doi.org/10.1111/1467-8322.00031

Truter, I. (2014). To Teach Is to Touch a Life Forever… *SA Pharmaceutical Journal, 81,* No. 3. https://journals.co.za/doi/abs/10.10520/EJC152874

Tylor, E. B. (1871). *Primitive Culture: Researches into the Development of Mythology, Philosophy, Religion, Art and Custom.* John Murray

Vygotsky, L. (1986). *Thought and Language.* The MIT Press.

Wenger, E. (1998). *Communities of Practice: Learning, Meaning, and Identity.* Cambridge University Press. https://doi.org/10.1017/CBO9780511803932